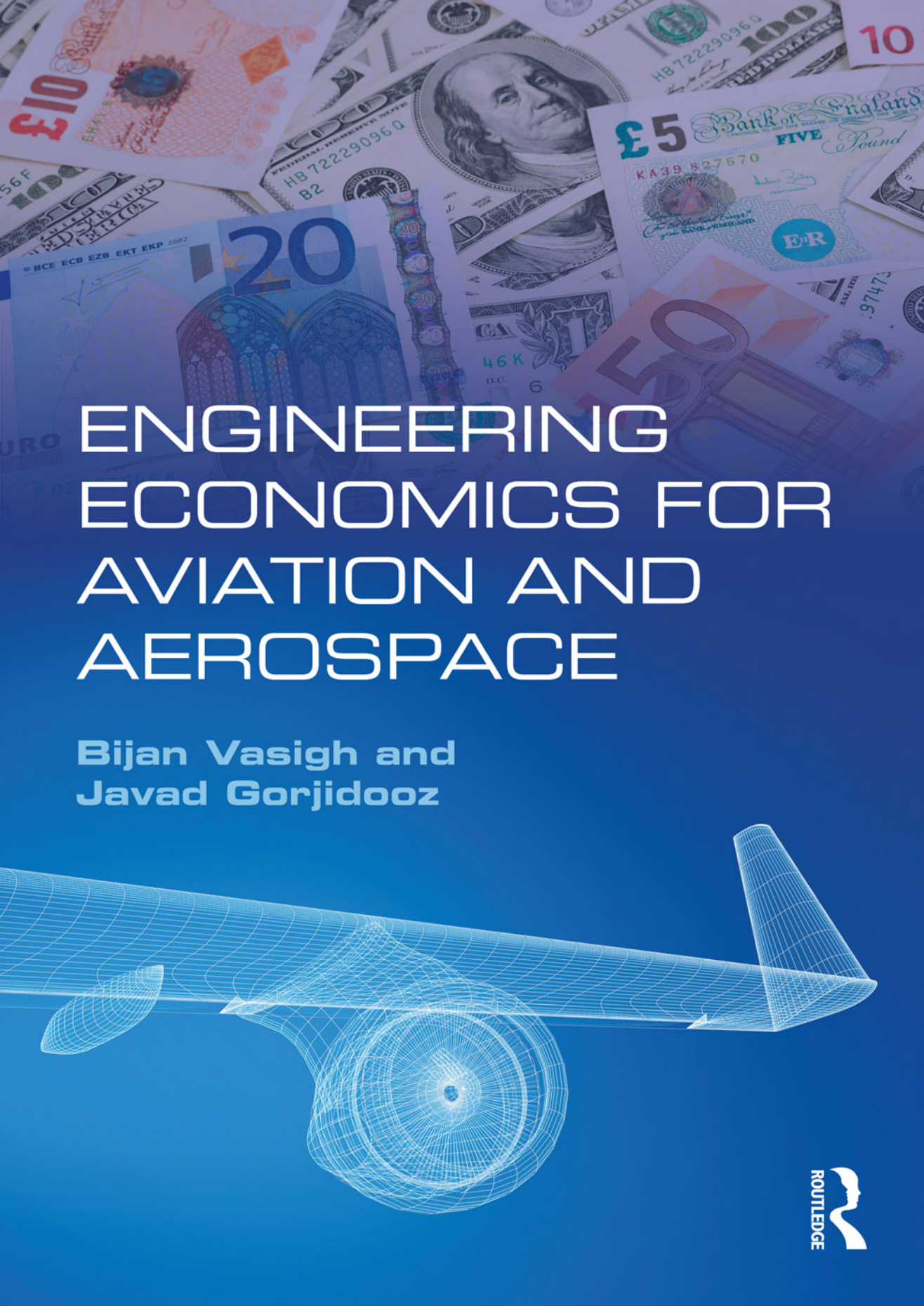
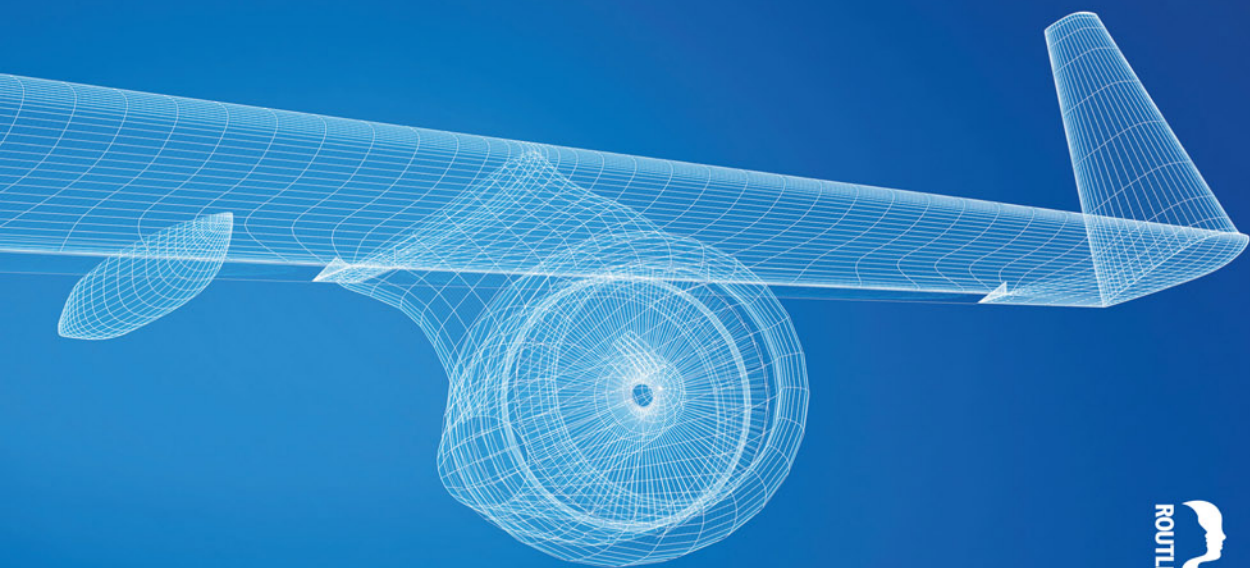




ENGINEERING ECONOMICS FOR AVIATION AND AEROSPACE

**Bijan Vasigh and
Javad Gorjidooz**



Engineering Economics for Aviation and Aerospace

For all engineers and practitioners, it is essential to have a fundamental understanding of cost structure, estimating cash flows, and evaluating alternative projects and designs on an economic basis. *Engineering Economics for Aviation and Aerospace* provides the tools and techniques necessary for engineers to economically evaluate their projects and choices.

The focus of this book is on a comprehensive understanding of the theory and practical applications of engineering economics. It explains and demonstrates the principles and techniques of engineering economics and financial analysis as applied to the aviation and aerospace industries. Time value of money, interest factors, and spreadsheet functions are used to evaluate the cash flows associated with a single project or multiple projects. The alternative engineering economics tools and techniques are utilized in separate chapters to evaluate the attractiveness of a single project or to select the best of multiple alternatives.

Most of the engineering economics and financial mathematics books available in the market take either a pure theoretical approach or offer limited applications. This book incorporates both approaches, providing students of aviation and industrial economics, as well as practitioners, with the necessary mathematical knowledge to evaluate alternatives on an economic basis.

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Bijan Vasigh and Javad Gorjidooz

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Contents

<i>List of Figures</i>	xiii
<i>List of Tables</i>	xvii
<i>Preface</i>	xxi
<i>Foreword by Brian Pearce</i>	xxii
<i>Foreword by Zane Rowe</i>	xxiv
<i>Foreword by Dr Barry Humphreys CBE</i>	xxvi
<i>Acknowledgments</i>	xxviii
<i>List of Abbreviations</i>	xxix
1 Introduction to the Air Transport Industry	1
Airline Industry before Deregulation	2
Airline Industry after Deregulation	3
Airline Profitability	6
Air Transport Industry	8
Aircraft Manufacturers	10
Jet Engine Manufacturers	14
Aircraft Leasing Companies	16
Service Providers (MRO, Ground Handling, Catering)	17
Airports	18
Air Navigation Service Providers (ANSP)	19
Global Distribution System (GDS) and Computer Reservation System (CRS)	19
Freight Forwarders and Integrators	22
Airline Mergers and Bankruptcies	23
Summary	25
Discussion Questions and Problems	27
2 Foundations of Engineering Economics and Finance	31
Definition of Engineering Economics	32
Interest and Time Value of Money	33
Simple Interest	34
Compound Interest	36
The Connection between Interest Rates and Their Determinations	38
Short-term Interest Rates	39
Long-term Debt and Interest Rates	42
Future Value of a Single Present Amount	47
Present Value of a Single Future Sum	49
Future Value of an Unequal Cash Flow Series	52
The Rule of 72	53
The Rule of 100	54
Summary	55
Discussion Questions and Problems	55

3	Annuities and Gradient Series	58
	Annuities or Uniform Series	59
	Future Value of an Annuity	61
	Annuity of a Future Value	63
	Present Value of an Annuity	64
	Annuity of a Present Value	68
	Amortization and Amortization Schedule	69
	Annuity Due	71
	Capitalized Cost	71
	Arithmetic Gradient Series	73
	Present Value of an Arithmetic Gradient Series	73
	Annuity of an Arithmetic Gradient Series	76
	Future Value of an Arithmetic Gradient Series	78
	Geometric Gradient Series	79
	Present Value of a Geometric Gradient Series	79
	Annuity of a Geometric Gradient Series	81
	Future Value of a Geometric Gradient Series	81
	Shifted Cash Flow Series	82
	Using Spreadsheets for Equivalences Computation	84
	Summary	87
	Discussion Questions and Problems	88
4	Nominal and Effective Interest Rates	92
	Nominal Versus Effective Interest Rates	93
	Calculating Effective Interest Rate	94
	Continuous Compounding	95
	Effective Rate to Nominal Rate	97
	Comparing Compounding Periods and Payment Periods	99
	Computing Equivalences Involving Only Single-Amount Factors	99
	Computing Equivalences when Payment Period Equals	
	Compounding Period	101
	Computing Equivalences when Payment Period Exceeds	
	Compounding Period	102
	Computing Equivalences when Payment Period is Less Than	
	Compounding Period	104
	Using Spreadsheets for Effective Rate and Equivalences Computation	105
	Summary	108
	Discussion Questions and Problems	109
5	Analysis of Present Worth	112
	Measuring Project Cash Flows	113
	First Cost (P)	113
	Net Annual Cash Flow (NACF)	114
	Terminal Cash Flow (TCF)	115
	Evaluation of Single Projects	116
	Bond Valuation	120
	Types of Project Alternatives	127
	Mutually Exclusive (ME) Projects	127

Independent Projects (IP)	128
Do-nothing Alternative	128
Evaluation of Mutually Exclusive Equal-life Alternatives	128
Evaluation of Mutually Exclusive Different-life Alternatives	130
Study Period Approach	131
Least Common Multiple or Replacement Chain Approach	132
Capitalized Cost or Equivalent Annual Annuity Approach.	133
Evaluation of Independent Projects	135
Using Spreadsheets for Present Worth Computation	137
Life Cycle Costing (LCC)	138
Summary	140
Discussion Questions and Problems	141
6 Analysis of Annual Worth	146
Motivation for Using Annual Worth Analysis	147
Evaluation of Single Projects	148
Capital Recovery	152
Annuity of Net Annual Cash Flows	153
Evaluation of Mutually Exclusive Projects	154
Annual Worth Analysis of Equal-life Alternatives.	155
Annual Worth Analysis of Non-Equal-life Alternatives	155
Evaluation of Independent Projects	157
Annual Worth of Permanent Projects	159
Using Spreadsheets for Annual Worth Computation	160
Summary	161
Discussion Questions and Problems	162
7 Analysis of Rate of Return.	167
Rate of Return Calculation	168
Interpretation of Rate of Return Values	170
Trial and Error Method.	170
Rate of Return of Bonds.	173
Evaluating Mutually Exclusive Projects	175
Incremental Cash Flows	176
Cost-based Alternatives	178
Revenue-based Alternatives	180
Equal-life Service Requirement.	181
Evaluating Independent Projects	184
Complications with Rate of Return	186
Multiple Rates of Return Scenario	187
ROR Reinvestment Rate Assumption	188
Modified Rate of Return Analysis.	188
Using Spreadsheets for Rate of Return Computation.	189
Summary	191
Discussion Questions and Problems	191
8 Public Sector Projects and Benefit to Cost Analysis	195
Public Sector versus Private Sector Projects	196

Benefit/Cost Analysis of Single Projects	200
Evaluation of Cost-based Mutually Exclusive Alternatives	202
Evaluation of Benefit-based Mutually Exclusive Alternatives	204
Evaluation of Independent Projects	208
Modified Benefit and Cost Analysis	210
Using Spreadsheets for Benefit/Cost Ratio Computation	212
Summary	213
Discussion Questions and Problems	214
9 Analysis of Break-even and Payback Period	219
Introduction to Break-even Analysis	220
Elements of Break-even Analysis	221
Break-even Analysis of a Single Project when Interest is Equal to Zero	223
Break-even Analysis between Two Alternatives with Interest Rate of Zero	225
Break-even Analysis of a Single Project with Interest Rate Greater than Zero	226
Break-even Analysis of Mutually Exclusive Alternatives with Interest Rate Greater than Zero	228
Airline Break-even Load Factor	229
Introduction to Payback Period Analysis	231
Conventional Payback Period Analysis	232
Discounted Payback Period Analysis	234
Using Spreadsheets for Break-even Analysis and Payback Periods	235
Summary	237
Discussion Questions and Problems	238
10 Uncertainty, Risk Analysis, and Simulation	242
Decision Making	243
Probability and Business Decisions	244
Cumulative Probability Distribution	246
Joint Probability Distribution	246
Measuring Risk and Reward	249
Sample Space	250
Expected Return or Mean	251
Variance	253
Standard Deviation	253
Calculating Variance and Standard Deviation Using Spreadsheets	254
Normal Distribution	258
Calculating Normal Distribution Probability Using Spreadsheets	260
Standard Normal Distribution	260
Portfolios	265
Portfolio Beta	266
Capital Asset Pricing Model (CAPM)	267
Security Market Line (SML)	269
Sensitivity Analysis	272
Break-even Analysis	272
Monte Carlo Analysis (Simulation)	275

Summary	279
Discussion Questions and Problems	279
11 Analysis of Depreciation, Replacement, and Abandonment	283
Introduction	284
Depreciation Methodologies	285
Straight Line Depreciation (SL) Method	286
Declining Balance Depreciation (DB).	289
Modified Accelerated Cost Recovery System (MACRS)	290
Sum-of-the-Years Digits (SYD) Method	292
Comparison of Different Approaches to Depreciation	294
Using Spreadsheets for Depreciation Computations	297
Basics of a Replacement Decision.	303
Impairment	306
Abandonment	308
Summary	311
Discussion Questions and Problems	312
12 Engineering Cost Estimation	317
Definition of Cost	319
Fixed Costs (FC)	319
Variable Costs (VC).	320
Total Costs (TC).	321
Cubic Cost Function	324
Marginal Costs (MC)	324
Average Total Cost (ATC)	325
Average Fixed Cost (AFC).	328
Average Variable Cost (AVC)	329
Sunk Costs	331
Opportunity Costs.	332
Relationship between Economic and Accounting Cost	334
Cost of Debt	335
Cost of Equity	335
Weighted Average Cost of Capital (WACC).	335
ROIC vs WACC	335
Cost Divisions and Categories	336
Direct Costs	337
Indirect Costs	338
Learning Curve	338
Estimating Costs	341
Cost Estimation Techniques	343
Unit Cost Method	344
Cost Index Method	346
Cost Capacity Equation	349
Cost Factor Method.	351
Indirect Cost Estimation and Allocation	352
Airline Cost Structure.	354
Operating Costs (OC).	354
Non-operating Costs (NOC).	358

Summary	358
Discussion Questions and Problems	359
13 Analysis of Financial Statements	363
The Income Statement: Measuring a Firm's Profits	364
Revenue, Sales, or Income	365
Cost of Goods Sold (COGS)	367
Gross Profit	368
Operating Expenses	369
Depreciation	369
Operating Profit	370
Financial Costs	370
Net Income	372
The Balance Sheet: Measuring a Firm's Book Value	377
Current Assets	377
Fixed Assets (Non-current Assets)	382
Total Liabilities	383
Current Liabilities	383
Non-current Liabilities	384
Stockholders' Equity	384
Statement of Cash Flow	386
Cash Flow from Operating Activities	386
Cash Flow from Investing Activities	387
Cash Flow from Financing Activities	387
Measuring Free Cash Flow	390
Common Size Financial Statements	390
Common Size Income Statement	390
Common Size Balance Sheet	392
Summary	395
Discussion Questions and Problems	396
14 Analysis of Financial Ratios	400
Liquidity Ratios	402
Current Ratio (CR)	402
Quick Ratio (QR)	404
Asset Management Ratios	405
Accounts Receivable Turnover Ratio (ART)	405
Average Collection Period (ACP)	406
Inventory Turnover (ITO)	407
Fixed Asset Turnover Ratio (FAT)	408
Total Asset Turnover Ratio (TAT)	408
Financial Management Ratios	410
Debt Ratio (D/A)	410
Debt to Equity Ratio (D/E)	412
Times Interest Earned Ratio (TIE)	413
Profitability Ratios	413
Gross Profit Margin (GPM)	414
Operating Profit Margin (OPM)	415

Net Profit Margin (NPM)	415
Return on Total Assets (ROA)	417
Return on Equity (ROE)	418
Dividend Ratios	418
Dividend Payout Ratio (DPR)	418
Dividend Yield	419
Market Value Ratios	420
Market to Book Ratio	420
Price–Earnings Ratio (P/E Ratio)	421
DuPont System Ratios	421
Return on Assets (ROA)	422
Return on Equity (ROE)	422
Window Dressing	425
Altman Insolvency Index	427
Key Financial Ratios for the Airline Industry	428
Available Seat Mile (ASM)	429
Revenue Passenger Mile (RPM)	429
Cost per Available Seat Mile (CASM)	429
Revenue per Available Seat Mile (RASM)	430
Revenue per Revenue Passenger Mile (RRPM) or Passenger Yield	431
Average Stage Length (ASL)	432
CASM (Stage Length Adjusted)	432
Average Daily Block Hour Utilization	432
Load Factor (LF)	433
Break-even Load Factor (LF_{b-e})	433
Summary	434
Discussion Questions and Problems	435
15 Fundamentals of Decision Theory	438
Decision-making Process	440
Decision Trees	441
Nodes	442
Branches	443
Decision Environments	445
Decisions under Certainty	447
Decisions under Uncertainty	447
Decisions under Risk	454
The Economic Theory of Information	456
Expected Value of Perfect Information (EVPI)	456
EV with PI	456
Queueing Theory	458
Customer Arrivals	459
Arrival Characteristics	462
Waiting Line Characteristics	462
Service Facility Characteristics	464
Cost–Capacity Tradeoff	467
Summary	468
Discussion Questions and Problems	469

Appendix A 473

Appendix B 529

Appendix C 531

Appendix D 533

Appendix E 535

Appendix F 537

Glossary 564

Index 575

Figures

1.1	Top Four US Domestic Carriers' Market Share	4
1.2	Return on Capital Invested in Airlines	5
1.3	Airlines Profit per Passenger	7
1.4	World Airline Profitability over Last Two Decades	7
1.5	Aviation Value Chain	9
1.6	US Commercial Aircraft Manufacturer Mergers	11
1.7	Airbus vs Boeing: Number of Deliveries	12
1.8	Bombardier vs Embraer: Number of Commercial Aviation Deliveries	13
1.9	Aircraft Manufacturing Industry Outlook	14
1.10	Jet Engine Manufacturers' Market Share	15
1.11	Market Share of Top 10 Aircraft Leasing Companies Globally	17
1.12	Top 10 World Airports by Passenger Traffic (in 100s) and Total Aircraft Movements	19
1.13	Global Distribution System (GDS)	20
1.14	Evolution of US Airline Industry	26
2.1	Historical Data of US Federal Fund Rates	40
2.2	LIBOR for 12-month Loans Based on US Dollars	41
2.3	Bond Rates and Maturity for Various Airlines and Manufacturing Companies	46
2.4	Cash Flow Diagram for a Given P and Finding F at the End of Year 5	48
2.5	Cash Flow Diagram for a Given F at the End of Year 5 and Finding P	50
2.6	Future Value of a Stream of Unequal Cash Flow Series	52
2.7	Present Value of a Stream of Unequal Cash Flow Series	53
3.1	Annuity Cash Flow Diagram	61
3.2	Annuity Cash Flow Diagram	66
3.3	Amortization Schedule Depiction	70
3.4	Arithmetic Gradient Series	73
3.5	Arithmetic Gradient Series	74
3.6	Arithmetic Gradient Series	75
3.7	Arithmetic Gradient Series	76
3.8	Geometric Gradient Series	80
3.9	Shifted Cash Flow Series	83
3.10	Shifted Cash Flow Series	84
3.11	Calculating the Future Value Using Excel Spreadsheets	85
3.12	Calculating the Future Value Using Excel Spreadsheets	85
3.13	Calculating the Present Value Using Excel Spreadsheets	85
3.14	Calculating the Annuity of a Future Amount Using Excel Spreadsheets	86
3.15	Calculating the Annuity of a Present Amount Using Excel Spreadsheets	86
3.16	Calculating the Time Period Using Excel Spreadsheets	86
3.17	Calculating the Interest Rate Using Excel Spreadsheets	86

3.18	Calculating the Present Value of an Arithmetic Gradient Series Using Excel Spreadsheets	87
4.1	Effective Interest Rates for Various Nominal Rates	96
4.2	Nominal Rates for Various Effective Interest Rates	98
4.3	Annuity Series Cash Flows	103
4.4	Calculating Semi-annual Effective Rate Using Excel Spreadsheets	106
4.5	Calculating Quarterly Effective Rate Using Excel Spreadsheets	106
4.6	Calculating Monthly Effective Rate Using Excel Spreadsheets	106
4.7	Calculating Daily Effective Rate Using Excel Spreadsheets	107
4.8	Calculating Semi-annual Nominal Rate Using Excel Spreadsheets	107
4.9	Calculating Quarterly Nominal Rate Using Excel Spreadsheets	108
4.10	Calculating Monthly Nominal Rate Using Excel Spreadsheets	108
4.11	Calculating Daily Nominal Rate Using Excel Spreadsheets	108
5.1	Bond Valuation	124
5.2	Calculating the Value of the Bond Using Excel Spreadsheets	124
5.3	Calculating the Value of the Bond Using Excel Spreadsheets	125
5.4	Calculating NPV Using Excel Spreadsheets	138
6.1	Cash Flow Diagram	149
6.2	Cash Flow Diagram	150
6.3	Cash Flow Diagram	150
6.4	Cash Flow Diagram	151
6.5	Cash Flows for One Life Cycle	156
6.6	Cash Flows for Two Life Cycles	157
6.7	Annual Worth Computation Using Excel Spreadsheets	161
6.8	Annual Worth Computation in Excel Spreadsheets	161
7.1	Cash Flow Diagram	169
7.2	Cash Flow Diagram	170
7.3	Present Worth as a Function of the Interest Rate	172
7.4	Endpoints for Interpolating the ROR	172
7.5	Multiple Rates of Return	188
7.6	Rate of Return Computation in Excel Spreadsheets	190
8.1	Benefit/Cost Computation in Excel Spreadsheets	213
9.1	Linear Costs Relations	222
9.2	Nonlinear Cost Curves	223
9.3	Nonlinear Cost Curves and Total Revenue	223
9.4	Linear Costs and Revenue Functions	224
9.5	Break-even Point (Q_{BE})	225
9.6	Break-even Quantity between Two Alternatives	227
9.7	Average Achieved Load Factor and Break-even Load Factor Trend	231
9.8	Calculation of Break-even Quantity with Zero and Non-Zero Interest Rate Using Excel	236
9.9	Calculation of Conventional and Discounted Payback Periods with Zero and Non-Zero Interest Rate Using Excel	237
10.1	Normal Cumulative Distribution (Probability of filling equal or less than . . .)	248
10.2	Inverse Cumulative Distribution (Probability of filling equal or more than . . .)	248
10.3	Data and Formula Entry for Variance Calculation	255

10.4	Result Obtained for Variance Using Excel Formula	256
10.5	Calculation of Standard Deviation Using Excel Formula	257
10.6	Result Obtained for Standard Deviation Using Excel Formula	257
10.7	Normal Distribution for Returns	258
10.8	Normal Distribution	259
10.9	Normal Distribution ($\pm 1\sigma$)	259
10.10	Normal Distribution ($\pm 2\sigma$)	259
10.11	Normal Distribution ($\pm 3\sigma$)	259
10.12	Normal Distribution Using Excel Function	261
10.13	Standard Normal Distribution	262
10.14	Normal Distribution of Ticket Prices	264
10.15	Normal Distribution with Varying Standard Deviations	264
10.16	Beta for Selected World Airlines as of March 2016	268
10.17	Security Market Line (SML)	270
10.18	Shifts in Security Market Lines of Three Different Portfolios	271
10.19	Break-even Point Based on Initial Assumptions	274
10.20	Sensitivity Analysis Scenario 1—Increase in the Fixed Cost	274
10.21	Sensitivity Analysis Scenario 2—Increase in the Variable Cost	275
10.22	Sensitivity Analysis Scenario 3—Increase in the Market Price	275
10.23	Generating a Random Number in a Given Range Using Excel Formula	276
10.24	Probability Density Distribution	277
10.25	Correlation Coefficients	278
10.26	Inputs Ranked by Effect on Output	278
11.1	Comparison of Different Depreciation Methods	297
11.2	Calculating Straight Line Depreciation Using Excel Spreadsheets	298
11.3	Calculating Annual Asset Value Change under Straight Line Depreciation Using Excel Spreadsheets	298
11.4	Calculating Declining Balance Depreciation Using Excel Spreadsheets	299
11.5	Calculating Annual Asset Value Change under Declining Balance Depreciation Using Excel Spreadsheets	300
11.6	Calculating Double Declining Balance Depreciation Using Excel Spreadsheets	300
11.7	Calculating Annual Asset Value Change under Double Declining Balance Depreciation Using Excel Spreadsheets	301
11.8	Calculating Depreciation Using MACRS Method	302
11.9	Calculating Depreciation Using Sum-of-the-Years Digits Method	302
11.10	Calculating Annual Asset Value Change under Sum-of-the-Years Digits Depreciation Method Using Excel Spreadsheets	303
11.11	Possible Changes in Economic Service Life	305
11.12	Impairment Measurement under IAS 36	306
12.1	Short-run Cost Curves	323
12.2	Average Fixed, Variable, Marginal, and Total Cost Curves	323
12.3	Marginal Cost and Average Total Cost	327
12.4	Calculating Marginal Cost Using Excel Spreadsheets	327
12.5	CASM Comparison for Q3, 2015	328
12.6	Average Fixed Cost Curve	329
12.7	Minimum AVC with Change in Quantity	331

12.8	ROIC, WACC, and Debt Ratio	337
12.9	Learning Curve	340
12.10	Sample Learning Curve for Boeing 787	341
12.11	Airline Operating Cost Index	355
12.12	Labor Costs of Selected US Airlines, 2015	357
12.13	Fuel Costs of Selected US Airlines, 2015	357
13.1	Ancillary Revenue for Global Airlines	375
14.1	Current Ratio for Various Airlines (2015)	403
14.2	Total Asset Turnover for Various Airlines (2015)	410
14.3	Debt Ratio for Various Airlines (2015)	411
14.4	Return on Total Assets for Various Airlines (2015)	417
14.5	Price–Earnings Ratio for Various Airlines in USD (2015)	421
14.6	ROA Using DuPont System for Various Airlines (2015)	423
14.7	ROE Using DuPont System for Various Airlines (2015)	426
14.8	Z-scores for Manufacturing Companies in Aviation	428
14.9	CASM (Cents) for Various Airlines (2015)	430
14.10	RASM (Cents) for Various Airlines (2015)	431
14.11	RRPM (Cents) for Various Airlines (2015)	431
14.12	Average Stage Length (Miles), US Carriers (2015)	432
14.13	Average Daily Block Hour Utilization, US Carriers (2015)	433
14.14	Break-even Load Factors (%) for Various Airlines (2015)	434
15.1	Decision Tree	442
15.2	Decision Tree with Terminal Values	444
15.3	Decision Tree with Probabilities and Terminal Values (Complete Tree)	445
15.4	Exponential Probability Distribution	460
15.5	Probabilities for Poisson Distribution	461
15.6	Single and Multiple Server Aircraft Landing at the Airport	464
15.7	Cost vs Capacity	468

Tables

1.1	Top 10 Lessors by Fleet Value	16
2.1	Simple Interest Rate Calculation	36
2.2	Compound Interest Calculations	37
2.3	Difference Between Accrued Simple and Compound Interests	38
2.4	Overview of Different Governmental Loans	44
2.5	Compound Interest Factors for 6 Percent	49
2.6	Compound Interest Factors for 10 Percent	51
3.1	Compound Interest Factors for 5 Percent	62
3.2	Compound Interest Factors for 10 Percent	64
3.3	Present Value of an Annuity Calculation	66
3.4	Compound Interest Factors for 6 Percent	67
3.5	Amortization Schedule for EZjET	69
3.6	Compound Interest Factors for 8 Percent	75
3.7	Compound Interest Factors for 8 Percent	77
4.1	Effective Annual Interest Rates for Selected Nominal Rates	96
4.2	Nominal Annual Interest Rates for Selected Effective Annual Rates	98
5.1	Present Worth Analysis Accept and Reject Criteria	117
5.2	Forecast of Costs and Revenues for Development of B787 Dreamliner	119
5.3	Net Cash Flows per Year for B787 Development	119
5.4	Bond Valuation	123
5.5	Cash Flows for Both Robotics and Non-robotics	129
5.6	Cash Flows for Both Alternatives	132
5.7	Cash Flows for Both Alternatives under Replacement Chain Approach	133
5.8	DirectJet Airlines Independent Projects Cash Flows	136
5.9	DirectJet Airlines Possible Bundles, Total Investments, and PWs	137
5.10	Net Annual Cash Flows	138
5.11	Life Cycle Costing	140
6.1	Accept and Reject Criteria for Annual Worth Analysis	148
6.2	Mutually Exclusive Alternatives	155
6.3	Cash Flows for Cessna 150	156
6.4	Majestic Byproducts Independent Projects	158
7.1	Accept and Reject Criteria for Rate of Return	170
7.2	Information for Interpolation	173
7.3	Mutually Exclusive Projects of Different Sizes	176
7.4	Mutually Exclusive Projects with Cash Flows Pattern Differential	177
7.5	Accept and Reject Criteria for the Δ Project	177
7.6	Cash Flows for Projects A and B and the Δ Project	178
7.7	Cash Flows for Projects C and D and the Δ Project	178
7.8	Mega Storage Facility Options in Hanoi, Vietnam	179
7.9	Mega Storage Facility Options in Hanoi, Vietnam	179

7.10	Airline Expansion Plan	180
7.11	Airline Incremental Cash Flows for $\Delta_{\text{CAE-DN}}$ Project	180
7.12	Airline Incremental Cash Flows for $\Delta_{\text{LAE-CAE}}$ Project	181
7.13	Legacy Regional Airport Projects	182
7.14	Legacy Regional Airport Incremental Cash Flows for $\Delta_{\text{WMS-DN}}$ Project	182
7.15	Legacy Regional Airport Incremental Cash Flows for $\Delta_{\text{JFS-WMS}}$ Project	183
7.16	Legacy Regional Airport Independent Projects	184
7.17	Cash Flow Stream of an Unconventional Project	187
7.18	Comparing Interest Rates and Their Corresponding PW	187
7.19	Accept and Reject Criteria for Modified Rate of Return	189
7.20	Net Annual Cash Flows	190
8.1	Economic Impacts (Benefits) of Commercial Airports in the US in 2013	198
8.2	Accept and Reject Criteria for Benefit/Cost Ratio	201
8.3	Accept and Reject Criteria for the Δ Project	203
8.4	Cash Flows for Both Alternatives	203
8.5	Cash Flows for Both Alternatives and Incremental Cash Flows for the Δ Project	204
8.6	Legacy Regional Airport Mutually Exclusive Projects	205
8.7	Legacy Regional Airport Mutually Exclusive Projects in AW Values	205
8.8	Legacy Regional Airport Projects in Order of Their Total Costs	206
8.9	Comparison of Do-nothing with New Economy Parking Garage	206
8.10	Comparison of New Economy Parking Garage and Concourse B Expansion	207
8.11	Comparison of Concourse B Expansion and Concourse A Expansion	208
8.12	Legacy Regional Airport Independent Projects	209
8.13	Food Court Project for Gator International Airport	211
9.1	Examples of Fixed and Variable Costs	222
9.2	Cash Flows for Both Turboprop and Turbofan	228
9.3	NACFs	233
9.4	Cumulative NACFs	233
10.1	Cumulative Probability Distribution	247
10.2	Probabilities Assigned to Defective Items	252
10.3	Mean and Variance	254
10.4	Variance and Standard Deviation	255
10.5	Calculation of Portfolio Returns Variance	257
10.6	Probability Profile for Different Ticket Prices	262
10.7	Calculation of Expected Value of the Ticket	262
10.8	Calculation of Variance	263
10.9	Investment Summary	267
10.10	Calculation of Portfolio Beta	267
10.11	Required Rate of Return under Different Market Premium	271
10.12	Components of Cost of the Project	277
10.13	Summary of the Analysis	278
11.1	Annual Depreciation, Straight Line Method	287
11.2	Straight Line Example with Salvage Value	288
11.3	Annual Depreciation, DDB Depreciation Method	290
11.4	Annual Depreciation, DDB Method (150%)	291

11.5	MACRS Depreciation Rates	292
11.6	MACRS Depreciation Expense Calculation	293
11.7	Sum-of-the-Years Digits Depreciation Method	294
11.8	Comparison of Different Approaches to Depreciation (SL and MACRS)	295
11.9	Comparison of Different Approaches to Depreciation (SYD and DDB)	296
11.10	Ownership Cost vs Operating Cost	305
11.11	MACRS Depreciation Schedule	307
11.A1	Double Declining Balance Depreciation Method	315
12.1	Short-run Cost Schedule	322
12.2	Jet Services International Project Costs	326
12.3	DirectJet Accounting Costs	333
12.4	DirectJet Net Gain	333
12.5	Jet-Services ROIC and WACC Estimates	336
12.6	Cobb–Douglas Data Assumption	342
12.7	Regression Analysis Results	343
12.8	Components and Unit Cost Factor	345
12.9	Calculation of Estimated Total Cost	346
12.10	CPI for All Urban Consumers as of December	347
12.11	PPI for Aircraft Engine and Engine Parts as of January	347
12.12	PPI for Construction as of December	347
12.13	PPI for Wholesale Trade Industries as of December	348
12.14	ENR Construction Cost Index	348
12.15	Possible Values for Correlating Exponent	349
12.16	Available Seat Miles and Load Factor for Each Route	353
12.17	Computation of Revenue Passenger Miles	353
12.18	Indirect Costs Allocation	354
12.19	Operating Expenses for Delta Air Lines	356
13.1	Boeing Income Statement, 2015	366
13.2	COGS Section of Boeing Income Statement, 2015	367
13.3	Jet Propulsion Depreciation Schedule	370
13.4	DirectJet Schedule of Earnings and Taxes Paid	372
13.5	DirectJet Schedule for Loss Carryforward and Loss Carryback	372
13.6	Delta Air Lines Income Statement, 2015	373
13.7	Annual Financial Disclosures of Ancillary Revenue	375
13.8	MyJet Airlines Financial Data	376
13.9	General Engine Manufacturing Company Financial Data (a)	376
13.10	General Engine Manufacturing Company Financial Data (b)	377
13.11a	Balance Sheet for Delta Air Lines	378
13.11b	Balance Sheet for Delta Air Lines	379
13.12	Metro Jet Statement of Retained Earnings	386
13.13	Statement of Cash Flows for Delta Air Lines, 2015	388
13.14	Statements of Cash Flow Highlights, Southwest Airlines	389
13.15	Delta Air Lines Common Size Income Statement, 2015	391
13.16	Phoenix Aviation Income Statements	393
13.17	Delta Air Lines Common Size Balance Sheet, 2015	393
14.1	MyJet Airlines Financial Data	405
14.2	Total Asset Turnover for Various Airlines	409

14.3	MyJet Balance Sheet Data	412
14.4	DirectJet Financial Data	416
14.5	ROA Using DuPont System for Various Airlines	423
14.6	ROE Using DuPont System for Various Airlines	425
14.7	Direct Jet Current Balance Sheet	426
14.8	Direct Jet New Balance Sheet	427
14.9	Altman Z-score for Selected Manufacturing Companies	428
14.10	Break-even Load Factors for Various Airlines	434
15.1	Nodes and Branches	443
15.2	DirectJet Payoff Table	448
15.3	DirectJet Payoff Table—Maximax Approach	449
15.4	DirectJet Payoff Table—Maximin Approach	449
15.5	DirectJet Payoff Table—Minimax Approach	450
15.6	DirectJet Payoff Table—Equally Likely Approach	451
15.7	AvParts Example—Decision under Uncertainty	452
15.8	AvParts Example—Decision under Uncertainty, Maximax Approach	452
15.9	AvParts Example—Decision under Uncertainty, Maximin Approach	452
15.10	AvParts Example—Decision under Uncertainty, Equally Likely Approach	453
15.11	AvParts Example—Decision under Uncertainty, Regret Analysis	453
15.12	AvParts Example—Decision under Risk	454
15.13	Risk as Variance Example	455
15.14	Probabilities for Exponential Distribution	460
15.15	Cost–Capacity Tradeoff	467

Preface

For all engineers and practitioners, it is essential to have a fundamental understanding of estimating cash flows, costing projects, and evaluating alternative projects and designs on an economic basis. This book provides the tools and techniques necessary for engineers to economically evaluate their decisions and designs. For example, the supersonic Concorde aircraft was considered to be the most sophisticated aircraft, flying at twice the speed of sound, and known by many people as an aviation icon and an engineering phenomenon. But the very high costs of research and development, in addition to expensive operating costs made the commercial operation of Concorde uneconomical. After producing 20 aircraft, in 1979, the program was terminated because of the lack of demand by airlines due to these high operating costs. Most of the engineering economics and financial mathematics books available in the market have taken either a pure theoretical approach or a limited application approach without justification. We have written this book by incorporating both of the approaches synchronously, theory and well justified application.

The focus of this book is on comprehensive understanding of the theory and on practical applications of engineering economics. The objective is to explain and demonstrate the principles and techniques of engineering economics and financial analysis as applied to the aviation and aerospace industries. Time value of money, interest factors, and spreadsheet functions are used to evaluate the cash flows associated with a single project or multiple projects. The alternative engineering economics tools and techniques are utilized in separate chapters to evaluate the attractiveness of a single project or to select the best of multiple alternatives. Mutually exclusive cost-based and revenue-based alternatives and independent projects are covered throughout the book. This textbook is designed to be used at the undergraduate level, as well as being a tool for self-study, and a reference for the professional working within the aviation and aerospace industry.

This book is intended to provide students and practitioners with the necessary mathematical knowledge and techniques to evaluate, analyze, and solve engineering economics-related problems especially in the field of aviation and aerospace.

For students, a sophomore or higher level is recommended to thoroughly understand the tools and techniques of engineering economics. A background in calculus is not necessary; however, a basic knowledge of college algebra, pre-calculus, or calculus for the purpose of math maturity makes the material more meaningful and therefore easier to learn and apply.

For practitioners, this book explains the concepts and applications carefully and clearly which assists readers to understand the material and helps them to apply the tools and techniques to engineering economics situations.

Foreword by Brian Pearce

This is a book for people involved in the operation of airlines and aerospace companies who need to take commercial decisions or just need to understand how the accountants and financial analysts look at the success or otherwise of their projects and their companies. Most of the books on financial analysis don't deal with the specific challenges of air transport. Most of the books on air transport economics are not written with the engineer in mind. Bijan Vasigh and Javad Gorjidoz have filled the gap with this excellent book, which takes a structured journey through all the key concepts and approaches that an engineer needs so as to understand what makes financial sense.

Economics matters for the engineer, and others involved in operations or development. The author reminds us of the great engineering projects, like Concorde, that failed financially. Aerospace and the airline industry is littered with failures (as well as successes) because the economics of major projects or major business initiatives have not been sufficiently well understood. Lockheed's attempt to challenge the wide-body aircraft market with the L-1011 TriStar in the 1970s and 1980s ended in expensive failure and the company's withdrawal from the commercial aircraft market. There are numerous examples of much smaller projects which may have been engineering successes but were financial failures. So, understanding the economics of engineering and operational projects, and being able to show how they may succeed financially, is important.

This is a fascinating industry, and it is worth understanding it from an economics as well as an engineering perspective. There are not many industries that have expanded three to four times as much as the global economy over the past 40 years; where efficiency improvements have more than halved the cost, in real terms, of providing the product; where investment is constantly attracted; and yet where the airline operators have, until recently, routinely destroyed billions of dollars of shareholder value each year. But those companies in the airline supply chain and aerospace industries do rather better. And the wider economic benefits of air transport are clear. This is an important industry. Air transport has both enabled and benefited from the globalization of the world economy. Connecting cities and countries around the world has brought tremendous value not just to air travelers and shippers but also to the wider economy, as connectivity has opened new markets and facilitated the benefits of specialization in the businesses that cluster in well-connected cities. Understanding how economics and finance shape the outcome of major projects and businesses will give students not only intellectual satisfaction but, perhaps more importantly, the practical tools to help ensure engineering and operational projects are commercially successful.

The authors make good use of the main tools of the financial analyst from modern financial economics. You will be led through the techniques and thought processes of the financial analyst, shown how future cash flows from your engineering project can be appropriately discounted into a present value that can be compared against costs,

how to measure and interpret breakeven points and payback periods, and much more. For those from the public sector there is a guide to cost benefit analysis which looks beyond the narrower interests of the private company. I think of particular value are sections on risk, the known unknowns you can plan for at least probabilistically, and uncertainty, the unknown unknowns which can't be planned for, except by creating flexibility in your project or business model. Finally, the chapter on decision theory is of great value to the practical engineer or operational person who may be used to dealing with a solid and measurable world and may now have to make commercial choices in the face of much risk and uncertainty. How do you do it? The authors carefully guide you through the techniques and offers several tools of great practical use.

There are many books about air transport. There are few that are really clear and useful for the practical engineer or operations person in the airlines or aerospace business. This is one that is.

Brian Pearce
Chief Economist, International Air Transport Association
Visiting Professor, Cranfield University

Foreword by Zane Rowe

In the fast paced world of global competition, massive amounts of data, and limited capital, making well-informed decisions at all levels of the organization is an imperative part of business.

Bijan Vasigh and Javad Gorjidoz have a strong track record of bringing financial and economic principles to life using the aviation and aerospace industry as a backdrop. While these principles could be applied to many sectors, there aren't too many that have the global influence, high stakes, and tumultuous track record of the aviation industry. The aviation history books are filled with turbulence, capital destruction, colorful characters, and questionable decision making on a grand scale.

This industry has had a profound impact on how we live our lives today. The decisions building up to the many achievements the industry has celebrated create an enthralling backdrop to understanding the economic and financial principles that have made such a big impact on globalization and our quality of life. For aspiring engineers and students having a blend of technical expertise with the economic and financial knowledge is a powerful quality that will serve them well in any professional capacity. Engineering Economics for Aviation and Aerospace builds upon that strength, offering the reader an understanding of economic and financial principles behind the projects and decisions on a dynamic stage filled with numerous tradeoffs and competing opportunities.

While many engineering feats are graded by their size and scale rather than economic viability, this textbook helps the reader understand tradeoffs that need to be made in a world of many competing projects and ideas. Understanding these principles will allow for better decision making and ultimately help differentiate companies and communities.

The authors tackle a variety of fundamental financial tools and techniques from estimating cash flows to evaluating alternative projects and designs. Included are explanations to assist in understanding return metrics to help measure and compare numerous projects. The authors include analyses of public projects, where the public benefits and associated costs are drivers of success.

Evaluating risk management and analyzing uncertainty in multiple outcomes are factors also addressed in addition to key elements and decisions in analyzing project costs. Ultimately the authors elevate the view to the company level covering key ratios that measure the health of the business and industry.

There are a number of ways to learn about good financial decision making—and the authors have brought to life key economic and financial principles that keep the reader engaged and help the reader make better decisions. The disciplines covered in this text can be applied to most industries and financial decisions. This textbook will animate engineers and those wanting to understand financial metrics and decision making at all levels of the organization.

Engineering Economics for Aviation and Aerospace is a powerful combination of financial and economic theory with its practical application. The reader will learn

about the impact, complexity, and importance of good decision making. Ultimately, this book helps a new generation of students and engineers understand how to make good financial trade-offs in their discipline in the exciting field of aviation and aerospace, having a pronounced impact on the world we'll be a part of in the future.

Zane Rowe

Chief Financial Officer and Executive Vice President of VMware

Foreword by Dr Barry Humphreys CBE

There are many books on both engineering and economics, but texts which bring together the two academic disciplines are rarer. Even more rare is to then apply those disciplines to the aviation and aerospace industries. The fact that there is a need for such an application is aptly demonstrated at the beginning of Bijan Vasigh and Javad Gorjidoz's book when the authors quote the example of the Anglo-French supersonic Concorde. This aircraft was a superb piece of engineering, and those who designed and built it were rightly proud of what they achieved. As the authors say, it was "an aviation icon and an engineering phenomenon." Unfortunately, it was also an economic disaster. High development, production and operating costs, combined with adverse environmental impacts, meant that when the programme was finally cancelled in 1979, only 20 aircraft had been produced, and even they had effectively to be given away. There were no customers willing to pay an economic price, and the financial losses to the British and French taxpayers were enormous. Rarely has the importance of combining an understanding of engineering with one of economics been better illustrated.

However, the authors of this book go beyond that. They also seek to address the subject matter from both a theoretical and a practical perspective. The economic and financial theory needed by engineers to carry out their jobs is set out clearly and in detail. The authors then show, issue by issue, how the theory can be applied to everyday projects and challenges. The use of financial formulae, the difference between nominal and effective interest rates, and analyses of present worth, annual worth, rate of return, break-even and payback periods, depreciation, replacement and abandonment, financial statements and ratios and decision theory—all are covered, with repeated references to their use in practice.

The focus of the book is very much on US experience, although the principles discussed can easily be applied elsewhere. Undergraduate students of engineering and economics will certainly benefit from the information carefully assembled by the authors, but practitioners in the aviation and aerospace industries also have much to gain from the book. The objective is stated to be to provide a comprehensive understanding of the theory and practical application of engineering economics. This is successfully achieved. Reading the book will help to ensure that students and practitioners do indeed have the necessary mathematical knowledge and techniques to, as the authors say, "evaluate, analyze and solve engineering economics related problems in the field of aviation and aerospace."

Aviation is seen by many as being a very interesting, even sexy, industry. It attracts many enthusiasts at all levels. Unfortunately, and perhaps at least partly as a result, it also has an appalling financial record. Since the Wright Brothers first flew, airlines as a group worldwide have actually destroyed more capital than they have created, hence their long record of bankruptcies and government bail-outs. Similarly, aircraft manufacturers have often built the wrong aircraft at the wrong time, with sometimes disastrous results for their viability. It is not surprising that today there is only a handful of companies capable of building a modern jet aircraft. And airports

are not immune from criticism either. They have a very mixed record when it comes to investments, with more than a few decisions resulting in “white elephants” of little use to airlines or the travelling public. If this book by Gorjidooz and Vasigh can help in just a small way to reduce the likelihood of poor investment decisions in the future, it will have served a very useful purpose. It is extremely important that aviation engineering students in particular have a full understanding of the economic and financial characteristics of the industry they are about to enter. Then perhaps when an engineering project as brilliant as Concorde is next developed, it might also prove to be financially viable.

Dr Barry Humphreys CBE

Formerly Director of External Affairs and Route Development at Virgin Atlantic Airways and Chairman of the British Air Transport Association.

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Bijan Vasigh
Javad Gorjidooz

Abbreviations

ACI	Airports Council International
ACMI	Aircraft, Crew, Maintenance, and Insurance
AFC	Average Fixed Cost
AMEX	American Stock Exchange
AOC	Air Operator's Certificate
AOPA	Aircraft Owners and Pilots Association
ASL	Average Stage Length
ASM	Available Seat Miles
ATA	Air Transport Association
ATC	Air Traffic Control
ATI	Air Transport Intelligence
BTS	Bureau of Transportation Statistics
CAPM	Capital Asset Pricing Model
CASM	Cost per Available Seat Mile
CCC	Cash Conversion Cycle
CD	Certificate of Deposit
CEO	Chief Executive Officer
CFO	Chief Financial Officer
CPA	Certified Public Accountant
CR	Current Ratio
CRS	Computer Reservation Systems
DCF	Discounted Cash Flow
DFL	Degree of Financial Leverage
DOC	Direct Operating Cost
DOL	Degree of Operating Leverage
DOT	Department of Transportation
EAC	Equivalent Annual Cost
EBIT	Earnings before Interest and Taxes
EOQ	Economic Order Quantity
EPS	Earnings per Share
EU	European Union
FAA	Federal Aviation Administration
FASB	Financial Accounting Standards Board
FC	Fixed Cost
FFP	Frequent Flyer Programs
FIFO	First In, First Out
FTSE	Financial Times Stock Exchange
FV	Future Value
GAAP	Generally Accepted Accounting Principles
GDP	Gross Domestic Product
GECAS	GE Commercial Aviation Services

IATA	International Air Transport Association
ICAO	International Civil Aviation Organization
IFRS	International Financial Reporting Standards
ILFC	International Lease Finance Corporation
IOC	Indirect Operating Cost
IPO	Initial Public Stock Offering
IRR	Internal Rate of Return
IRS	Internal Revenue Services
JAA	Joint Aviation Authorities
JIT	Just-in-time Inventory Strategy
LCC	Low-cost Carrier
LIBOR	London Interbank Offered Rate
MACRS	Modified Accelerated Cost Recovery System
MARR	Minimum Acceptable Rate of Return
MC	Marginal Cost
MIRR	Modified Internal Rate of Return
MRO	Maintenance, Repair, and Overhaul
NASDAQ	National Association of Securities Dealers Automated Quotations
NOC	Non-operating Cost
NPIAS	National Plan of Integrated Airport Systems
NPV	Net Present Value
NWC	Net Working Capital
NYMEX	New York Mercantile Exchange
NYSE	New York Stock Exchange
O&D	Origin and Destination
OAG	Official Airline Guide
P/E	Price-to-Earnings Ratio
PI	Profitability Index
PV	Present Value
QR	Quick Ratio
RASM	Revenue per Available Seat Mile
ROA	Return on Assets
ROE	Return on Equity
ROI	Return on Investment
RPM	Revenue Passenger Mile
RRPM	Revenue per Revenue Passenger Mile
RRTM	Revenue per Revenue Ton Mile
RTM	Revenue Ton Miles
SEC	Security Exchange Commission
SML	Security Market Line
TC	Total Cost
TFC	Total Fixed Cost
TOC	Total Operating Cost
VC	Variable Cost
WACC	Weighted Average Cost of Capital

1

Introduction to the Air Transport Industry

Everything not invented by God is invented by an engineer.

Prince Philip, Duke of Edinburgh



By generating a total of 58.1 million jobs globally, the air transport sector is one of the world's most vital industries. Air transportation provides an economical and reliable mode of moving passengers and cargo with no comparable substitute especially for the long haul market. Aircraft manufacturers leverage innovative technologies such as the fly-by-wire program to improve the economic efficiency and eco friendliness of commercial aviation. Airlines, aircraft and engine manufacturers, airports, air traffic management services, civil aviation regulators all work in tandem to provide sustainable growth. A key task in any manufacturing facility is to continuously reduce cost. Engineering economics is a branch of economics that applies economic principles to engineering projects. An important task of the engineer economist is to continuously seek new ways to improve efficiency and manage cost. With the techniques of engineering economics, a jet engine manufacturer by using meaningful economic theory evaluates the cost and benefits of spending money to develop more fuel-efficient and environment-friendly engines. Jet aircraft in service today are well over 70 percent more fuel efficient per seat mile than aircraft in the 1960s.¹ Air transport continues to create tremendous value for its customers and others in the value chain such as airports, aircraft manufacturers, jet engine producers, and travel facilitators. The world's 1,397 airlines operate a total fleet of 25,332 aircraft,¹ by serving 17,678 commercial airports globally.²

In this chapter we review the following:

- ◆ Airline Industry before Deregulation
- ◆ Airline Industry after Deregulation

- ♦ Airline Profitability
- ♦ Air Transport Industry
 - Aircraft Manufacturers
 - Regional Aircraft Manufacturers
 - Jet Engine Manufacturers
 - Aircraft Leasing Companies
 - Service Providers (MRO, Ground Handling, Catering)
 - Airports
 - Air Navigation Service Providers (ANSP)
 - Global Distribution System (GDS) and Computer Reservation System (CRS)
 - Online Travel Agents (OTA)
 - Freight Forwarders and Integrators
- ♦ Airline Mergers and Bankruptcies
- ♦ Summary
- ♦ Discussion Questions and Problems

Airline Industry before Deregulation

Total deregulation would allow anybody to fly any route, a situation that is unlikely ever to occur.
New York Times Magazine, May 9, 1976

In 1938 in the United States, the Civil Aeronautics Act transferred federal responsibilities for non-military aviation from the Bureau of Air Commerce to a new independent agency, the Civil Aeronautics Authority (Bunke, 1954). The Civil Aeronautics Act of 1938 created the Civil Aeronautics Board (CAB) and the Civil Aeronautics Administration. The Civil Aeronautics Board was responsible for issuing and overseeing aircraft and pilot certification and suspension. In addition, this act issued air carrier route certificates and regulated airline fares. The newly established CAB also became responsible for investigating aircraft accidents. Before deregulation of the airline industry, there were 16 trunk airlines in the United States. Trunk was the generic name for the common carriers that provided scheduled national and international air service. Government assistance through protection, subsidy, promotion, and regulation was thought to be necessary to permit the new industry to develop. For example, at that time railroads were still the dominant form of both passenger and freight transportation, so railroad companies were specifically prohibited from having a financial benefit in an airline. The fear was that very rich and powerful railroad firms would crush the competition from the embryonic airline industry.

Forty years later, by 1978, six of the original 16 trunk airlines had left the market and no new trunk airline had been allowed to come into existence. The ten that had survived accounted for 90 percent of the air carrier market in the United States.

During the era of regulation, the airline industry experienced a very stable existence and this was mainly attributed to the control CAB had over the airline industry (Kahn, 1988). The fixed nature of prices and routes meant that airlines were assured of a certain profit and the only form of competition that existed was “non-price competition”. Since the prices were fixed for any particular route, airlines purely had to compete based on quality of service and other non-price related factors. The extreme regulation by CAB also made it really hard for any new airline to enter the industry since the cost of operating the airline would not be feasible for a newcomer.

In 1978, the supersonic commercial jet era was two years old (Concorde flights between Europe and the United States had begun in 1976), and the CAB machinery of airline regulation had slowed down significantly.³

Airline Industry after Deregulation

The United States Airline Deregulation Act of 1978 was a dramatic event in the history of economic policy. It was the first thorough dismantling of a comprehensive system of government control since the Supreme Court declared the National Recovery Act unconstitutional in 1935.

Alfred Kahn

Deregulation refers to the Airline Deregulation Act of 1978 (92 Stat. 1705). This law amended the historic Federal Aviation Act of 1958 in the area of economic regulation of airlines by the federal government (NewMyer, 1990). The deregulation legislation was the result of several years of industry, academicians', and congressional debate over the role that the CAB played in the economic regulation of the airline industry. The CAB's primary role was to protect the industry, through government intervention and regulation. This protection included various CAB actions which limited competition among airlines, including pricing controls, route controls, and caps on the formation of new airline companies. By the mid-1970s, the aforesaid restrictions seemed inappropriate to most industry observers, consumer advocates, and politicians. The resulting Airline Deregulation Act of 1978 was signed into law by President Jimmy Carter with an effective date of October 24, 1978. The following provisions were included:

1. Eventual elimination of the Civil Aeronautics Board.
2. Decontrol of domestic airline air fares and routes.
3. A 180-day limit on the response of the CAB to almost any issue (including requests for the economic certification of new airlines).
4. An “essential air service” program to provide federally subsidized air service to rural areas.
5. Labor Protection Provisions for the purpose of providing job protection for employees displaced by deregulations.

6. Changes in the final approval process for airline mergers (Wolfe and NewMyer, 1985).

Airline deregulation opened the airline industry to intense competition since there was no longer any control over the prices or routes airlines could fly. The birth of low-cost carriers took place during the post-airline deregulation era. Along with the entry of new low-cost carriers came the foray of revenue management into the airline industry and the development of hub-and-spoke (HS) airline networks. Managing costs and focusing on revenue maximization became the primary focus of all the airlines. This proved extremely costly for the legacy carriers since they could not cope with all their legacy costs. A previously stable industry suddenly looked extremely unstable due to the sudden volume of changes being introduced.

The market share controlled by the top four major carriers in the United States began to become more concentrated toward the mid-2000s as a result of mergers and bankruptcies. This can be clearly seen from the data in Figure 1.1.⁴

The subsequent rise in market share of the top four carriers (Southwest, Delta, United, and American Airlines) is attributed to the fact that all the legacy carriers have adapted to the idea of revenue management, and also to the fact that many of the top carriers were involved in mergers as a result of possible bankruptcies. One of the low-cost carriers, Southwest, is currently one of the top four market shareholders which is testament to the impact deregulation has had on the airline industry in the United States. A decade ago, passengers could choose from a variety of large and medium-sized carriers; today the choices are much more limited. With fewer competitors, there is a higher probability that the air fares will go up and passenger services slide.

Among those in the supply chain, only the airline industry has found it difficult to generate an adequate level of profits. Financially, the return on capital for investors in the airline industry is extremely poor. For years airlines were generating returns far below their cost of capital, even though they had enjoyed a steady growth since 2012. However, the situation has recently improved. Thanks to increased load factors,

Figure 1.1 Top Four US Domestic Carriers' Market Share

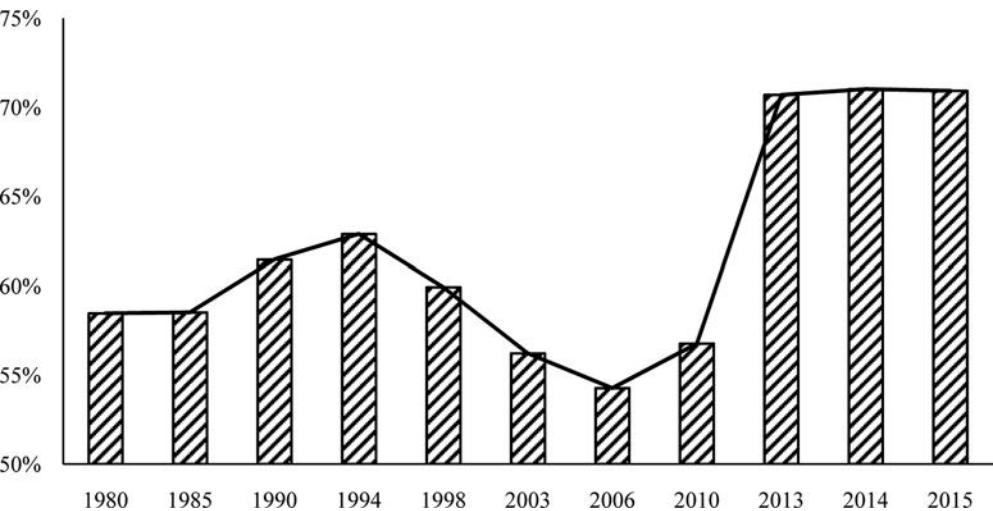
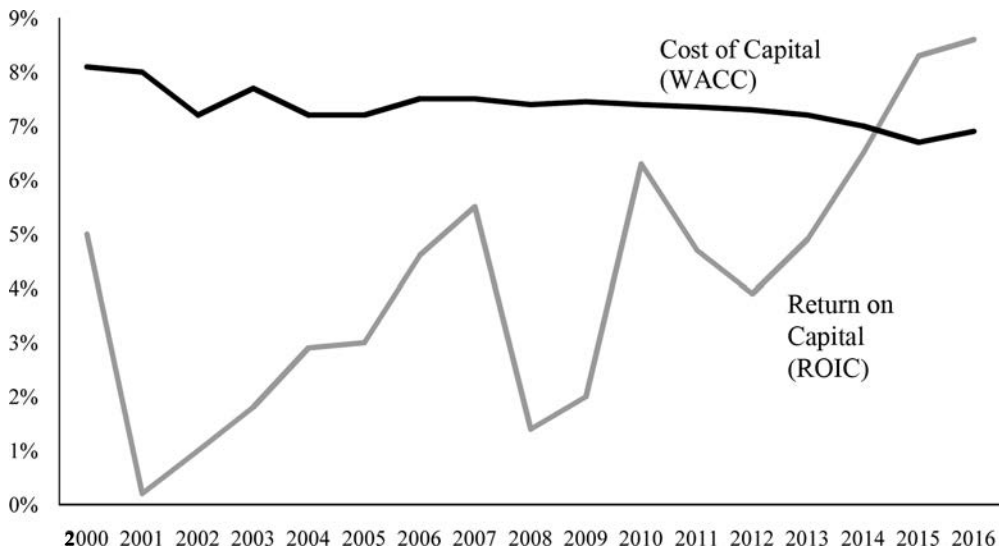


Figure 1.2 Return on Capital Invested in Airlines

falling fuel prices, consolidation and growing ancillary revenues, airlines earned above weighted average cost of capital (WACC) returns in the last couple of years and fetched 8.3 percent return on invested capital (ROIC) in 2015, which did, for the first time, adequately reward equity owners. In the year 2016, this return is expected to be 8.6 percent, which is 1.7 percent above WACC.⁵ The details can be seen in Figure 1.2.⁶

Air transport managers are routinely faced with the challenge of making choices when selecting one decision over another. That is why engineering economics can be applied by the management to improve productivity, efficiency, and profitability.

However, in the US this was not the case prior to the Airline Deregulation Act of 1978. Before the act was enforced, the federal government was responsible for assigning routes, minimizing competition, and setting fares, ensuring that the airlines remained profitable. Airline Regulation proved to be highly rigid and passengers were being forced to pay higher fares particularly in the early 1970s with the oil crisis of 1973. The Airline Deregulation partially transferred control over air travel from the governmental to the market segment. Post deregulation, airlines were forced to reduce cost, increase revenue and remain competitive. The deregulation of 1978 resulted in the infusion of a number of start-up carriers competing with legacy airlines for passengers and markets. The low costs offered by the start-up airlines such as People Express led to the fall of previously major carriers like Continental and Northwest by forcing them into bankruptcy. An imminent effect of the bankruptcies is airline consolidation, with many of the major airlines merging to form powerful coalitions. Some of the popular mergers were Delta Air Lines and Northwest Airlines, and America West and US Airways which in turn merged with American Airlines. The airline industry in the US has now condensed into just four major carriers and numerous low-cost carriers.

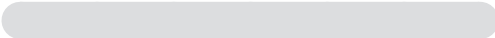
Deregulation in the US was quickly followed by other European and Asian countries doing the same within the span of a few years. A crucial step in Europe was also the privatization of airlines and airports as part of the deregulation. Air transport privatization and deregulation was pioneered by Great Britain under the Thatcher Government with its privatization of British Airlines and the British Airports Authority in 1987. Privatization was intended to provide impetus for airlines and airports to be operated more efficiently and competitively.

Managing risks is a strategic challenge for the air transport industry. One of the other influences airline deregulation had on the aviation industry as a whole was the increase in number of aircraft leased as opposed to being bought, which was not the case prior to deregulation. As of 2015, as many as 40 percent of the aircraft around the world are leased and that number is expected to continue to rise.

Airline Profitability

Let's keep things in perspective. Apple, a single company, earned \$13.6 billion in the second quarter of this year. That's just under half the expected full-year profit of the entire airline industry. We don't begrudge anyone their business success. But it is important for our stakeholders, particularly governments, to understand that the business of providing global connectivity is still a very tough one.

Tony Tyler, Director General and CEO, IATA



According to the International Air Transport Association (IATA), the industry's return on capital is expected to exceed the industry's cost of capital by 1.7 percent, with total net profits of \$36.3 billion, in 2016.⁷ This is a good news story, and the lower oil prices are giving airlines significant cost advantages. On proposed revenues of \$717 billion, the industry is expected to achieve a 5.1 percent net profit margin in the year 2016.⁷ This significant strengthening reflects the net impact of several global factors including stronger global economic prospects, record load factors, lower fuel prices, and greater ancillary revenue. As a result, unbundling ticket prices is becoming an indispensable part of airlines' revenue stream today. Airline ancillary revenue was projected to be \$59.2 billion worldwide in 2015. The top four US airlines saved a combined \$11.1 billion in fuel costs in the first nine months of 2015.⁸

Over half the global net profit is expected to be generated by airlines based in North America (\$19.2 billion). "For North American airlines, the margin on earnings before interest and taxation (EBIT) is expected to exceed 12%, more than double that of the next best performing regions of Asia-Pacific and Europe."⁹ According to IATA, the industry made \$9.31 profit per passenger on average in 2015, and is expected to earn \$9.59 per passenger in 2016.¹⁰ Profitability varies greatly from one region to another. The most profitable airlines are in North America, and the least belong to African carriers. The details on airline performance by region and industry as a whole can be found in Figure 1.3.¹⁰

Figure 1.3 Airlines Profit per Passenger

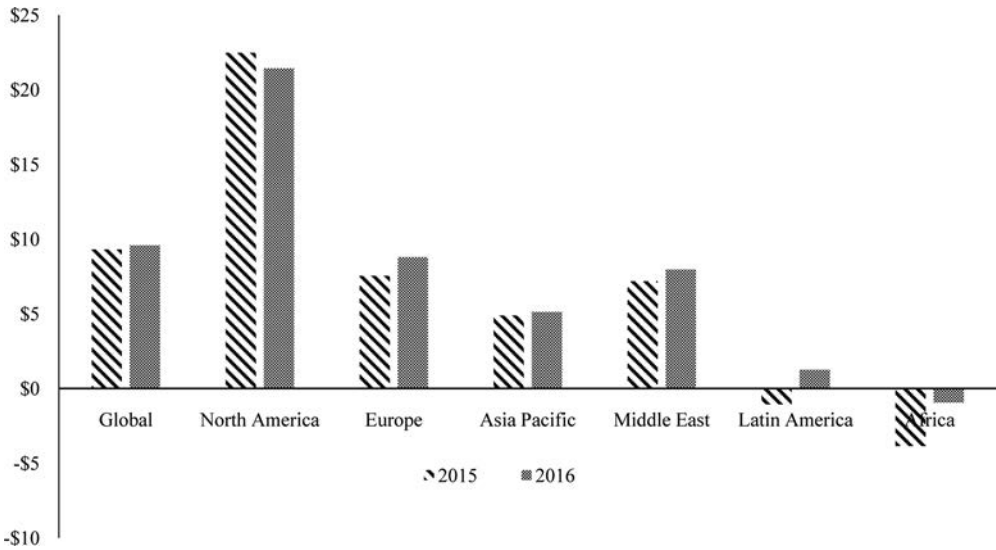


Figure 1.4 World Airline Profitability over Last Two Decades

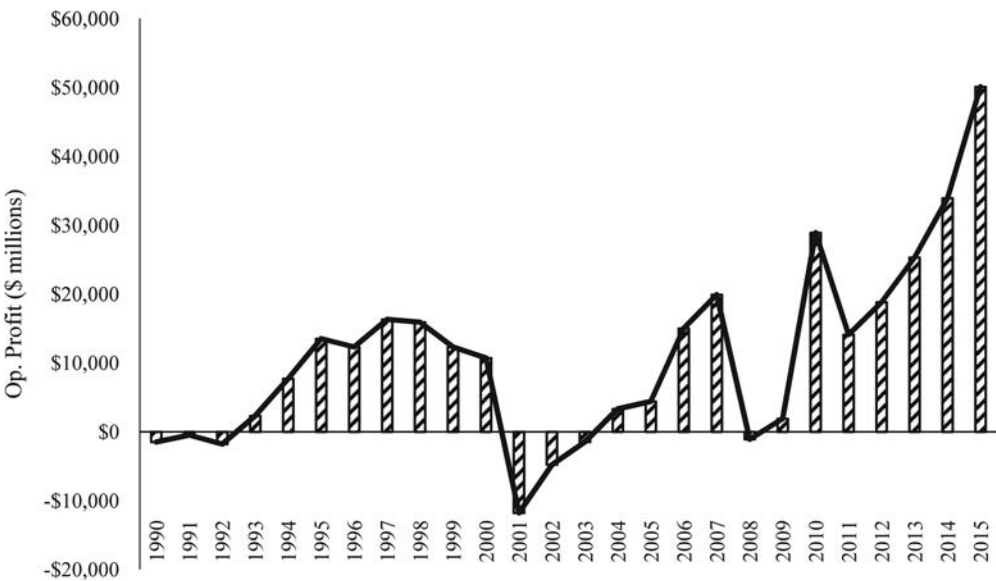


Figure 1.4¹¹ presents the combined operating profits of all the airlines in the world since 1990. It can be noted that the operating profits follow a cyclical pattern to a large extent. In the last two decades the two most significant events which impacted the profitability were the 9/11 attacks and the economic meltdown of 2008; although there has been an excellent recovery post the economic meltdown and the industry is only expected to grow further in the coming years.

The global airline industry has transformed itself with major gains in efficiency. This is clearly evident in the record high passenger load factor of 85 percent for the year 2015.¹² The result is a hard-earned 4.6 percent average net profit margin.¹³ On average, airlines retained \$9.31 for every passenger carried in 2015. Much of this growth has been driven by low-cost carriers (LCCs), which control some 25 percent of the worldwide market as of 2015 and which have been expanding rapidly in emerging markets; and some of the growth also came from continued gains by carriers in developed markets.

Post 2010, and especially in the last years, airlines have enjoyed massive amounts of profits which can be clearly seen in Figure 1.4. For example, American Airlines reported a profit of \$6.3 billion (that is a 50 percent change over the last year) with a record pretax profit margin of 15 percent by the end of 2015. Some of the main reasons to which such profits can be attributed are:

- ♦ Consistent drop in fuel prices
- ♦ Lower labor costs
- ♦ Airline bankruptcies
- ♦ Airline consolidation

In the case of American Airlines, record profits were achieved primarily because the airline's biggest expense—fuel—had dropped significantly. By the end of 2015, fuel costs on mainline operations dropped 41 percent in total compared to last year. Fuel costs as a percentage of total operating expenses (mainline operations) were 33.2 percent in 2014 and 21.6 percent in 2015.

Although the current state of the airline industry is looking bright, airline economists are unanimously worried about its future and about the sustainability of the current levels of profit. The main areas of concern for the economists are:

- ♦ Expectation of a rise in fuel prices as opposed to the current trend of falling prices.
- ♦ Expectation of an increase in labor costs as a result of:
 - Higher demand for pilots and flight attendants.
 - The large number of pilot retirements forecast for the near future.
 - Attrition, which refers to the act of reduction in staff and employees in a company through normal means, such as retirement and resignation, but also involving not filling the vacant position. This removal of a position entirely increases the pressure on current employees and could lead to them claiming a pay rise, thereby increasing labor costs.

Air Transport Industry

Airlines have moved away from bundling ticket options by opening new revenue streams for the sale of individual ancillaries. Not only do these retail models bring new ways for airlines to be competitive and market their brands, they're improving the passenger

experience by letting customers personalize and upgrade in-flight services and order them à la carte.

Marisa Garcia, Apex, November, 2015

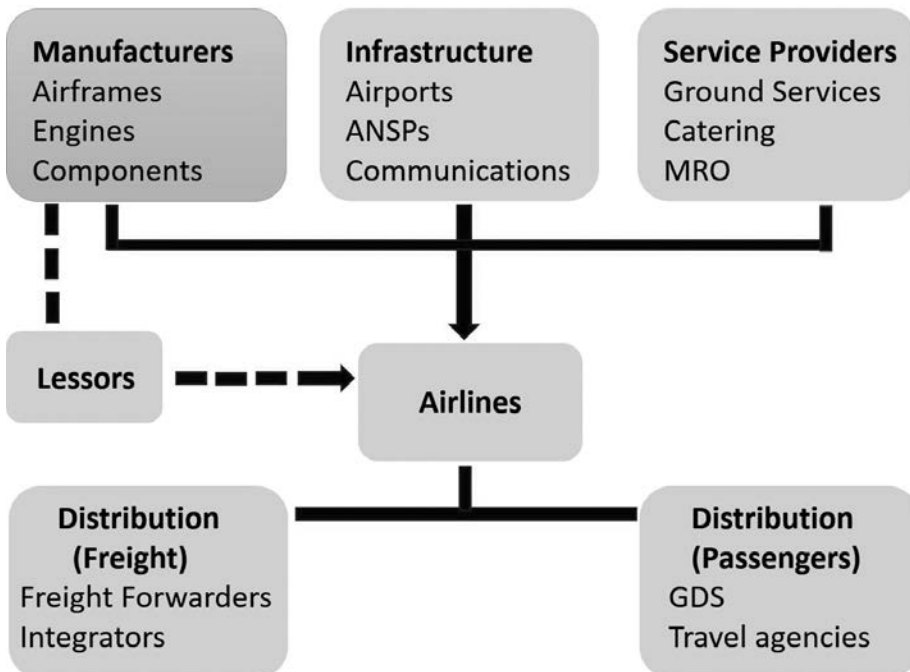
The transport industry has evolved over the last 40 years. Technological changes and productivity growth have spurred the all-time high number of passengers of 3.64 billion in 2014.¹⁴ In the year 2016 this number is expected to rise up to 3.8 billion passengers traveling over some 54,000 routes.¹⁵ The industry has transformed into a comprehensive dynamic environment with a large number of participants involved in complicated decision-making processes and continuously increasing complexity of operations.

The commercial aviation value chain represents a network of interlinked entities with the airline industry being the central node, as can be observed from the Figure 1.5.¹⁶ It can be subdivided into the upstream flow of suppliers, and downstream flow of distributors.

The upstream sector of the aviation value chain consists of:

- ◆ Manufacturers (producers of airframes, engines, parts, and components)
- ◆ Infrastructure providers (airports, air navigation service, and communication providers)

Figure 1.5 Aviation Value Chain



Source: (Tirethway and Markhvida, 2013)

- ♦ Service providers (catering, MRO, ground handlers)
- ♦ Aircraft leasing companies

The downstream sector of the aviation value chain consists of:

- ♦ Distribution of freight (freight forwarders and integrators such as UPS, FedEx, and DHL)
- ♦ Distribution of passengers (distribution of airline products through various global distribution systems [GDSs], travel agencies, and tour operators) (Tretheway and Markhvida, 2013)

All of these aviation value chain participants are discussed further in this chapter.

The introduction of Very Large Body (VLB) aircraft like Boeing B747 and Airbus A380 has increased passenger-carrying capability significantly. On the manufacturing side, the industry consists of just a few aircraft and jet engine manufacturers.

Aircraft Manufacturers

The commercial aircraft industry requires a very large investment to develop and manufacture aircraft, avionics, and engines (Vasigh et Al., 2012). Since the dawn of aviation, many aircraft manufacturing companies have gone out of business or merged with other peers. In Europe, Junkers was a major German aircraft manufacturer. Junkers manufactured some of the best-known aircraft over the course of its 50-plus years in business, including commercial aircraft (F-13), helicopters, and military jets. A total of 322 F-13s were produced, mainly between 1920 and 1925, and most went into service in Germany and Russia (Kay and Couper, 2004). Junkers built the first all-metal airplane using a tubular structure protected with corrugated sheet iron. In 1969 Junkers was acquired by Messerschmitt-Bölkow-Blohm, a German aerospace company, and the Junkers name disappeared.

The aircraft manufacturing industry has been dominated by two main entities, namely, Boeing and Airbus. After World War II, four prominent players emerged in the business of building commercial jets: Boeing, McDonnell, Douglas, and Lockheed. Douglas manufactured commercial aircraft that could also be used as military aircraft (DC-6). McDonnell Aircraft and the Douglas Aircraft Company merged in 1967.¹⁷ McDonnell Douglas produced a number of well-known commercial and military aircraft such as the MD-80, MD-90, and DC-10 airliner, and F-15 Eagle air-superiority fighter jets. In 1982, McDonnell Douglas decided that all new aircraft models would have the designation MD. In 1996, McDonnell Douglas announced plans to build a 400-seat jumbo jet to

Aviation Snippets

An 'aerospace cluster' is where an aircraft manufacturer sets up shop and then acts as a magnet for parts suppliers, logistics and distribution companies, research organizations, design companies, training establishments, maintenance and support organizations and hundreds of related businesses—pulling in millions of dollars in investment from around the world and generating thousands of well-paid jobs. The world's first and largest aerospace cluster (started by Boeing) is located in Washington State, in the northwest of the USA.

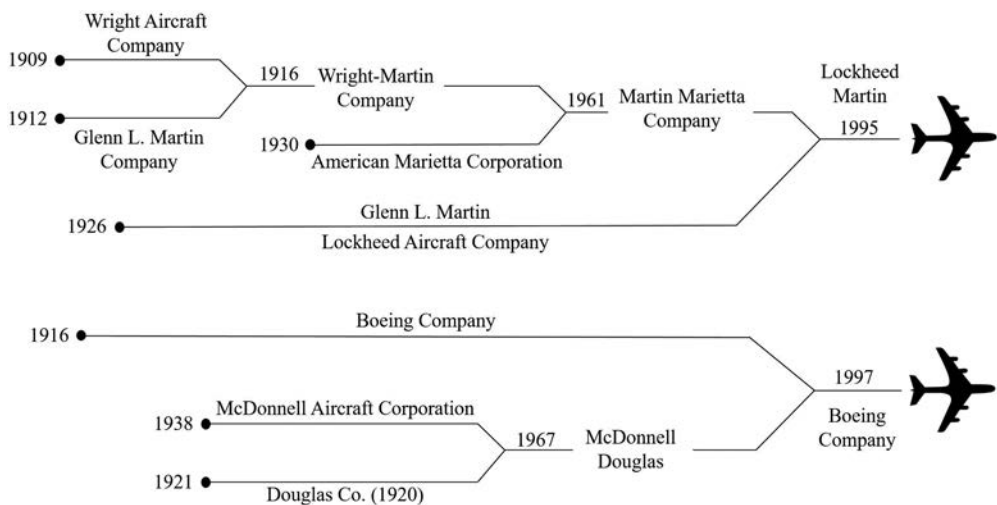
challenge the monopoly status of Boeing's 747 in that category. The MD-11, built from 1990 until 2001, was the last commercial aircraft of McDonnell Douglas during the 1990s before the company was acquired by Boeing (Gray, 1996), marking the end of the McDonnell Douglas commercial aircraft production.¹⁸

A wide assortment of commercial aircraft was produced by Lockheed including Electra, Super Constellation, and L-1011 TriStar. Lockheed lost \$2.5 billion on the L-1011 program between 1968 and 1984, by producing a total of 250 aircraft.¹⁹ The collapse of the L-1011 program led to Lockheed's exit from the commercial aircraft manufacturing business, at which point Lockheed decided to focus exclusively on military aircraft (Sandler and Hartley, 2007). The Boeing Company was founded by William Boeing in 1916. Boeing's real start into the commercial aviation business came through the mail. The US Postal Service awarded Boeing a contract to fly the mail between San Francisco and Chicago in 1927 (Vasigh et al., 2012). Boeing developed the Model 40A aircraft to fly this route to carry mail. The growth of Boeing has been immense since then and the only major competition Boeing currently faces is that from Airbus.

Boeing's announcement of a planned \$13 billion merger with McDonnell Douglas in 1996 raised major concerns about the market power of the combined firm among industry observers (Tucker, 1996).

Boeing started to design passenger aircraft throughout the late 1920s and early 1930s, progressively building aircraft that could hold more passengers and cargo. In 1952, Boeing began to build a prototype of the 707, or Dash 80 for short. The Dash 80 became Boeing's first commercial jetliner with both military and government applications. A total of 916 Boeing 707s were delivered to customers from 1958 to 1982 (Eden, 2008). Before the introduction of the 707, Douglas and Lockheed aircraft were preferred by US airlines. That all changed after the 707 became the first US manufactured jetliner and Boeing became the premier aircraft manufacturer for

Figure 1.6 US Commercial Aircraft Manufacturer Mergers²⁰



Source: Compiled by the authors from Boeing and Lockheed-Martin data

Adapted from Aircraft Finance: Strategies for Managing Costs in a Turbulent Industry (Vasigh et al., 2012)

airlines around the world. In addition to the 707 itself, Boeing designed and produced the 727, 737, 747, 757, 767, 777, and 787. The latest Boeing aircraft that entered into the market was the 787 (The Dreamliner) in 2011.

Airbus was formed as a consortium of smaller aircraft manufacturers in mainly Germany and France. It was founded in 1970 but was operating more as an alliance until the owners decided to create the European Aeronautic Defence and Space Company (EADS) in 2000. EADS was just recently rebranded to the more commonly known subdivision Airbus.²¹ With the delivery of the A380, Airbus is currently the manufacturer of the largest commercially used airliners. However, this specific aircraft has been below sales target. Especially the political structure of the company, with ownership and production in several countries, often leads to debate when it comes to economic downturns and possible shutdowns of facilities.

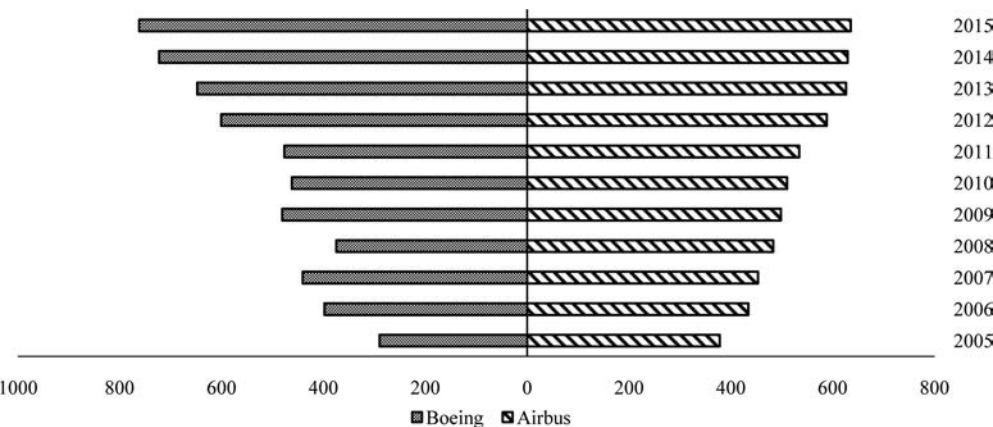
The first Airbus aircraft was the A300, a twin engine with its first flight in 1972. Today, Airbus is producing its bestseller, the A320 family (A318, A319, A320, A321) which is also available with a new engine option (the A320neo). In the wide-body segment, Airbus is still offering the A330, its new A350 with composite material and its flagship, the A380. With the A380, Airbus has created the largest commercially used aircraft which, in a full-economy seating, would fit more than 850 people. Sales of the A380 have been slower than expected with only Emirates ordering a significant number.²²

Today, the market for civil aircraft is almost evenly split between these two giants in aircraft manufacturing. Airbus has passed the milestone of more than 1,000 aircraft booked in 2015 to date, with 1,007 net orders. Boeing secured its position as the top airplane maker in the world for another year, surpassing Airbus in 2015 by delivering 762 jets last year, that is 39 more aircraft than the previous year.²³

In 2014, Boeing delivered a total of 723 jets compared to Airbus's 629.

Figure 1.7 shows the growth of both Airbus and Boeing over the past decade based on the number of overall deliveries made by the two manufacturers. Another important aspect, as mentioned above, is the even growth enjoyed by both companies, making the aircraft manufacturing industry an almost perfect duopoly.

Figure 1.7 Airbus vs Boeing: Number of Deliveries (2005–2015)



Due to the impressive performance of the A300B4 model, the A300 and A310 became the first family of twin-engine aircraft to become ETOPS (Extended-Range Twin Operations)-certified.²⁴

Regional Aircraft Manufacturers

In recent times, smaller companies such as Bombardier and Embraer have started to encroach on the market share that Boeing and Airbus enjoyed in the past. Embraer is the world's third largest commercial aircraft manufacturer after Boeing and Airbus. The advent of Bombardier and Embraer into the 140 seat range of aircraft has proved to be crucial in entering the narrow-body market. The number of deliveries made by these two relatively small manufactures can be seen in Figure 1.8. Their gradual entry into the narrow-body aircraft market can be further projected into the future, based on the volume of orders received by these manufacturers. Bombardier delivered 84 commercial aircraft worldwide in 2014 and 76 commercial aircraft in 2015.²⁵ Bombardier has been focusing on developing larger aircraft to compete against the 737 and A320 series. Embraer delivered 92 commercial aircraft in 2014 and 101 commercial aircraft in 2015.²⁶

It can be seen from Figure 1.8²⁷ that the existing duopoly is under serious threat in one particular segment of the manufacturing market, the narrow-body aircraft. Since Europe and North America have been dominating the aircraft manufacturing market, several countries have started initiatives to create their own manufacturer to increase independence. Bombardier (CRJ series and C series) and Embraer (ERJ 145 series) have already managed to capture considerable market share and further entrants into the market are expected in the near future, such as Mitsubishi Aircraft Corporation with their Mitsubishi Regional Jet (MRJ), Sukhoi Superjet (SSJ-100), and Commercial Aircraft Corporation of China (Comac).

The MRJ has secured 223 firm orders, with the biggest single order from US regional airline group SkyWest. The MRJ is planned for delivery in 2017 to Japan's ANA airlines.

Additionally, the SSJ-100 is a new Russian aircraft model, featuring fully electronic fly-by-wire control system, with a strong focus on engine maintainability.

Figure 1.8 Bombardier vs Embraer: Number of Commercial Aviation Deliveries (2005–2015)

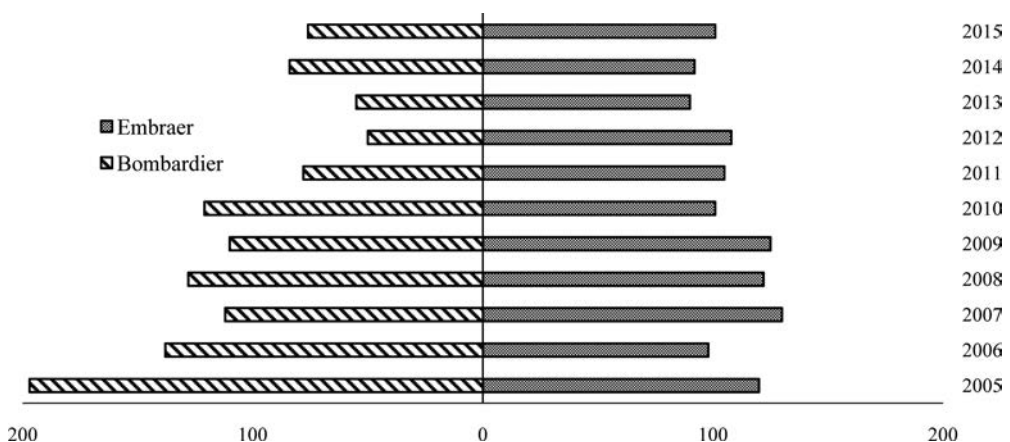
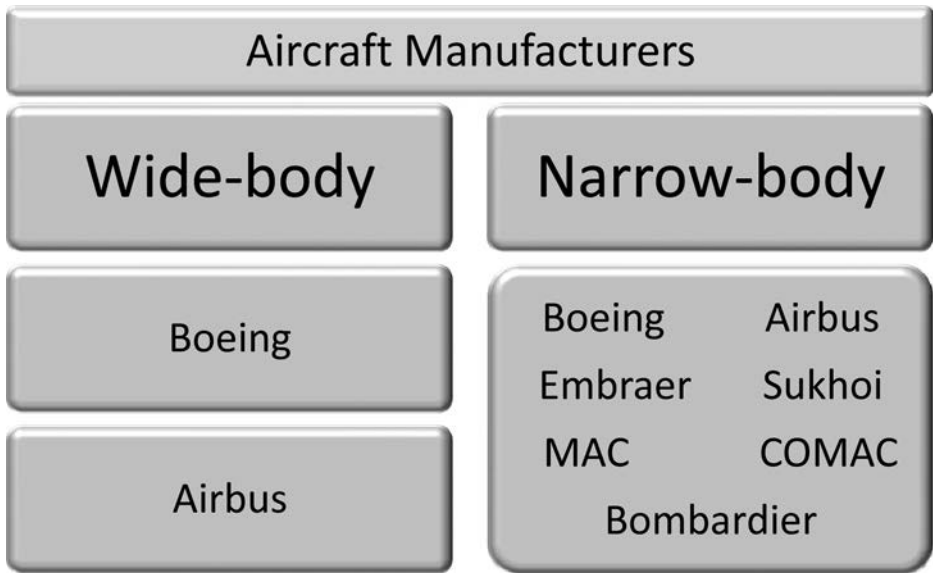


Figure 1.9 Aircraft Manufacturing Industry Outlook



The first two SSJ-100 entered service with a Mexican airline in 2013. The Russian company is also targeting the Pacific Rim, Africa, Latin America, and Australia for sales of its jets.

The Chinese Comac offers the C919, a short to medium range commercial aircraft, and a turbofan short to medium range regional (ARJ-21), with which they are competing with the A320 and the B737. The company has received 517 orders for the C919 from domestic and international airlines.

Mitsubishi, a Japanese company, has also recently entered the market and is producing the Mitsubishi Regional Jet (MRJ). The aircraft is a twin-engine regional jet with a seating capacity of 78–90 passengers. Mitsubishi Aircraft has received orders for a total of 407 units of the MRJ from six overseas and domestic airlines including All Nippon Airways and Japan Airlines.

The company had scheduled to deliver the first MRJ in April to June 2017, but recently revealed that it was reviewing the entire development plan following the commencement of flight-testing. Consequently, the first delivery of the MRJ has been delayed to mid-2018.

Jet Engine Manufacturers

It wasn't until the jet engine came into being and that engine was coupled with special airplane designs — such as the swept wing — that airplanes finally achieved a high enough work capability, efficiency and comfort level to allow air transportation to really take off.

Joseph F. Sutter, Boeing Commercial Airplanes

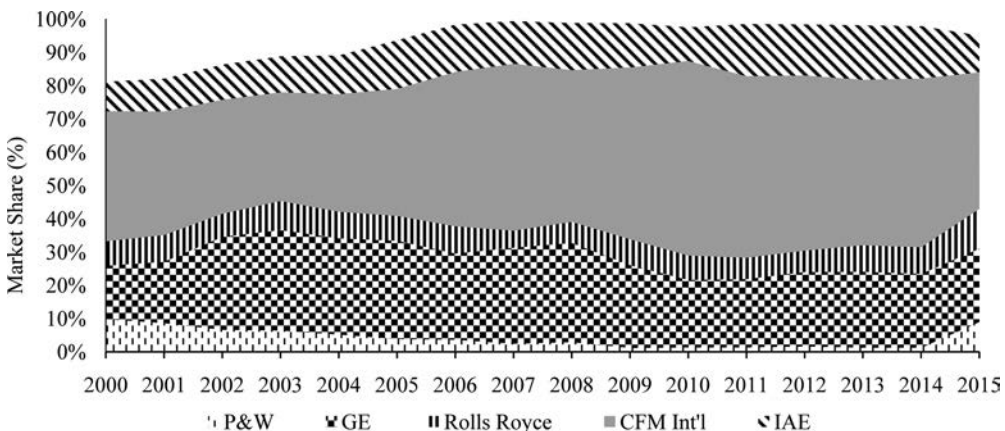
The world jet engine market is dominated by four gargantuan manufacturers producing engines for commercial airlines around the world. The development of new engines requires a huge initial investment in labor, capital, equipment, and technology. Jet engine manufacturers have made significant improvements in fuel efficiency, reducing the cost of fuel for the aircraft operators in the recent decades. Engines constitute a significant part of the total cost of aircraft ownership and operation. Advances in aircraft engines have contributed to higher fuel efficiency in the commercial fleet. Since 1980, the average fuel burn per aircraft seat mile has been reduced significantly. Consequently, the complexity and the technological advances have led to the formation of a large, consolidated, oligopolistic industry with the following firms:

- ◆ General Electric (GE)
- ◆ Rolls-Royce
- ◆ CFM International (CFMI)
- ◆ International Aero Engines (IAE, mainly Pratt & Whitney)

CFM International is a partnership between General Electric and Snecma, the French state-owned engine manufacturer. International Aero Engines was formed in 1983 between Pratt & Whitney (P&W), Rolls-Royce, Daimler-Benz, Fiat, and Japan Aero Engines. Rolls-Royce sold all its shares in IAE to Pratt & Whitney in 2012. CFM has generally been the provider of engines for narrow-body aircraft. P&W, meanwhile, has a smaller presence in the commercial aircraft market but is a big player in the military contracting field. The consortiums created an interesting situation where manufacturers could be both partners and competitors concurrently.

Figure 1.10²⁸ displays the worldwide market share for commercial jet engines in terms of deliveries for the past 15 years. The growing influence of consortiums such as CFM International and IAE can be clearly seen from the graph. CFM International has the largest market share, showing that the influence of GE and its other members has been significant. Also, the diminishing market share of Pratt & Whitney indicates that P&W has focused more of its resources toward IAE, resulting in the increasing market share for IAE.

Figure 1.10 Jet Engine Manufacturers' Market Share



Aircraft Leasing Companies

Aircraft leasing is becoming prominent in the aviation industry since many airlines are starting to appreciate the advantages of leasing, such as reduced financial risk and greater flexibility. Over half of the commercial aircraft coming into the market are leased, and smaller airlines lease an even higher percentage of their fleet. A lease is a contract by which the lessor grants the airline the right to use the aircraft in exchange for a series of lease payments. Major leasing companies are generally capable of providing maintenance services, financial, and technical services. AerCap and GE Capital Services (GECAS) are the two global leaders in aircraft leasing with a fleet of approximately 2,929 aircraft. Table 1.1 provides the total fleet value of the top 10 leasing companies:

Why do airlines lease their aircraft?

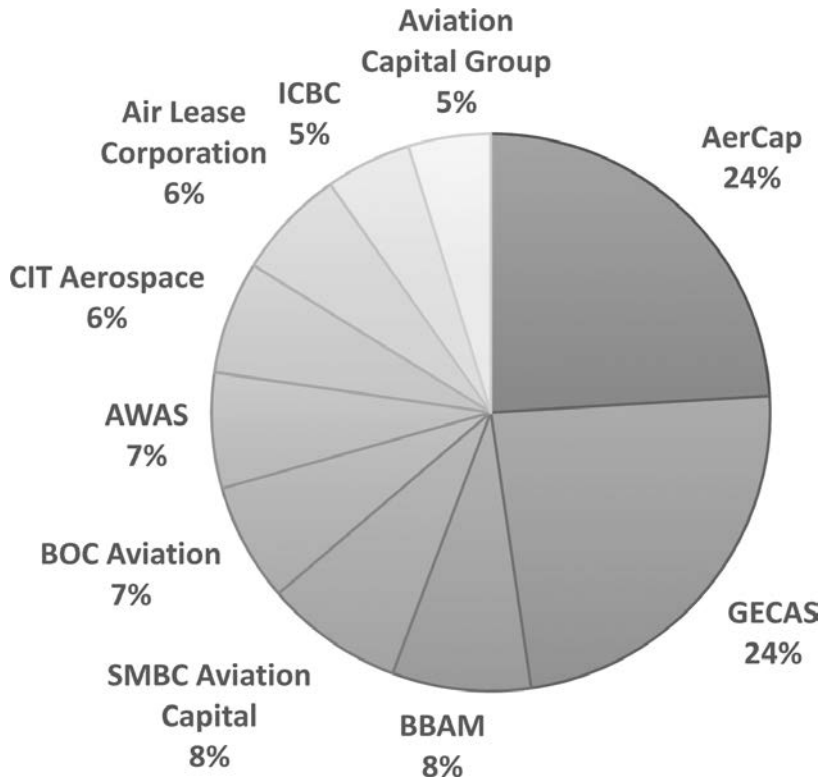
- ♦ Airlines are uncertain about the future demand
- ♦ Easier for the start-ups
- ♦ Leasing companies may provide service and maintenance on favorable terms
- ♦ Lessees and Lessors face different tax situations
- ♦ Conserving liquidity

Table 1.1 Top 10 Lessors by Fleet Value

Rank 2015	Leasing Company	Total Fleet Value (\$M)	Total Fleet
1	AerCap	33,105	1,305
2	GECAS	32,507	1,624
3	BBAM	11,060	292
4	SMBC Aviation Capital	11,037	386
5	BOC Aviation	9,445	246
6	AWAS	9,222	313
7	CIT Aerospace	9,004	312
8	Air Lease Corporation	8,806	235
9	ICBC	6,858	168
10	Aviation Capital Group	6,561	272

Source: Compiled by the authors from Flightglobal, 2015.

In 1973, Steven Háy co-founded the aircraft leasing business that became International Lease Finance Corporation (ILFC); he led the company from its start in 1973 until his departure in 2010.²⁹ In 2014, AerCap completed its acquisition of ILFC from American International Group (AIG). The transaction provided AIG \$3 billion in cash and 97.6 million AerCap shares (Johnsson and Tracer, 2014). The combined company is the largest aircraft leasing firm in the world by fleet value, with a fleet of over 1,305 aircraft (Table 1.1). GECAS is the second largest commercial aircraft and

Figure 1.11 Market Share of Top 10 Aircraft Leasing Companies Globally

engine lessor in the world by fleet value, and it has a fleet of 1,624 aircraft, used by 245 airlines around the world.

In 2010, S. Hazy, after terminating his relationship with ILFC, founded Air Lease Corporation (ALC). As of 2015, Air Lease reported that it owns 235 Airbus, Boeing, Embraer, and ATR aircraft (Table 1.1).

Service Providers (MRO, Ground Handling, Catering)

Aircraft maintenance has been crucial since flight's inception. As safety and airworthiness is the key to successful commercial aviation, Maintenance, Repair and Overhaul (MRO) services are the cornerstone of the industry. MRO services became commercially available in the early 1970's with the trend of airline technicians to do third party contractual repairs. As arguably the most highly regulated global industry, MRO services underscore the safety levels and confidence in the aviation industry.

Jet Aviation

Ground handling can be described as the services provided to aircraft, passengers, and cargo at an airport. It is an essential service, without which aircraft cannot depart,

and includes all the activities that take place around the aircraft and in the terminal building and/or cargo warehouse with regard to the load being carried on the aircraft.

Catering (procurement and service of in-flight and club meals) is another important service requirement in the airline industry and the quality of this service can be a significant factor in retaining customer loyalty and satisfaction.

Airports

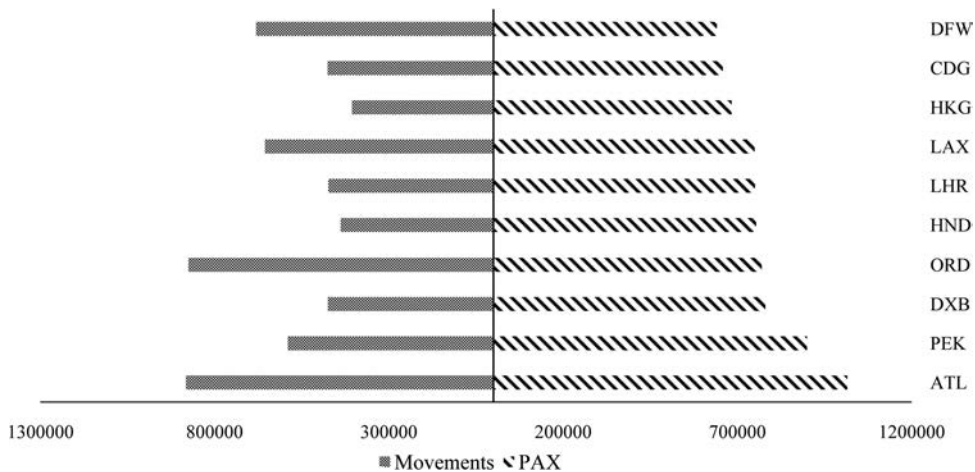
According to the Federal Aviation Administration (FAA), airports are categorized as commercial service, primary, cargo service, reliever, and general aviation. Generally, commercial airports are publicly owned airports that receive scheduled passenger service, and have at least 2,500 passenger enplanements annually.³⁰ Cargo airports are airports that are served by aircraft providing air transportation of only cargo with a total annual landed weight of more than 100 million pounds.³⁰ Airports designated by the FAA to relieve congestion at commercial service airports and to provide improved general aviation access to the overall community are called reliever airports. According to the Airports Council International (ACI) World Airport Traffic Report, there are 17,678 commercial airports in the world, and 41,788 civilian and military commercial airports around the world in total.³¹ This is an indication of the volume of passengers and the importance of proper connectivity around the world.

The trend toward airport privatization began in the United Kingdom with the Transport Act of 1981. The Act established the framework for privatizing British Airports by creating a holding company. The Thatcher administration privatized seven commercial airports and transferred the ownership to private companies via the British Airport Authority (BAA). The airports privatized are London Heathrow (LHR), Gatwick (LGW), Stanstead (STL), Southampton (SOU), Edinburgh Airport (EDI), Aberdeen Airport (ABZ), and Glasgow Airport (GLA).

Privatization of airports is ensuring that the airports function in an efficient manner since there is now adequate motivation for proper management of the airport. Although privatized airports have a few disadvantages, the process of privatization has helped airports earn a bigger role in the air transport industry value chain. Worldwide airport passenger numbers increased to over 6.7 billion in 2014. Figure 1.12³² presents the world's busiest airports of 2015. Atlanta (ATL) remains the world's busiest airport on the globe with 101.5 million passengers. With a rapidly growing economy, China's civil aviation industry enjoys significant growth. Beijing Airport (PEK) with 89.9 million passengers is ranked the second among the top 10 busiest airports, followed by Dubai International Airport with 78 million passengers as the third busiest airport around the world.³²

Aviation Snippets

Paris's three main airports—Charles de Gaulle, Orly and Le Bourget—are home to 1,000 different businesses employing 115,000 people. Helping businesses and local authorities benefit from growth at the airport is an integral part of the Aéroports de Paris (AdP) corporate social responsibility strategy.

Figure 1.12 Top 10 World Airports by Passenger Traffic (in 100s) and Total Aircraft Movements

Source: Compiled by the authors from several databases, 2016.

Air Navigation Service Providers (ANSP)

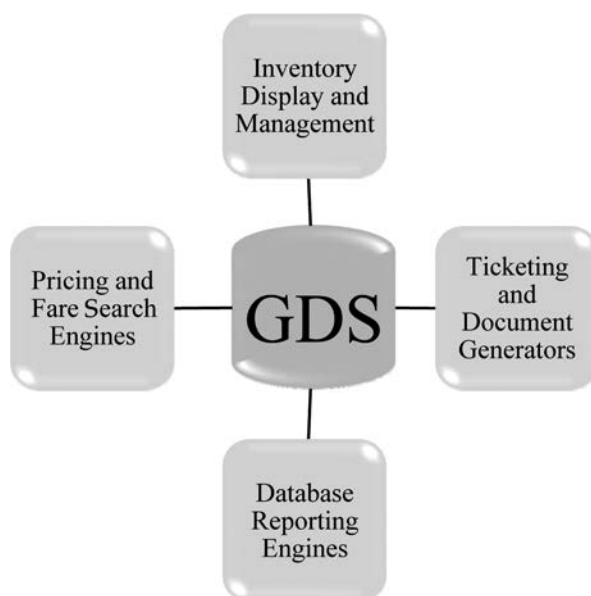
Air Navigation Service Providers (ANSPs) have historically operated as government departments, financed by taxes and charges. This structure reflected the role of ANSPs as a form of policing of the skies. In the late 1980s, under pressure from a radically expanding air transport industry, the Oceanic region started a more general trend in the sector with the commercialization of ANSPs in New Zealand and Australia in 1987 and 1988, respectively. Commercialization has intensified in particular in Europe, where a large majority of states now operate air traffic control (ATC) in a business-oriented way. However, Europe is still in the process of aligning the various, previously national, navigation centers into the European navigation system. Both Africa and Asia have witnessed a significant increase in commercialization. In the Americas, apart from Canada, the traditional governmental model has continued to prevail.

Global Distribution System (GDS) and Computer Reservation System (CRS)

GDS is a global computerized reservation network used as a single point of access for reserving airline seats, hotel rooms, and rental cars by reservation and travel agents. There are a number of GDSs that distribute airline schedules, seat availability by booking class, and other relevant booking class information worldwide. Individual airlines can use the GDS to see their own published information but more importantly to monitor their competition. The primary companies dominating the global distribution systems market are:

- ◆ Amadeus³³
- ◆ Sabre
- ◆ Travelport

Amadeus, Sabre, and Travelport are among the top global distribution systems available today. The airline reservation system, commonly known as ARS, is generally considered to be the user side system for any airline or GDS.³⁴

Figure 1.13 Global Distribution System (GDS)

GDSs have four integrated functional components (Figure 1.13).

Similar to the airline reservation system, the passenger name record is maintained in the GDS system. Reservation Systems Services are defined as services provided by computerized systems that contain information about air carriers' schedule, availability, fares and fares rules, and through which reservations can be made or tickets may be issued. Some airlines are not very enthusiastic advocates of GDS and would prefer to conduct direct sales with travelers; however, the overhead of maintaining such networks compels them to deal with these firms.

United Airlines was the first airline to install their in-house CRS, called Apollo, in travel agencies in 1976. American Airlines quickly followed suit and by the end of the year 130 reservation offices received the Sabre system.

European airlines began developing their own GDSs. These systems included Amadeus and Galileo³⁵ and were very similar to the systems developed in the United States. The GDS called Abacus

Aviation Snippets

After realizing how bookings via mobile devices were increasing rapidly in every travel sector, TAROM, Romania's national carrier, chose Amadeus for a mobile solution which offered excellent functional integration, a strong focus on usability and a seamless integration process.

Prior to its adoption of Amadeus Mobile Solutions, many of TAROM's customers were either unaware that they could make bookings on their mobile devices, or not overly confident of its capabilities. TAROM quickly created strong customer awareness of its mobile offering through a communications campaign featuring an attractive commercial "mobile-only" flights offer.

was developed by a consortium of Southeast Asian Airlines. TravelSky is the name of the GDS used in China. The concentration which took place in the CRS sector has left only four global players, three US-owned and US-based ones, i.e., Sabre, Worldspan, and Galileo, and a European one, Amadeus. Especially low-cost carriers avoid the GDS fees and only offer tickets through their own distribution channel. However, Ryanair, as the exemplary European LCC, also has recently started to offer bookings through GDS. Lufthansa started an initiative to reduce dependency on the reservation system providers by adding a GDS fee to all GDS bookings, making their own distribution network 8–16 Euros cheaper per ticket.

While a CRS is focused on the reservation process for the customer, the GDS takes a more global approach by managing and containing all relevant booking information, reservation details, and pricings. Being the main distribution channel for many airlines, GDS is a business model while the CRS is rather the technical possibility to book tickets.

An added component of the GDS or CRS is the network of Online Travel Agents who utilize these services on behalf of the different airlines and also manage to charge a fee for the service they provide.

Online Travel Agents (OTA)

Worldwide online travel sales are estimated to reach US\$590 billion, and global travel and tourism to reach \$2,840 billion, by 2017.³⁶ Online travel agencies like Expedia, Priceline, and TripAdvisor are generating huge revenue with gross reservations crossing the \$35 billion mark. Expedia, launched in 1996 by Microsoft and sold in 2001, has become one of the most successful electronic-commerce sites.³⁷ Expedia recently acquired Travelocity from Sabre Corporation for \$280 million.³⁸ In November 2014, Expedia completed its acquisition of Australia-based Wotif Group, and in September 2015 completed its acquisition of Orbitz Worldwide for \$1.6 billion.

Expedia, through brands such as Trivago, Hotels.com, and Hotwire, as well as its eponymous operation, earned \$39.4 billion in gross bookings in 2014. Expedia uses multiple global distribution systems like Amadeus and Sabre reservation systems for flights and for hotels, Worldspan and Pegasus, along with its own hotel reservations, and consolidators' reservations. The biggest travel companies globally include:

- ◆ Expedia
- ◆ Carlson Wagonlit
- ◆ Priceline
- ◆ TUI
- ◆ American Express

Expedia-owned Brands

- ◆ Expedia
- ◆ Expedia.com
- ◆ CarRentals.com
- ◆ Expedia Affiliate Network
- ◆ Expedia Local Expert
- ◆ HomeAway
- ◆ Hotels.com
- ◆ Hotwire Group
- ◆ Orbitz
- ◆ Travelocity
- ◆ Trivago
- ◆ Venere.com
- ◆ Wotif.com

Travel agents are companies/individuals that render services for passenger travel, such as advice and planning services, services related to arrangement of tours, accommodation, passenger and baggage transportation, and ticket issuance services. These services are provided on a fee or contract basis.

Freight Forwarders and Integrators

A freight forwarder is a person or a company that organizes shipments for individuals or corporations to get goods from the manufacturer or producer to the market, customer, or final point of distribution. A forwarder does not move the goods but acts as an expert in the logistics network. A forwarder contracts with carriers to move cargo ranging from raw agricultural products to manufactured goods.

Another freight distributor in the aviation value chain is an integrator, a company providing door-to-door delivery of freight and a whole range of services associated with that such as planning, tracking, shipping, clearance, complex supply chain management solutions, and other general and customized services. Some of the most well-known US integrators are DHL, UPS, and FedEx.

In the commercial aviation sector, just about every member in the aforementioned value chain enjoys a decent level of profits. One of the enduring ironies of the aviation industry is that the companies that actually move passengers from one place to another, the most crucial link in the chain, struggle to break even.

This comparative failure of airlines in the aviation industry value chain can be attributed to the following major factors:

- ♦ The bargaining power of suppliers is high, with powerful labor unions especially at hub operations, concentrated oligopolies in aircraft and engine manufacturing, local monopolies at airports and increasing concentration in the supply of services. For example, the aircraft manufacturing industry is currently a duopoly with Boeing and Airbus covering a majority of the industry. The consolidation of the aerospace industry in North America reached a climax on December 15, 1996 when the then Boeing CEO Phil Condit and McDonnell Douglas CEO Harry Stonecipher announced the two aerospace giants would merge in a \$13.3 billion stock-for-stock transaction. Airbus, on the other hand, was formed out of a consortium of four major manufacturers in Europe, making Airbus nearly a monopoly in the European market although many small manufacturers exist there as well. This duopoly among aircraft manufacturers ensures that the bargaining power with them remains high.
- ♦ The bargaining power of the GDSs is very high, since the three major players are insulated from competition by their market power. The three major GDSs in the industry currently are Travelport, Amadeus, and Sabre. Similar to aircraft manufacturers, this concentrated oligopoly that exists between the GDSs ensures a high bargaining power for them.
- ♦ Buyer bargaining power is also high, largely because of the perceived commoditization of air travel and low switching costs. This is an important factor since this is the reason airlines have a restriction on the possible revenue they could generate.
- ♦ The threat of substitute services is medium and rising, with improving technology for web-conferencing and competition from high speed rail on

- short haul markets, particularly in Europe where the rail infrastructure is more accessible and taking the train is more prevalent among people.
- ◆ The threat of new entrants is moderate, with easy entry into many markets, though considerable investment costs in the beginning, and with easy access to distribution channels and limited incumbency advantages.
 - ◆ Rivalry among existing competitors is high, partly because of the economics (high sunk costs per aircraft, low marginal cost per passenger, perishable product, and limited economies of scale), but also because of government constraints restricting consolidation through exit or cross-border merger.
 - ◆ Airports, which are another big component in the aviation industry value chain, play a vital role in restricting the possible profits of airlines since most major airports in the world are quasi-monopolistic, meaning, despite the existence of possible surrounding airports, some of the airports dominate the market in certain regions of the world. The best example to demonstrate this is London Heathrow Airport (LHR).

To remain in contention in such a competitive industry and to sustain the growth that they have experienced in recent times, airlines need to evolve further and try to incorporate decisive changes in some aspects of their operation.

Airlines could invest more resources in analyzing customer preferences which would in turn enable them to build a more solid customer base leading to increased market share and profits. Also, airlines could engage in digitization to reduce operating costs. For example, systems that can enable real-time resource planning and allocation drive greater utilization. Tech-enabled engines can notify maintenance and operations centers of performance issues while an aircraft is still in the air, and request that a replacement part be waiting when it lands. Such measures can reduce downtime and greatly improve performance, driving down costs while also bolstering passenger satisfaction through more frequent on-time arrivals and departures. Airline managers need to be ruthless in cutting costs in areas that are not relevant to safety, reputation, branding, or customer value. Finally, airlines should continue to seek partnerships that will be more targeted and synergistic than the traditional alliances that now dominate. Those alliances allow route sharing on a broad basis, but they aren't tailored narrowly enough to allow airlines to strategically fill in specific gaps. For example, Qantas, which was already part of the one-world alliance, recently forged such a targeted partnership with Emirates. Qantas did not have the traffic to fly profitably to multiple cities in Europe, although there was significant demand among its loyal customer base. Emirates, by contrast, had sufficient demand to access a large number of destinations in Europe, but it did not have a loyal base of local customers in Australia. The new partnership gave Qantas access to many more destinations in Europe, and let Emirates access an extremely loyal base of customers.

Airline Mergers and Bankruptcies

If a capitalist had been present at Kitty Hawk back in the early 1900s, he should have shot Orville Wright. He would have saved his progeny money. But seriously, the airline business has been extraordinary. It has eaten up capital over the past century like almost no other

business because people seem to keep coming back to it and putting fresh money in. You've got huge fixed costs, you've got strong labor unions and you've got commodity pricing. That is not a great recipe for success. I have an 800 (free call) number now that I call if I get the urge to buy an airline stock. I call at two in the morning and I say: "My name is Warren and I'm an aeroholic". And then they talk me down.

Warren Buffet

The airline industry is very competitive, susceptible to external shocks, and, as a result, fearfully turbulent. Since 1978, more than 250 new airlines have been started in the US, and nearly all of them failed. The terrorist attacks such as the recent events in Paris and Brussels may weaken near-term or potentially mid-term passenger demand. A lower oil price makes it more economical to operate smaller planes and serve more destinations. The current decline in the oil price has helped the airlines to achieve higher profits.

The air transport industry has advanced dramatically since its first flight in 1903, primarily improving after deregulation in 1978, which drastically altered the landscape and competitive environment for airlines. Delta Air Lines pioneered the hub-and-spoke model, which was subsequently replicated by the other major US carriers. While the legacy carriers initially benefited from deregulation, the hub-and-spoke model and new competition also exposed them to many drawbacks, namely their inability to adapt quickly to an ever-changing environment. Low-cost carriers took advantage of the inflexibility inherent in the model and adopted a point-to-point system which allows for greater versatility in a volatile market. The period between 2000 and 2010 was particularly rough for the industry, which faced steep declines in travel, spikes in fuel prices, and labor disputes, all culminating in a record 20 airlines filing for either Chapter 7 or Chapter 11 bankruptcy (Tolkin, 2010).

By filing for Chapter 11 protections, a company gets time to devise a new business plan to pay off its creditors. A bankruptcy judge can impose a new contract—and new wage cuts—on a union, approve the transfer of pension obligations to the federal government, and agree to eliminate some debts altogether. There are also a few disadvantages associated with bankruptcy protection. The high cost of bankruptcy is one of the biggest factors. The debtor must pay for its own legal advisers as well as for those of its creditors; the total cost of an airline bankruptcy can be hundreds of millions of dollars. If the company can't find enough cash to pay back

Aviation Snippets

Russia's second largest passenger carrier UTair scored another win in its efforts to achieve financial sustainability after federal authorities extended sovereign guarantees to help meet its obligations to creditors. Under a government decree, UTair was able to secure up to Rb9.5 billion (\$135 million) in borrowings to finance restructuring and current operational activities. This aid package covered half of an Rb19 billion syndicated loan that it has obtained from a pool of 11 domestic banks for a term of seven years.

Flightglobal, December 2015

loans and cover legal fees, it gets liquidated (terminated).

Bankruptcies are a way for capital to reorganize in a more efficient way. They do help to move the industry to a more efficient structure where consumers get good services at reasonable prices and airlines can actually make money. Although, airlines are never going to consistently make money year after year because it is an industry that is inherently volatile. A change in the structure of airline companies will result in airlines being able to more effectively, on average, make money.

The most common consequence of a bankruptcy in the airline industry is a “merger” or “consolidation” of two or more carriers in order to rescue operations of the firm facing bankruptcy. A clear indication of that trend is depicted in Figure 1.14 which gives an account of all the major airline mergers in the past 15 years which ultimately streamlined the major carriers in the US, from 10 airlines to four airlines.

Airline mergers definitely have a number of pros and cons associated with them. For a passenger, an airline merger could mean improved services and less congestion in airports due to reduced number of carriers. Although, mergers promote the sense of monopoly and this in turn could result in a price hike making it one of the biggest disadvantages to the passenger. From the airlines’ point of view, a merger would mean a large-scale expansion making the merged entity more competitive in markets not previously engaged. The drawback for the airlines arises from the process of merging two large companies and managing the employees and assets in a cost-effective and efficient manner. The merger of American Airlines and US Airways could be the last major merger in the domestic aviation sector in the US, since the Department of Justice (DOJ) would definitely consider any further mergers among the major carriers to be large-scale collusion and prevent them from happening. A monopoly market is definitely discouraged since that would grant ultimate power to the airlines, resulting in a drastic increase in prices and fall in levels of service.

In the United States, the four majors controlled 79 percent of their market, while Europe’s top four airlines controlled 47 percent capacity in 2014. In the year 2015 United States Department of Transportation reported that four major carriers accounted for 66 percent of the market share domestically.³⁹

Aviation Snippets

Alaska Air Group to acquire Virgin America, creating West Coast’s premier carrier

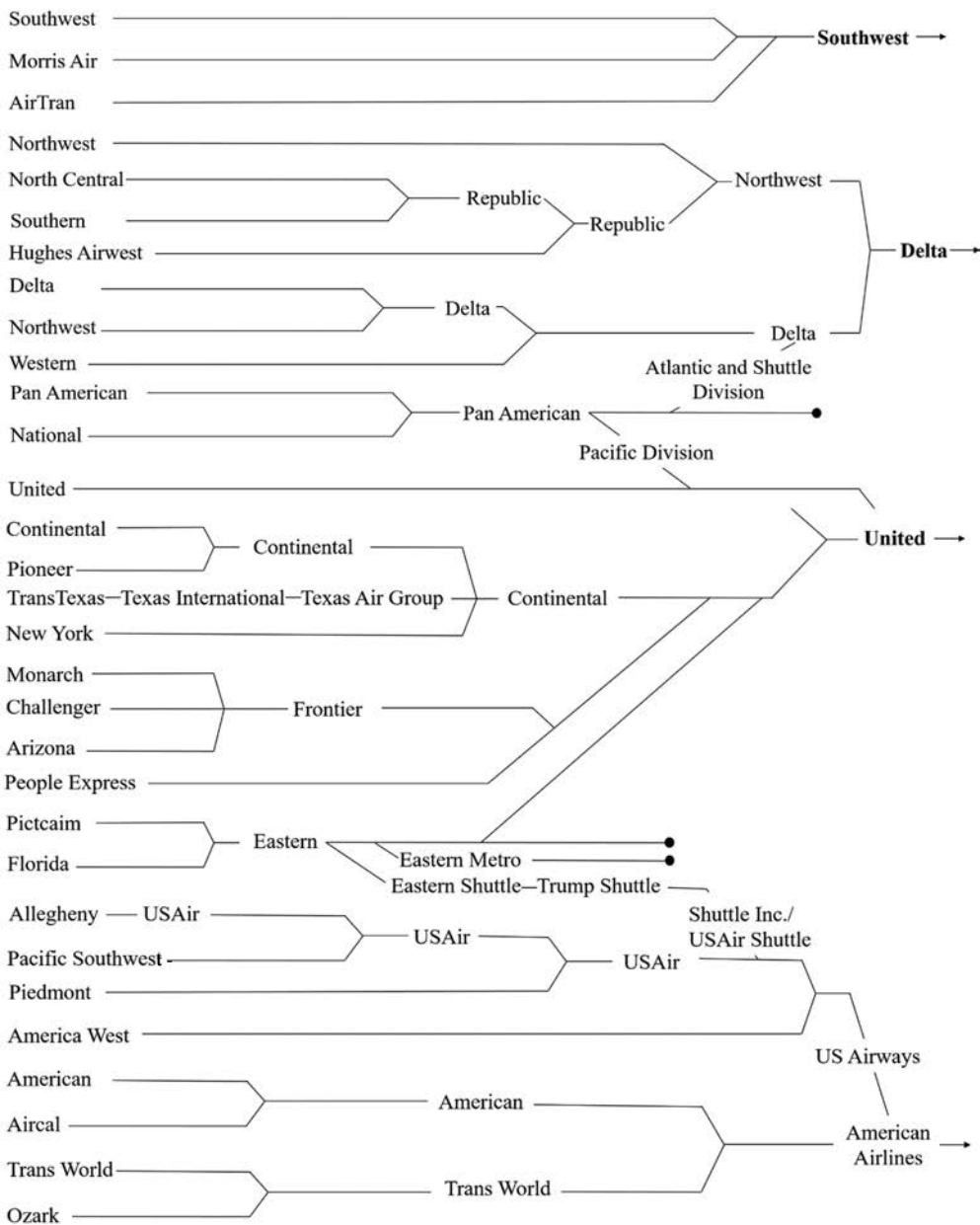
Alaska Air’s \$2.6 billion cash deal to buy rival Virgin America Inc. aims to wring \$225 million in annual net synergies once the two businesses are combined. The price amounts to a 51% premium to Virgin America’s share price a week before the deal’s announcement on Monday, according to Dealogic. By contrast, year to date, the average U.S. acquisition was an already expensive 37%. The combination is set to create the fifth-largest U.S. airline by traffic, dethroning JetBlue Airways Corp. from the spot.

Wall Street Journal, April 2016

Summary

This chapter provides a basic insight into the airline industry and its evolution. The Civil Aeronautics Board controlled a regulated industry prior to 1978 and this period

Figure 1.14 Evolution of US Airline Industry



Source: Adapted from Introduction to Airline Economics (Vasigh, Fleming, and Tacker, 2013).

in civil aviation was considered to be the most stable phase, since most of the airlines made profits and were able to sustain their respective markets. The lack in flexibility of prices and the lack of motivation to provide better service prompted the execution of the Deregulation Act of 1978. Post Deregulation, the airline industry became very unstable due to the entry of low-cost carriers forcing many major carriers to either change their current style of operation or leave the market. Many legacy carriers were impacted by the legacy costs incurred by them which the new entrants were not faced with. After a period of turbulence, the airline industry now seems to have perfectly understood the role of revenue management and cost management. While profits have fluctuated a great deal, the industry has enjoyed steady growth, falling prices, and market concentration. The growth of Southwest Airlines in the domestic US market is testament to the growth of low-cost carriers in the commercial aviation industry. All the airlines have been enjoying a good level of profits for the past four to five years. This is mainly attributed to the fall in fuel prices and the increase in global demand for air travel. Some of the consequences of the turbulent period in the airline industry were the numerous airline bankruptcies and some major airline mergers which happened as a result of some of those bankruptcies. Over the 40-year span, the industry has transformed into a comprehensive dynamic environment with a large number of participants involved in complicated decision-making processes and continuously increasing complexity of operations. Nowadays, the commercial aviation value chain represents a network of interlinked entities with the airline industry being the central node. It can be subdivided into the upstream segment consisting of various suppliers (manufacturers, infrastructure and service providers, and lessors), and downstream segment of freight and product distributors.

This book provides an overview of a variety of engineering economic subjects that commonly are practiced in the transport industry. Engineers play an increasingly important role in companies, as evidenced in aircraft and engine manufacturers. Engineering economics is closely associated with financial management with a focus on problem solving at operational levels.

Discussion Questions and Problems

- 1.1 What impact has deregulation had on commercial airline profitability?
- 1.2 What are the main purposes of airline Computer Reservation Systems (CRSs)?
- 1.3 Would you expect aircraft and jet engine manufacturers to have the same degree of control over price? Why?
- 1.4 What is the difference between airline CRS and GDS?
- 1.5 Discuss the pros and cons of airline consolidations.
- 1.6 Explain the main functions of Global Distribution Systems.
- 1.7 What is required for the airline industry to continue its profitability?
- 1.8 Explain what the major network airlines should do to minimize the competitive threat posed by the low-cost airlines.
- 1.9 Why does the airline industry continue to dominate the world transportation system?

- 1.10 What has been the impact of the introduction of wide-body aircraft on economic development in the world?
- 1.11 From an engineering point of view, how can airlines reduce the aircraft operating cost and increase revenue?
- 1.12 What is the economic rationale behind airline mergers?
- 1.13 Describe engineering economics and define its role in decision making.
- 1.14 Has the airline deregulation been good for the airline industry as a whole?

Notes

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- 9 IATA, *Airline Profitability Strengthens Further*, Press Release No. 26, published June 8, 2015.
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- 29 Company Annual Reports. 2010.
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- 31 Airports Council International (ACI), Annual Report, 2015.
- 32 Airports Council International (2016). ACI Releases Preliminary World Airport Traffic Rankings.
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- 35 Now owned by Travelport.
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2

Foundations of Engineering Economics and Finance

The immediate effect of the deficit is to make you feel good, like when you go on a trip and pay later. You feel good, and then you get a hangover. The deficit makes you feel good – until you pay later.

Franco Modigliani



Nowadays, most companies that hire engineers are expecting them not only to participate in the decision-making process by evaluating the projects' technical aspects but also to provide an economic assessment of projects of different sizes. Engineers are expected to perform cost analyses, compare alternatives, and make decisions by considering both the technical and the financial virtue of projects. *Engineering economics* equips engineers with a collection of skills and techniques that will allow them to analyze, evaluate, and compare alternatives on an economic basis. In practice, engineering economics involves decision making based on cash flows associated with a given project, life of the project, and interest rates. In this chapter, we discuss the fundamental concepts of engineering economics and introduce the basics of evaluating the financial merit of projects. We will make a distinction between simple and compound interest, calculate the future value of a single present amount, the present value of a future single sum, and the future and present values of streams of non-equal amounts. The concept of time value of the money will also be discussed:

- ◆ Definition of Engineering Economics
- ◆ Interest and Time Value of Money
 - Simple Interest
 - Compound Interest

- ◆ The Connection between Interest Rates and Their Determinations
 - Short-term Interest Rates
 - Discount Rate
 - Federal Fund Rate
 - The Prime Rate
 - London Interbank Offered Rate (LIBOR)
 - Treasury Bills (T-Bill)
 - Certificate of Deposit (CD)
 - Long-term Debt and Interest Rate
 - Treasury Notes
 - Treasury Bonds
 - Municipal Bonds (Munis)
 - Revenue Bonds
 - Corporate Bonds
- ◆ Future Value of a Single Present Amount
- ◆ Present Value of a Single Future Sum
- ◆ Future Value of an Unequal Cash Flow Series
- ◆ The Rule of 72
- ◆ The Rule of 100
- ◆ Summary
- ◆ Discussion Questions and Problems

Definition of Engineering Economics

Engineering economics is a study of the decision-making process to evaluate one or multiple projects and assess their economic viability based on factors such as cash flows, life of each project, and the interest rates. These comparisons can be accomplished using engineering economics tools and techniques that simplify the evaluation of alternatives. Therefore, engineering economics can be defined as *a science that equips engineers with a collection of skills, tools, and techniques to allow them to analyze, evaluate, and compare alternatives on an economic basis*. In real life, there are a number of other factors that impact the decision-making process. These factors may include: safety, health, and welfare of the public; environmental impacts as they affect the public; ethical issues related to how well the decision-making process was conducted; political issues impacting the decision-making process, and public acceptance. For instance, let us consider the Heathrow airport expansion project. Heathrow airport is one of the major international airports and one of the busiest airports in Europe and in the world. This airport is owned by the private Heathrow Airport Holdings. During 2014, Heathrow had over 470,000 flights¹ (roughly 1,290 flights a day) and served 73.4 million passengers, mostly international (93 percent). Now, this busy airport that is desired by many airlines and passengers has only two runways. The expansion of Heathrow airport has been discussed in the past few years. The expansion project, which includes building a third runway and another terminal, is expected to increase the capacity of the airport which in turn will lead to a notable increase in the airport revenue. However, the

decision on this project is not based merely on the expected financial gain in the future but also on other aspects of the project and how they will impact the environment and community. Factors such as carbon emission, noise pollution, and the creation of new direct and indirect jobs are just some of those that are considered in addition to economic factors for the decision on the merits of the expansion project.

Interest and Time Value of Money

Starting a new project or expanding an existing business requires capital. Often, companies have to acquire the needed capital from investors to carry out new projects. On the other hand, investor motivation to provide the companies with needed capital is to recover the initial investment with some additional payment known as interest.

The rate of interest acts as a link between income-value and capital-value.

Irving Fisher

Consider the scenario of renting a car from a rental company. If you don't have a car and you need one for business or pleasure, you borrow one from the rental company for a given period of time. You are expected to return the car to the rental company at the end of the lease period and pay for the privilege of using the car during that period. Similarly, when companies need to borrow money from investors, they are expected to return the capital (same as returning the rental car at the end of the lease period) and pay for the privilege of having control of the invested capital for the given period. Often, the amount of payment investors expect (i.e., interest) is related to the amount of capital they invested in a company.

In 2012, Delta Air Lines acquired the Trainer refinery facility which was on the brink of closing. The main motivation of this rather unconventional decision was to protect the airline against the refining margins.² During that time, fuel was one of the highest cost items for the airline and while the jet fuel prices are largely dictated by price of oil, the refining margins may at times be significant. According to Delta Air Lines, acquiring the refinery cost them \$150 million.³ Thus, Delta Air Lines needed to get \$150 million capital from investors who expect their money to be returned to them in the future with interest. For Delta Air Lines, the interest payment is the cost of getting the capital for buying an oil refinery.

There are two types of interest: simple interest and compound interest. In simple interest, the interest is charged simply to the principal borrowed. In compound interest, however, one is charged interest on the principal in addition to the interest on the amount of interest the borrower owes to the lender at each period, as if it is another loan. We will discuss the details of these two concepts further in this chapter. To understand and distinguish between simple interest and compound interest, one would need to know the concept of time value of money. The whole basis of time value of money is the fact that a dollar in hand today is worth more than a dollar to be received at some time in the future. It should be noted that this difference is not simply due to inflation but it is because if you have the money today, you could invest it and earn interest. Thus, when a borrower defers the payment of interest to the lender in a long-term loan, it is as if the borrower borrowed additional money from the lender and the lender is entitled to gain interest on this additional amount.

In understanding the time value of money and interest, we need to note that the time value of money is not the same for everyone. Consider the rental car example. For a common person or a tourist, having a car for a day provides convenience of transportation. For a common person who is working part-time as an Uber driver, having a car for a day is the equivalent of additional revenue by providing transportation services to others. On the other hand, for a professional taxi driver, a car is the main tool of business and having a car for a day often results in a higher revenue than the previously mentioned groups.⁴ The same analogy applies for companies. The time worth of money for each company highly depends on its ability to generate revenue based on the available capital. Similarly, on the other side of the spectrum, lenders expect a different amount of interest for foregoing their capital for the loan period. The expected interest, indeed, depends on factors such as economic conditions, market size, risk, demand for loan, and loan period. That is why, for instance, different banks announce different interest rates for real estate loans.

Simple Interest

Simple interest is the scenario when interest is only paid based on the principal borrowed at the end of the loan period. Under simple interest, borrowers provide no additional payment during the course of the loan and lenders only receive the principal and the interest at the end of the loan period.

Example 2.1

If you deposit \$1,000 in a savings account at the beginning of the year and receive \$1,060 at the end of the year, you have earned \$60 in interest in one year. The interest rate you earned on this savings account is 6 percent per year. The interest rate is calculated by dividing the dollar amount of interest over the principal and multiplying it by 100 to get it in percentage.

$$\text{Interest Rate (\%)} = \frac{\$60}{\$1000} \times 100$$

$$\text{Interest Rate} = 6\%$$

$$i = \frac{I}{P} \times 100$$

Where: i = Interest rate expressed in percentage
 I = Dollar amount of interest accrued in one year
 P = Principal (Present value)

Simple interest is the interest paid on the principal only and can be calculated for one year as follows: Interest = principal $\times$ annual interest rate

$$I = P \times i$$

Example 2.2

If you deposit \$1,000 in a savings account that is paying a simple interest rate of 6 percent per year, you will earn \$60 in interest at the end of the first year as follows:

$$I = \$1,000 \times 6\% = \$60$$

For a multi-year period, simple interest is calculated using the principal only and the total simple interest accrued over a multi-year period is computed as follows:



$$I = P \times i \times n$$

Where:

n = the number of time periods, usually in years

Example 2.3

You deposit \$1,000 in a savings account paying a simple interest rate of 6 percent for four years. How much interest do you owe four years later?

Solution:

By using the above equation, the amount of interest earned after four years can be calculated as:

$$I = \$1,000 \times 6\% \times 4$$

$$I = \$240$$

At the end of the four years, you have accumulated \$1,240 which is the sum of the principal and the accumulated four years of interest.

$$\text{Amount at the end of year 4} = \text{Principal} + \text{Accrued interest in four years}$$

$$\text{Amount at the end of year 4} = \$1,000 + \$240 = \$1,240$$

$$F = P + P \times i \times n$$

$$F = P [1 + (i \times n)]$$

Where:

F = the future value or the total accumulated amount at the end of the n th period

In Example 2.3 for simple interest, because interest is paid on the principal only, the interest accrued in preceding interest periods is ignored in calculating the interest for each period and interest is only calculated based on the original principal.

Example 2.4

DirectJet borrowed \$10,000,000 with simple interest rate of 5 percent and with total principal and interest due in five years. Calculate the annual interest and total interest paid on this loan.

$$\text{Annual Interest} = \$10,000,000 \times 0.05 = \$500,000$$

$$\text{Total interest at the end of year 5} = \$500,000 \times 5 = \$2,500,000$$

$$\text{Total interest and principal due at the end of year 5} = \$12,500,000$$

Table 2.1 Simple Interest Rate Calculation

Interest Rate	5%	=B\$3*B7		=D6+C7
End of Year	Loan Principal	Interest Due	Amount Owed	Amount Paid
0	\$10,000,000	0	\$10,000,000	0
1	\$10,000,000	\$500,000	\$10,500,000	0
2	\$10,000,000	\$500,000	\$11,000,000	0
3	\$10,000,000	\$500,000	\$11,500,000	0
4	\$10,000,000	\$500,000	\$12,000,000	0
5	\$10,000,000	\$500,000	\$12,500,000	\$12,500,000
Total Interest		\$2,500,000		

The principal and interest are paid at the end of year 5. The amount due at the end of each year increases by the constant amount of annual interest of \$500,000. The interest accrued at the end of each year does not earn interest in the following years because the interest is simple. Table 2.1 presents the details of annual interest, amount owed at the end of each year, and total amount due at the end of year 5.

Simple interest is one of the fundamental concepts in engineering economics. However, in practice, simple interest loans are not common and are usually offered for short-term periods. One example of simple interest loans is the fixed-rate short-term loan known as a payday loan, also called cash advance. Payday loans are small loans (e.g., \$50 to \$1,000) that are given for a short period of time (e.g., 14 days) and the borrowers are expected to return the principal with the interest (e.g., \$15 for \$100 loan). Despite the name, these loans are not technically related to actual paydays.

Compound Interest

In practice, interest is normally compounded, meaning interest is paid on the principal and the accrued interest at any given period. In other words, the interest at the end of each period is added to the principal and both principal plus interest earn interest in the next period, and that process continues through to the end of the time period. Note that in this scenario, there is no payback until the end of the loan period.

The interest for the first period is computed as follows:

$$\text{Interest for first period} = \text{Principal} \times \text{Interest}$$

$$I = P \times (P \times i)$$
$$I = P \times (1 + i)$$

The interest for the second period is computed as follows:

$$I = (P + (P \times i)) \times i$$

The interest for each period is computed as follows:

$$\text{Interest per period} = (\text{Principal} + \text{Accrued interest}) \times (\text{Interest rate})$$

Example 2.5

Legacy Regional Airport (LRA) is a vibrant airport that serves both general aviation and commercial airlines. LRA borrowed \$200 million from Bank One at 5 percent compound interest for four years to build a parking garage. Calculate the interest per year and total interest paid to Bank One by Legacy Regional Airport.

$$\text{Interest at the end of year 1} = \$200 \times 0.05 = \$10$$

$$\text{Total amount due at the end of year 1 or beginning of year 2} = \$200 + \$10 = \$210$$

$$\text{Interest at the end of year 2} = \$210 \times 0.05 = \$10.5$$

$$\text{Total amount due at the end of year 2} = \$210 + \$10.5 = \$220.5$$

$$\text{Interest at the end of year 3} = \$220.5 \times 0.05 = \$11.025$$

$$\text{Total amount due at the end of year 3} = \$220.5 + \$11.025 = \$231.525$$

$$\text{Interest at the end of year 4} = \$231.525 \times 0.05 = \$11.576$$

$$\text{Total amount due at the end of year 4} = \$231.525 + \$11.576 = \$243.10$$

Total interest paid at the end of year 4 on this loan is the difference between the total amounts due at the end of year 4 minus the loan principal.

$$\text{Total interest paid at the end of year 4} = \$243.10 - \$200$$

$$\text{Total interest paid at the end of year 4} = \$43.10 \text{ million}$$

To highlight the difference between simple and compound interest, let's rework Example 2.4 assuming interest is compounded annually. The detailed calculation of annual interest accrued on this loan is tabulated in Table 2.2. This table shows the effect of changing interest from simple to compound on annual interest accrued, total interest accrued, and total amount paid on this loan. The interest accrued at the end of each year earned interest in the following years because the interest was compounded.

Table 2.2 Compound Interest Calculations

End of Year	Loan Principal	Interest Rate	Interest Due	Amount Owed	Amount Paid
0	\$10,000,000	5%	0	0	0
1	\$10,000,000	5%	\$500,000	\$10,500,000	0
2	\$10,500,000	5%	\$525,000	\$11,025,000	0
3	\$11,025,000	5%	\$551,250	\$11,576,250	0
4	\$11,576,250	5%	\$578,813	\$12,155,063	0
5	\$12,155,063	5%	\$607,753	\$12,762,816	\$12,762,816
Total interest			\$2,762,816		

Table 2.3 Difference between Accrued Simple and Compound Interests

Year	Simple Interest	Compound Interest	Increased Interest
1	\$500,000	\$500,000	\$0
2	\$500,000	\$525,000	\$25,000
3	\$500,000	\$551,250	\$51,250
4	\$500,000	\$578,813	\$78,813
5	\$500,000	\$607,753	\$107,753
Total increased accrued interest due to compounding impact			\$256,816

Table 2.3 shows the accrued simple interests, accrued compound interests, and the increase in accrued interests between simple and compound interests per year and the total accrued interest increased due to compounding of interest on this loan.

As can be seen in Table 2.3, the difference between simple and compound interest increases exponentially as time passes. Accordingly, using compound interest is beneficial for investors (lenders) for longer-term loans.

The Connection between Interest Rates and Their Determinations

As mentioned in the beginning, private individuals, business entities and also governmental agencies need money for many different projects, and they will borrow money if they believe their return on capital will be higher than the cost of borrowing. But for airlines, airports, and manufacturers borrowing is a powerful way to raise money to finance the growth and expansion they need. The US Airport Infrastructure through 2019 requires a total investment of \$75.7 billion, and that will have to be financed through different debt instruments and user fees.⁵ At the end of the fiscal year 2016, the total government debt in the United States, including federal, state, and local, is expected to be \$22.4 trillion. Lending and borrowing form an important foundation of many corporate and private finances. There are many reasons why individuals, private companies, and public institutions borrow money. When a governmental agency runs a budget deficit to fund public expenditures, it needs to borrow money to finance public expenditures. Similarly, when an individual needs cash, one alternative is to borrow funds and cover the expenses.

Since the interest rate is the price or the cost of borrowing money, interest rates can vary for a variety of reasons just like prices for other products. Economic conditions, inflation rate, time to maturity, and the risk of default are a few examples. When you borrow a certain amount from a lender, you must repay the loan, plus interest, within a certain period of time. Interest rates can be divided into several different groups and categories. Whether we borrow money for a new project, expansion of existing facilities or liquidity for unforeseen expenses, we rely mainly on financial institutions. A financial institution is a firm that facilitates financial

transactions such as investments, loans and deposits between the lenders and the borrowers. Financial institutions bring surplus spending units (lenders of funds) and deficit spending units (net demanders of funds) together.



Globally, interest rates are controlled and managed by various forms of governmental agencies. In the United States, interest rates paid by a borrower to a lender are generally driven by the Federal Reserve System (Fed). Founded in 1781, the Federal Reserve is the central bank of the United States. To stimulate economic growth, many central banks have kept their policy interest rates very low. The central banks in Denmark, Sweden, Switzerland, and Japan, have decided to have a negative rate on commercial banks' excess funds held on deposit at the central bank.⁶ This move pushes banks to use that money instead of depositing it at the central bank. The negative interest rate aims to increase investments and trigger the economy as banks are incentivized to give out loans.

The major objectives of central banks around the world relate to the maintenance of a monetary and financial stability that promotes the following:

- ◆ Price stability
- ◆ Economic growth
- ◆ Full employment

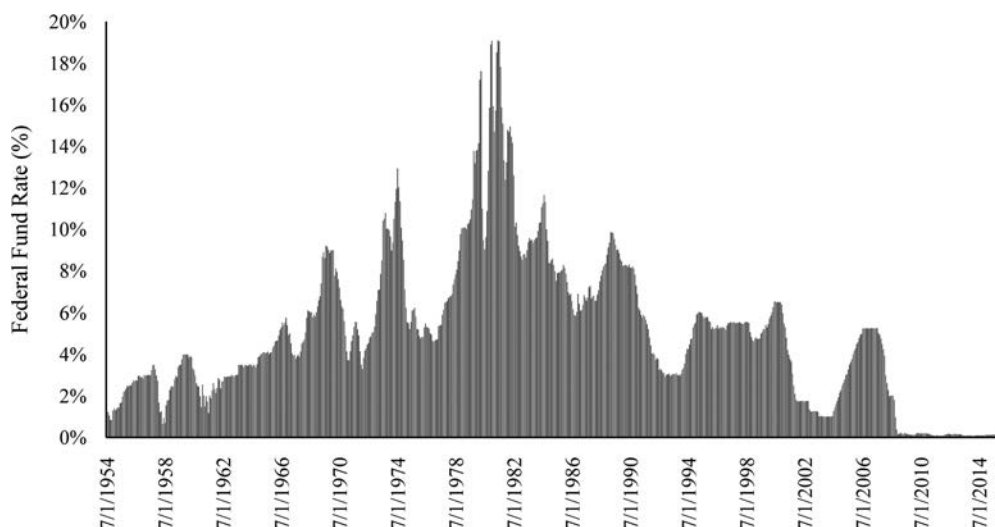
The Federal Reserve can affect the level of economic activities by changing short-term interest rates. By lowering the discount rate that banks pay on short-term loans from the Federal Reserve Bank, the Fed is able to increase the supply of money.

Short-term Interest Rates

Short-term interest rates include discount rates, interbank lending rates, the prime interest rate, Treasury bill rates, and Certificate of Deposit rates.

Discount Rate

Commercial banks and other depository institutions from time to time borrow money from the Federal Reserve Bank to cover temporary deposit drains.⁷ The discount rate is the interest rate charged by the Federal Reserve Bank to commercial banks and other depository institutions for loans received, and a discount window refers to credit facilities in which financial institutions go to borrow funds from the Federal Reserve. The Federal Reserve Banks offer three discount window programs to depository institutions: primary credit, secondary credit, and seasonal credit, each with its own interest rate.⁸ The Federal Reserve may change the discount rate to make temporary changes to the terms of the loans in order to encourage or discourage the borrowing of additional funds by financial institutions.

Figure 2.1 Historical Data of US Federal Fund Rates

Federal Fund Rate

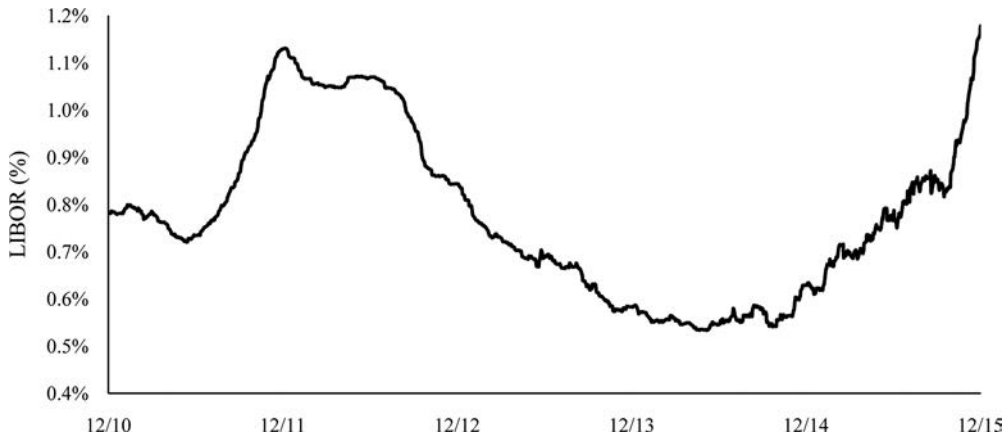
Commercial banks and other depository institutions are required to maintain a certain amount of deposit at the Federal Reserve Bank to meet their financial obligation. In the case that a depository institution expects to have more end of the day cash balance than it needs, the bank can lend the excess fund to another bank that expects to have a shortfall in its own balance. The federal fund rate is the interest rate at which depository institutions lend reserve balances to other depository institutions overnight. The lower the federal fund rate, the less expensive it is to borrow money, and the higher the federal fund rate, the more expensive the borrowing becomes. Federal fund transactions neither increase nor decrease money supply and bank reserves, rather they just redistribute reserves. Figure 2.1 presents historical federal fund rates from 1954 to 2008.

The Prime Rate

The prime interest rate is what commercial banks charge their most favored customers such as large corporations. The prime rate is a benchmark for interest rates on business and consumer loans, and it is often used as a benchmark for other loans like credit card and small-business loans. For example, a bank may charge a local business the prime rate plus two or three percentage points on a loan. The Federal Reserve has no direct control on the prime rate, but the change in prime rate tends to be closely correlated to changes in the target federal funds rate.

London Interbank Offered Rate (LIBOR)

Every day, the Intercontinental Exchange (ICE) publishes a common interest rate known as LIBOR. LIBOR, which stands for The London Interbank Offered Rate, is the estimation of the average of interest rates that leading banks in London are expected to be charged if they borrow from other banks. LIBOR is published through Thomson Reuters on a daily basis for loans of different time periods (e.g., one day,

Figure 2.2 LIBOR for 12 Month Loans Based on US Dollars

one week, three months or one-year loan time) and for different currencies (e.g., US dollar, British pound, Euro, Japanese yen). The LIBOR is often used by various lenders and borrowers to set interest rates. Figure 2.2 presents the LIBOR for borrowing for a loan period of one year based on the US dollar.

Additionally, to protect borrowers against unjust and unreasonably high interest (i.e., usury) many governments impose limits on maximum interest rates. In the US, states are responsible for setting the maximum interest rates.

Treasury Bills (T-Bills)

A Treasury bill is a short-term debt obligation backed by the US government. The role of government bond markets remains pivotal in virtually all of the major economies. Treasury bills have a face value of a certain amount, which is the amount paid at the maturity date. Every bill has a specified maturity date; when the bills reach their maturity date, the lender receives the face value back.⁹ Generally T-bills come with maturities of 4 weeks, 13 weeks, and 26 weeks. Treasury bills are sold in denominations of \$1,000 to \$5,000,000.

All of the T-bills are sold for less than their face value. For example, a T-bill with \$1,000 worth at maturity, may be sold at 5% discount for \$950. In this example, the interest earned on this T-bill is 5.26 percent, because \$50 is 5.26 percent of the \$950.

$$\text{Interest rate earned} = \frac{\$50}{\$950}$$

$$\text{Interest rate earned} = 5.26\%$$

As another example, a business firm could buy a 26-week T-bill with face value of \$1,000 for \$950.68 at the time of purchase, effectively earning an annualized yield of 10.24 percent on the investment.

Certificate of Deposit (CD)

Checking accounts and savings accounts are called demand deposits. A demand deposit is an account where the customer can deposit money and withdraw money

at any time, by going to the bank and completing a withdrawal. By law, the banks have to give the money immediately upon a request. Conversely, a time deposit is a depository account offered by financial institutions that pays interest if the money remains in the account for a specified period of time. A certificate of deposit is similar to a standard savings account with the exception that the owner cannot withdraw the funds before maturity. CDs state that the customer has a specified amount of deposit, usually for a given period of time at a fixed rate of interest, and are among the safest investments a person can make.

Example 2.6

Assume that the US government issues \$10 million worth of IOU¹⁰ (bonds) and agrees to redeem the bonds in six months (182 days). A financial institution buys this T-bill at a price of \$9,900,000. What is the interest on a yearly basis for this type of debt?

Interest Rate Charged

$$= \frac{\text{Face Value} - \text{Purchase Price}}{\text{Face Value}} \times \frac{365}{\text{Time until maturity (in days)}} \times 100\%$$

$$\text{Interest Rate Charged} = \frac{10,000,000 - 9,900,000}{10,000,000} \times \frac{365}{182} \times 100\%$$

$$\text{Interest Rate Charged} = 2\%$$

Typically, governments are charged a much lower price (interest) than businesses for borrowing money because their risk of default is zero. If the US government would run out of money, they would always be able to print more money and deflate their debt. This is different for the European Union due to their different monetary system with the European Central Bank. That is why countries like Greece are now paying a higher interest rate since they are so much in debt and the risk of default is high.

Example 2.7

A private customer of a local bank has excess cash in his savings account and would like to increase his return on that money. The bank is willing to offer him a higher interest rate if it can plan to have that money for a specific time period. Consequently, the customer and the bank open a certificate of deposit which allows the bank to invest that money for the agreed time and the customer slightly increases his return on his investment.

Long-term Debt and Interest Rates

The long-term interest rate is a central variable in economic growth and stability. Long-term debt matters to aircraft manufacturers, airlines, and engine manufacturers when they are planning to borrow money to start a long-term project as well as to the lenders weighing the risks and rewards of extending the lending time period. Long-term interest rates refer to interest rates on long-term financial securities, mainly bonds, maturing in about 10 years or more. Therefore, a bond is considered as a debt instrument in which an investor lends money to a borrower for a defined period of time at a set interest rate. There are three important things to know about any bond:

- ◆ *Par value (face value)*
Face (par) value refers to the stated value of the financial instruments at the time they are issued, and all bonds are redeemed at face value when they reach maturity. Corporate bonds normally have a par value of \$1,000, but this amount can vary greatly for government bonds. Par value is typically different from the market price.
- ◆ *Coupon rate*
The coupon rate is the interest rate that a bond pays to the bondholders. When a bond is issued, it promises to pay a fixed coupon payment until it matures. For example, a \$1,000 face value bond with a coupon interest rate of 7.5 percent pays a \$75 coupon payment annually.
- ◆ *Maturity date*
The maturity date of a bond is the date when the borrower returns the face value of the bond to the investor.

There are four basic kinds of bonds, all defined by who is selling the debt:

- ◆ US government bonds are called Treasury bonds
- ◆ Other government agencies
- ◆ Corporate bonds
- ◆ State and local governments bonds

Treasury Notes

The Treasury Department sells all bills, notes, and bonds at auction with a fixed interest rate. The difference between bills, notes, and bonds is the time until maturity.

- ◆ Treasury bills are issued for terms of less than a year.
- ◆ Treasury notes are issued for terms of 2, 3, 5, and 10 years.
- ◆ Treasury bonds are issued for terms of 20 to 30 years or more.

Treasury notes are an additional type of debt for the US government. Maturity is between one and 10 years but, unlike the T-bills, they do not only pay out the face value at the end but also pay out coupon payments every six months. The coupon rate (e.g., 5 percent of the face value) is determined before the note is sold. Treasury notes are well accepted investments as there is a large secondary market that adds to their liquidity.

Treasury Bonds

Treasury bonds (T-Bonds) are the most long-term bonds issued by the US government. T-Bonds mature in 20–30 years and pay out their face value at the end of maturity, in addition to the usual six-month coupon payments. If the Fed wants to increase the money supply, it buys government bonds with cash and therefore increases the cash in circulation or increases the overall money supply. Conversely, if the Fed wants to decrease the money supply, it sells bonds from its account, thus taking in cash and removing money from circulation. The escalating debt crisis in several European countries, such as Greece, has sent investors piling their money into the haven of government bonds in the US. Uncertainty over the European economy has caused broad swings in global financial markets.¹¹

Municipal Bonds (Munis)

The federal government is not the only public entity that issues debt securities: the states, counties, and other governmental agencies can also issue debt instruments including bonds to finance their projects. Municipal bonds offer slightly lower interest rates than the federal government bonds, but they have a few advantages compared to federal and corporate bonds. First, the interest on municipal bonds is exempt from federal taxes. Second, depending on the type of the project, municipal bonds may be secured by the state or local government agencies. State and local government agencies planned to issue a large class of municipal bonds in April and May of 2016.¹²

Muni funds can offer stability and income in a volatile world, but I don't think you should be preaching for excitement given these yields at the moment

Alliance Bernstein Investor,
April 2016

Revenue Bonds

The project will take at least two years and has a cost of \$200 million. The airport says the funding will come from future general airport revenue bonds and passenger facility charge revenues. . . . It's part of Destination CLT, a \$2.5 billion plan that will guide airport expansion into the 2020s.

Charlotte Douglas International Airport

Table 2.4 Overview of Different Governmental Loans

Type of US government loan	Characteristics
Treasury bills (T-bills)	◆ Matures after 28 days, 91 days, 182 days or 364 days ◆ Only pays at maturity, there are no coupon payments ◆ Issued by the federal government
Treasury notes (T-notes)	◆ Matures in 2–10 years ◆ Pays face value at maturity and coupon payments every six months ◆ Issued by the federal government
Treasury bonds (T-bonds)	◆ Matures in 20–30 years ◆ Pays face value at maturity and coupon payments every six months ◆ Issued by the federal government
Municipal bond	◆ Maturity can vary ◆ Pays face value at maturity and depending on time to maturity, coupon payments every six months ◆ Issued by state, county or city authority ◆ Special bond structures (e.g. revenue bonds) for specific projects are possible

Airports commonly turn to the capital markets to finance long-term construction projects. Airports finance construction of terminals, parking facilities by issuing revenue bonds. Landing fees, parking charges, and fuel fees provide the backing.

Corporate Bonds

Business firms issue bonds (Corporate Bonds) to raise money for capital investments. Bondholders in return receive regular, stipulated interest payments, set when the bond is issued. One of the most well-known risks in the bond market is the risk of default. The risk of default is the peril that the bond issuer will be unable to pay the contractual interest or principal. Some bonds are backed (secured) by physical assets, which can be liquidated in the case of default of payment. Secured corporate bonds are backed by collateral that the issuer may sell to repay you if the bond defaults before, or at, maturity. For example, a bond might be backed by a specific aircraft or plant and equipment. Secured bonds and securitization provide a dependable source of aircraft funding for the industry.

Typically, loans with a longer maturity date have a higher interest rate as they include a higher risk of default and a lack of liquidity. Examples are bonds issued by United Continental due in 2031 and bonds issued by JetBlue maturing in 2039. Other factors include whether a bond is callable. A callable bond gives the bond issuer the right to redeem the bond from the bondholders before maturity date. The callable provision would be exercised if the interest rates have declined since the bond was issued. The bond issuer calls its current bonds and reissues them at a lower interest rate.

Rating agencies grade bonds on a letter scale that indicates credit worthiness and risk. The three largest credit agencies are: Moody's Investors Service, Standard & Poor's, and Fitch. Credit rating agencies, also called rating agencies, are companies that assign credit ratings, which rate a debtor's ability to pay back debt by making timely interest payments as well as the likelihood of default.¹³

The following graphs (Figure 2.3) show different corporate bonds from aviation-related firms such as airlines or manufacturers. The graphs reveal that the later the maturity, the higher the

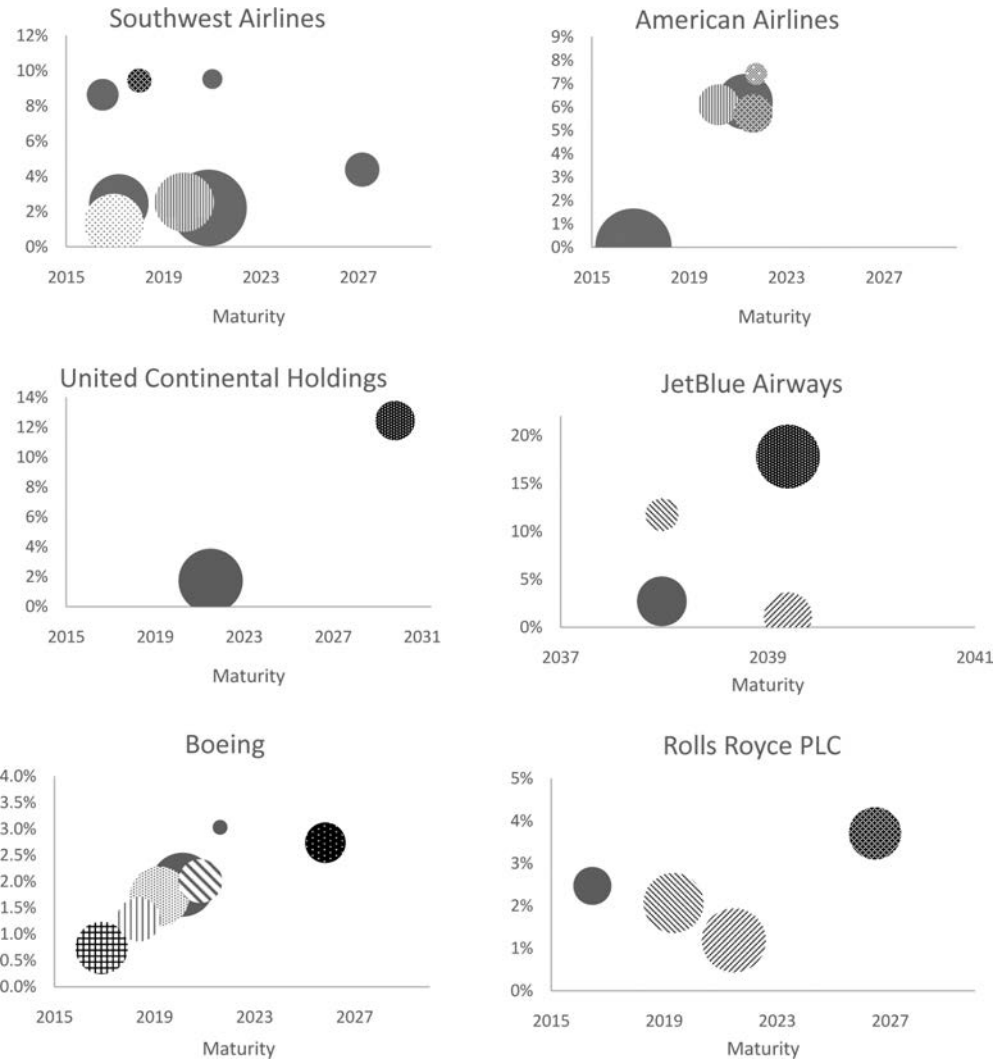
Aviation Snippets

Death of Eastern Airlines

In 1986, when Frank Lorenzo took over Eastern Airlines, the country's third largest carrier was in decline: its route structure was inherently weak, its costs were sky high, and its stiff-necked labor unions fought with each other almost as much as they fought management. To many in corporate America, Lorenzo seemed just the man to whip Eastern back into shape. Lorenzo, with the help of Michael Milken and Drexel Burnham Lambert's junk bond machine, built his tiny Texas Air Corporation into an empire. Indeed, with the addition of Eastern to Continental, Frontier, and People Express airlines, Lorenzo controlled the largest airline in the country, flying more than a fifth of the nation's air passengers on his planes. Lorenzo has been characterized as a ruthless empire builder, who uses a seemingly simple formula of slashing costs and offering lower fares. The problem, however, is that he had failed to parlay his formula into profitability. New York Air failed; Continental was consistently losing money; and Eastern was on the brink of extinction.

*Journal of Aviation/Aerospace
Education & Research (2000)*

Figure 2.3 Bond Rates and Maturity for Various Airlines and Manufacturing Companies



yield and thus the cost of borrowing. Additionally, firms with a much lower volatility than the airlines, here in the examples of Boeing and Rolls-Royce, pay much less for their debt than airlines. This is closely related to the associated risk as the airlines have a higher likelihood of defaulting on their debt.

Now that we have clarified the simple and the compound interest and introduced the different types of short- and long-term debt, we focus on the next two questions that may be raised by potential investors: future value of present investment and present value of future gain. These two questions are two sides of the same coin and deal with the investors' concern to estimate the outcome of their investment for a given period of time and a given interest rate.

Future Value of a Single Present Amount

In this section, we are interested in finding out the outcome of investment in a project with a given compound interest rate for a certain time. We assume that the only payback will happen at the end of the investment period.

To facilitate computation of future and present values of a single amount, we are using the following notations:

F	=	Future Value, future sum of money at the end of period n , or future worth
P	=	Present Value, single sum of money at the present time, or present worth
i	=	Interest rate per time period, compound rate, or discount rate
n	=	Number of time periods (usually years)
$(F/P, i, n)$	=	Future value of present amount interest factor at interest rate of i for period of n
$(P/F, i, n)$	=	Present value of future amount interest factor at interest rate of i for period of n

Assume we invest the present amount of P at the interest rate of i for a period of one year. The future value at the end of year 1 is equal to the principal (P) plus one year interest (P_i) and it is computed as follows:

$$F_1 = P + P \times i$$

$$F_1 = P \times (1 + i)$$

The future value at the end of two years is computed as follows:

$$F_2 = [P \times (1 + i)] \times (1 + i)$$

$$F_2 = P \times (1 + i)^2$$

The future value at the end of three years is computed as follows:

$$F_3 = [P \times (1 + i)^2] \times (1 + i)$$

$$F_3 = P \times (1 + i)^3$$

The future value at the end of four years is computed as follows:

$$F_4 = [P \times (1 + i)^3] \times (1 + i)$$

$$F_4 = P \times (1 + i)^4$$

The above relationship can be generalized to calculate the future value of a single present sum if it is compounded annually at rate of i percent for n years.

$$F_n = P \times (1 + i)^n$$

Example 2.8

A fixed base operator (FBO) currently set aside \$50 million to purchase equipment in five years. If the \$50 million is invested at the rate of 6 percent, how much will be accumulated in five years to purchase the equipment?

The cash flow diagram is shown in Figure 2.4. We will use this type of diagram throughout the book to visualize the cash flow in various time periods. As presented in Figure 2.4, the horizontal line represents time axes, and labels are the ends of each time period; that is, “1” represents the end of time period 1. The arrows represent the cash flows from the perspective of the party that we are trying to solve the problem for. Arrows above the time line represent positive cash flows (income or gain) and arrows below the time line indicate negative cash flows (cost and expenses).

For this problem, we draw the diagram from the perspective of the FBO. The FBO is setting aside \$50 million at the beginning of the time period. This expense is presented in Figure 2.4 as an arrow below the time line. During the five years, the length of the investment, there is no cash flow. However, at the end of the investment period, that is the end of the fifth year, the FBO collects the principal and compounded interest. This gain is presented by an arrow above the time line in Figure 2.4.

We are interested in finding the gain after five years of the investment, that is F_5 .

$$F_n = P \times (1 + i)^n$$

$$F_5 = \$50 (1 + 0.06)^5$$

$$F_5 = \$50 (1.3382)$$

$$F_5 = \$66.91 \text{ million}$$

The FBO will accumulate \$66.91 million to purchase the equipment.

In equation $F_n = P \times (1 + i)^n$ the expression $(1 + i)^n$ is simply a future value of the present amount interest factor and can be expressed as $(F/P, i, n)$. The pre-calculated values of these interest factors are tabulated in Appendix A. Therefore, this equation can also be expressed as follows:

$$F = P \times (F/P, i, n)$$

Figure 2.4 Cash Flow Diagram for a Given P and Finding F at the End of Year 5

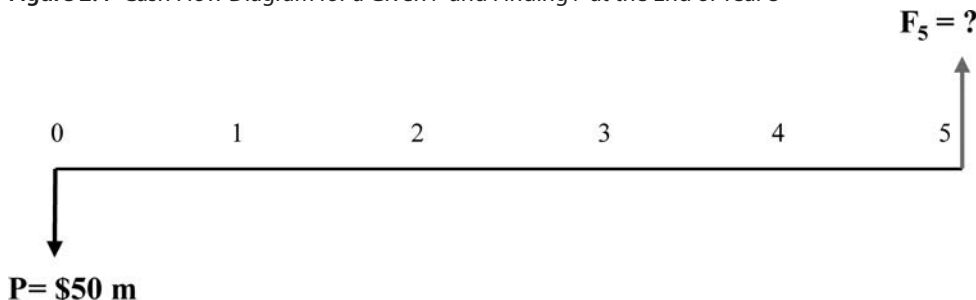


Table 2.5 Compound Interest Factors for 6 Percent

6%								
n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	F/A	A/F	P/A	A/P	P/G	A/G
1	1.0600	0.9434	1.0000	1.0000	0.9434	1.0600	0	0
2	1.1236	0.8900	2.0600	0.4854	1.8334	0.5454	0.8900	0.4854
3	1.1910	0.8396	3.1836	0.3141	2.6730	0.3741	2.5692	0.9612
4	1.2625	0.7921	4.3746	0.2286	3.4651	0.2886	4.9455	1.4272
5	1.3382	0.7473	5.6371	0.1774	4.2124	0.2374	7.9345	1.8836
6	1.4185	0.7050	6.9753	0.1434	4.9173	0.2034	11.4594	2.3304
7	1.5036	0.6651	8.3938	0.1191	5.5824	0.1791	15.4497	2.7676
8	1.5938	0.6274	9.8975	0.1010	6.2098	0.1610	19.8416	3.1952
9	1.6895	0.5919	11.4913	0.0870	6.8017	0.1470	24.5768	3.6133
10	1.7908	0.5584	13.1808	0.0759	7.3601	0.1359	29.6023	4.0220

Using Example 2.8, the future value at the end of year 5 is calculated as follows:

$$F_n = P(1 + i)^n$$

$$F_5 = \$50(1 + 0.06)^5$$

Or,

$$F_5 = \$50(F/P, 6\%, 5)$$

From Table A12 in Appendix A for 6 percent interest rate, which is partially shown here as Table 2.5, the value of the interest factor ($F/P, 6\%, 5$) is equal to 1.3382. We now plug the interest factor 1.3382 into the above relationship and compute the future value at the end of five years.

$$F_5 = \$50m(1.3382)$$

$$F_5 = \$66.91 \text{ million}$$

Present Value of a Single Future Sum

One of the critical measures for project evaluation and selection is to access the present worth of the future gain from performing a project (investment). Present worth is often calculated by reverse calculation of the future expected gain in a process called discounting. The discounting process is rather the reverse of compounding that was

introduced earlier in this chapter. To compute the present value of a single future sum, we simply solve the following equation for P , which is the present value.

$$F = P \times (1 + i)^n$$

$$P = F \times \frac{1}{(1 + i)^n}$$

Example 2.9

DirectJet's engineering department is planning to purchase power backup equipment in five years at a cost of \$500,000. How much should DirectJet set aside today earning 10 percent interest to accumulate \$500,000 in five years?

The easiest way to answer problems in economic analysis is to draw a cash flow time line or cash flow diagram. Figure 2.5 is a cash flow diagram showing just one outflow or disbursement of \$500,000 at the end of year 5.

$$P = F \times \frac{1}{(1 + i)^n}$$

$$P = F \times \frac{1}{(1 + 0.10)^5}$$

$$P = \$500,000 \times 0.6209$$

$$P = \$310,450$$

In the equation above, the term $\frac{1}{(1 + i)^n}$ is simply a present value of future amount interest factor and can be expressed as $(P/F, i, n)$. The pre-calculated values of these interest factors are tabulated in Appendix A. Therefore, equation $P = F \times \frac{1}{(1 + i)^n}$ can also be articulated as follows:

$$P = F \times (P/F, i, n)$$

Using Example 2.9, the present value (the amount that DirectJet should set aside today) is calculated as follows:

$$P = F \times \frac{1}{(1 + i)^n}$$

$$P = F (P/F, i, n)$$

$$P = \$500,000 (P/F, 10\%, 5)$$

Figure 2.5 Cash Flow Diagram for a Given F at the End of Year 5 and Finding P

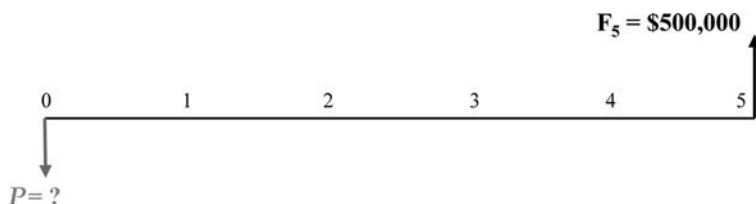


Table 2.6 Compound Interest Factors for 10 Percent

10%			10%					
n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	F/A	A/F	P/A	A/P	P/G	A/G
1	1.1000	0.9091	1.0000	1.0000	0.9091	1.10000	0	0
2	1.2100	0.8264	2.1000	0.47619	1.7355	0.57619	0.8264	0.4762
3	1.3310	0.7513	3.3100	0.30211	2.4869	0.40211	2.3291	0.9366
4	1.4641	0.6830	4.6410	0.21547	3.1699	0.31547	4.3781	1.3812
5	1.6105	0.6209	6.1051	0.16380	3.7908	0.26380	6.8618	1.8101

From Appendix A, Table A16, for 10 percent interest rate, which is partially shown here as Table 2.6, the value of the interest factor (P/F , 10%, 5) is equal to 0.6209. We now plug in 0.6209 in the above relationship and compute the present value.

$$P = \$500,000 (0.6209)$$

$$P = \$310,450$$

Example 2.10

Johnson Regional Airport (JRA) is planning to construct a new terminal and has received the following two cost proposals.

Proposal 1: Total cost of \$800 million payable at the beginning of the project.

Proposal 2: Total cost of \$1,200 million payable at the end of the project in four years.

If construction time is the same under both proposals and JRA earns 10 percent interest on its investment, which proposal is cost efficient?

There are two alternatives that provide solutions to this problem.

First, compare the cost of both proposals at the end of year 4 when the construction of the terminal is completed. The cost of proposal #2 at the end of year 4 is given and it is \$1,200 million. To calculate the cost of proposal #1 at the end of year 4, we use the following equation:

$$F_n = P(1 + i)^n$$

$$F_4 = \$800\text{m}(1 + 0.10)^4$$

$$F_4 = \$800\text{m}(1.4641)$$

$$F_4 = \$1,171.28\text{m}$$

Since the future value of the cost of proposal #1 in four years is less than the cost of proposal #2, proposal #1 is cost efficient.

Second, compare the cost of both proposals at the beginning of the construction. The cost of proposal #1 is given and it is \$800 million. To calculate the cost of proposal #2 at the beginning of the project, we use the following equation:

$$\begin{aligned} P &= F [1/(1 + i)^n] \\ P &= \$1,200\text{m} [1/(1 + 0.10)^4] \\ P &= \$1,200\text{m} [0.6830] \\ P &= \$819.6\text{m} \end{aligned}$$

Since the present value of cost of proposal #2 is greater than the cost of proposal #1, proposal #1 is cost efficient.

So far we have discussed the process of compounding and discounting cash flow to calculate the future value and present value from one another. However, in practice we often have situations in which we have to account for more than one payment. Fortunately, the process of dealing with more than one cash flow is straightforward and can be viewed as combinations of multiple single cash flows as we discuss in the next sections.

Future Value of an Unequal Cash Flow Series

To calculate the future value of a stream of non-equal cash flows at the end of n periods, we simply compute the future value of each cash flow at the end of the n^{th} period and then add them all together.

Example 2.11

Assume you plan to put some money into a savings account that pays 6 percent interest over the next four years. You will put into the account \$100 at the end of year 1, \$200 at the end of year 2, \$300 at the end of year 3, and \$400 at the end of year 4. How much money will you have in that account at the end of year 4?

Using equation $F_n = P \times (1 + i)^n$, first compute the future value of each cash flow at the end of year 4 and then simply add them together. Detailed calculations are presented in Figure 2.6.

Figure 2.6 Future Value of a Stream of Unequal Cash Flow Series

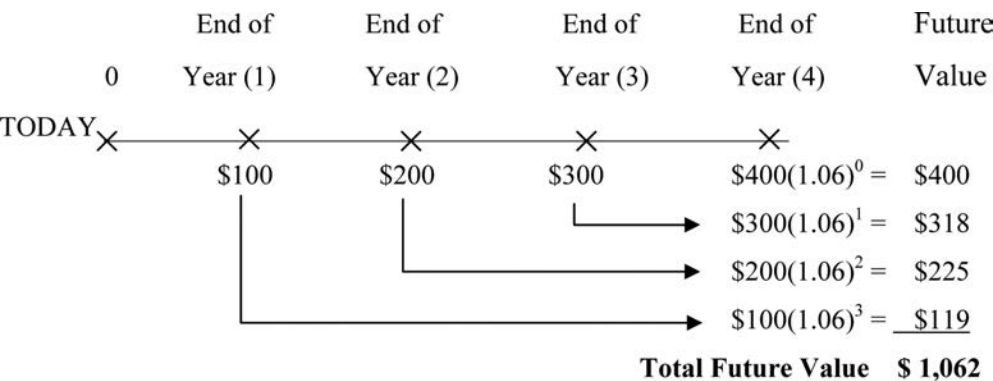
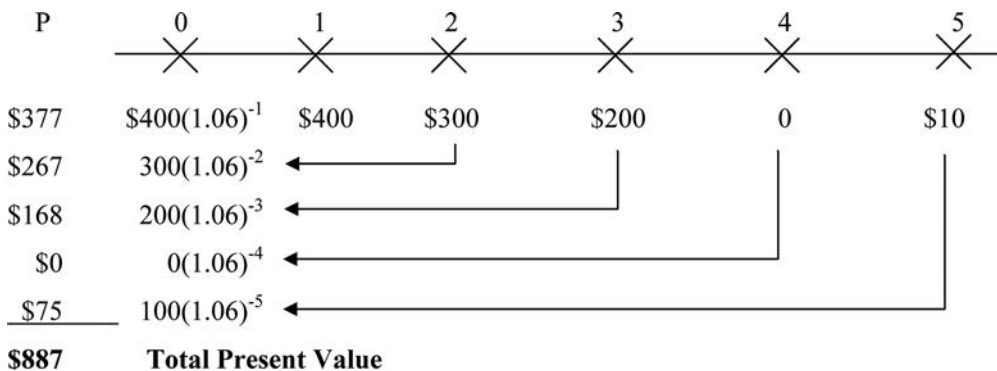


Figure 2.7 Present Value of a Stream of Unequal Cash Flow Series

Present Value of an Unequal Cash Flow Series

To calculate the present value of a stream of non-equal cash flows, we simply compute the present value of each cash flow at the time zero and then add them all together.

Example 2.12

Suppose you were offered a stream of payments: \$400 in one year, \$300 in two years, \$200 in 3 years, nothing in year 4, and \$100 in five years. Assume you earn 6 percent on your investment, what is the present value of this stream of payments? Equation

$P = F \times \frac{1}{(1+i)^n}$ can be articulated as follows:

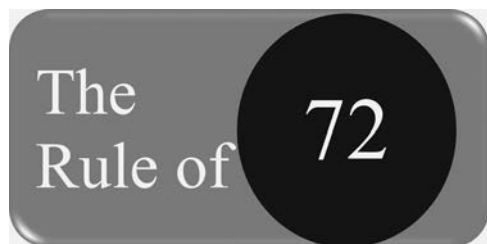
$$P = F \times \frac{1}{(1+i)^n}$$

$$P = F \times (1+i)^{-n}$$

Using this equation, first compute the present value of each payment and simply add them together at the end. Detailed calculations are presented in Figure 2.7.

The Rule of 72

The rule of 72 applies to compound interest rates and states that you can approximately determine how many years it will take to double a given sum of money at a given compound interest rate. It also states that you can approximately determine the interest rate that doubles a given sum of money within a given number of years, simply by dividing the number 72 by the compounded interest rate or by the number of years.



$$\text{Approximate number of years } (n) = \frac{72}{\text{Interest rate } (\%)}$$

$$\text{Approximate interest rate } (i) = \frac{100}{\text{Number of Years}}$$

Example 2.13

The Municipal agency currently has \$5 million in an investment account and needs \$10 million to complete a terminal renovation project. Given the agency earns 8 percent compound interest on its investment, approximately how many years will it take for Municipal to accumulate \$10 million to complete the project?

Using the equation above, it takes approximately nine years as follows:

$$n = \frac{72}{8} = 9 \text{ years}$$

Example 2.14

A regional airport is planning to complete an in-line system of conveyer belts and explosive detection machines for a baggage screening process in 10 years that will cost \$20 million. If the airport has currently \$10 million in an investment account, at what interest rate can the airport accumulate \$20 million in 10 years?

Using the equation above, at 7.2 percent the airport can double its current investment as shown below.

$$i = \frac{72}{10}$$

$$i = 7.2\%$$

The Rule of 100

The rule of 100 applies to simple interest rates and states that you can approximately determine how many years it will take to double a given sum of money at a given simple interest rate. It also states that you can approximately determine the simple interest rate that doubles a given sum of money within a given number of years. Simply divide the number 100 by the simple interest rate or by the number of years.

$$\text{Approximate number of years } (n) = \frac{100}{\text{Interest rate } (\%)}$$

$$\text{Approximate interest rate } (i) = \frac{100}{\text{Number of Years}}$$

Example 2.15

Rework Example 2.13 assuming the Municipal agency earns 8 percent simple interest instead of compound interest on its investment. Using equation above, it takes approximately 12.5 years as follows:

$$n = \frac{100}{8}$$

$$n = 12.5 \text{ years}$$

Example 2.16

Rework Example 2.14 and determine at what simple interest rate can the regional airport accumulate \$20 million in 10 years? Using the equation above, at 10 percent simple interest the regional airport can double its current investment as shown below.

$$i = \frac{100}{10}$$

$$i = 10\%$$

Summary

In this chapter, we introduce engineering economics and provide an overview of its application in evaluating investments and loans. Engineering economics is described as the study of the science that equips engineers with a collection of skills, tools, and techniques to allow them to analyze, evaluate, and compare alternatives on an economic basis.

Time value of money confirms that when you borrow money, you pay interest and the interest is synonymous to rent that you pay for using the borrowed money.

There are two types of interest: simple interest, which is charged based on the principal borrowed; and compound interest, which is charged based on the principal and accrued interest. In practice, interest is normally compounded. Since the interest rate is the price or cost of borrowing money, interest rates can vary for many reasons just like prices of other products. Economic conditions, inflation rates, time to maturity, and the default risk are a few examples. There are short-term and long-term interest rates. Short-term interest rates include discount rates, the federal fund rate (overnight interbank lending rate), the prime rate, the London interbank offered rate (LIBOR rate), Treasury bill rates, and certificate of deposit rates. Long-term interest rates include interest rates on Treasury bonds, Treasury notes, municipal bonds, corporate bonds, and mortgages.

This chapter derives the financial equations to compute the future value of a single amount, present value of a single future sum, and future and present values of streams of unequal amounts. It is fundamental that the concept of time value of money, the financial equations, and the use of interest factors are clearly understood, because the rest of this book is primarily based on these concepts.

This chapter also presents the difference between the rule of 72 and the rule of 100. The rule of 72 applies to compound interest and determines the approximate number of years to double a given sum of money at a given interest rate or to determine the approximate interest rate to double a given sum of money at a given time period. The rule of 100 operates similarly to the rule of 72, but applies to simple interest.

Discussion Questions and Problems

- 2.1 Jet Airways borrowed \$1,000,000 for one year to improve its liquidity. Jet Airways paid back the principal and interest of \$1,060,000 at the end

- of the year. What is the dollar amount of interest and interest rate on this loan?
- 2.2 You are investing \$10,000 in a certificate of deposit with a simple interest of 5 percent. How much interest will you accumulate in this CD at the end of four years?
 - 2.3 EZJET Flying operating out of Melbourne, Australia is considering borrowing \$5,000,000 with a simple interest rate of 6 percent with total principal and interest due in four years. How much interest will accrue at the end of each year and what is the total amount of interest due in four years?
 - 2.4 DirectJet Engineering is planning to purchase equipment in eight years costing \$14,000,000. It invested a lump sum of money in a certificate of deposit that pays 5 percent simple interest per year for eight years. How much did DirectJet Engineering invest in this certificate of deposit if the certificate is worth \$14,000,000 at the end of eight years?
 - 2.5 MyJet borrowed \$100 million at 7 percent compound interest for four years to expand its operations. Calculate the interest per year and total interest paid by the company in four years.
 - 2.6 DirectJet Engineering has invested \$100 million in a money market account that pays 8 percent interest per year. It is planning to use the accumulated amount at the end of six years to purchase equipment that supports the institution's growth. How much will be accumulated in this money market account in six years?
 - 2.7 FunJet is planning to expand its operation to Asia Pacific by ordering five Boeing 787 Dreamliners with delivery in five years. The B787's lower fuel consumption and superior long-haul capability is expected to improve the company's profitability as well. They have deposited one billion dollars in an investment account today and will receive \$1.34 billion in five years to pay for the five B787-10 airplanes. What annual compounded interest rate is FunJet earning in this investment?
 - 2.8 DirectJet Engineering needs \$7,320,500 to replace equipment and machinery to increase its operational efficiency. It has deposited \$5,000,000 in an investment account that pays 10 percent per year compounded annually. How many years would it take for DirectJet Engineering to accumulate at least \$7,320,500 in this investment account?
 - 2.9 You are investing \$10,000 in an account that pays 7 percent interest compounded annually. What is the future value of this investment at the end of five years?
 - 2.10 Jet Services Avionics (JSA) is planning to purchase a new warehouse at the airport in Phoenix five years from now at an estimated cost of \$2,000,000. How much should JSA set aside today earning 8 percent interest to accumulate \$2,000,000 in five years?
 - 2.11 DirectJet is planning to purchase new power backup equipment and has received the following two payment options:
 Option number 1: A payment of \$500,000 today.
 Option number 2: A payment of \$700,000 payable at the end of four years.
 If DirectJet uses a minimum acceptable rate of return (MARR) of 10 percent, which option is cost efficient?

- 2.12** In a consulting agreement, you will be receiving consulting fees of \$50,000, \$65,000, \$80,000, and \$100,000 at the end of each year for the next four years respectively. If you deposit these consulting fees into an investment account that pays 10 percent interest per year over the next four years, how much money will you have in that account at the end of year 4?
- 2.13** You are leasing a warehouse with annual lease payments of \$100,000, \$120,000, \$135,000, \$155,000, and \$180,000 at the beginning of each year for the next five years respectively. If you earn 8 percent on your investment, how much in lump sum do you have to deposit today to be able to make all lease payments for the next five years?
- 2.14** Johnson Regional Airport currently has \$500 million in an investment account that pays 12 percent per year compound interest. Johnson Regional Airport needs \$1,000 million to complete the construction of a new terminal. Assuming it earns 12 percent on future investments, approximately how many years will it take for Johnson Regional Airport to accumulate \$1,000 million to complete the project?
- 2.15** In Problem 2.14, assuming Johnson Regional Airport earns 12 percent simple interest instead of compound interest on its investment and assuming all other relevant information, approximately how many years will it take for Johnson Regional to accumulate \$1,000 million to complete the project?
- 2.16** DirectJet is planning to purchase two Airbus A350-1000 aircraft at the price of €320 million each in eight years. DirectJet currently has €320 million in an investment account. At approximately what compound interest rate can it accumulate €640 million in eight years to acquire the two A350-1000 aircraft?
- 2.17** In Problem 2.16, assume all relevant data, at approximately what simple interest rate can DirectJet accumulate €640 million in eight years to acquire the two A350-1000 aircraft?

Notes

- 1 Facts and Figures, Heathrow Media Centre.
- 2 How Cheap Oil Has Delta Air Lines Jet Fooled. *Forbes*, January 21, 2015.
- 3 Delta Buys Refinery to Get Control of Fuel Costs. *The New York Times*, April 30, 2012.
- 4 Uber and Lyft Have Devastated L.A.'s Taxi Industry, City Records Show. *Los Angeles Times*, April 18, 2016.
- 5 Airports Council International-North America (ACI-NA), March 3, 2016.
- 6 Why Use Negative Interest Rates? BBC News Services, February 15, 2016.
- 7 Structure and Functions, Federal Reserve Bank of Atlanta, Atlanta, GA, 2016.
- 8 Board of Governors of the Federal Reserve System, Monetary Policy, February 23, 2016.
- 9 The maturity structure of government paper including bonds spans a wide range.
- 10 An IOU is a signed document acknowledging a debt such as bonds.
- 11 *The Wall Street Journal*, June 29, 2015.
- 12 *Bloomberg*, April 18, 2016.
- 13 The Credit-Raters: How They Work and How They Might Work Better, *Business Week*, April 8, 2002.

3

Annuities and Gradient Series

Heathrow is one of the most heavily slot restricted airports in the world, and desirable slots are very valuable. Well, Oman Air just broke a record by buying the most expensive slot in Heathrow's history. Oman Air just purchased a prime London Heathrow slot from Air France/KLM for \$75 million. . . . If the third runway is shot down, I imagine it could drive up prices even more, while if it's approved I imagine it will immediately lower the value of slots.

Boarding Area, February, 2016



Many large engineering projects, such as building an airport or establishing an aerospace manufacturing plant, require fundraising. Many institutions borrow money from banks or issue bonds to partially finance their projects. These lending instruments allow the owners or managers to make periodic payments of constant amounts to pay off the loan. These payment streams of equal amounts are called annuities or uniform series.

In the previous chapter, we discussed the concept of time value of money and how investors expect to collect interest on their capital that they lend to borrowers. We focused on investment scenarios where borrowers only make one payment at the end of the loan period. In practice, however, investors expect to receive partial payment during the loan period. For instance, if you get a 30-year loan to buy real estate, the bank expects you to pay mortgage on a regular basis, let's say monthly, during the course of the loan. Clearly, considering the time value of money, the regular paybacks to investors change everything and we need a new approach to evaluate these types of loans.

The constant cash flow that takes place in regular time intervals is called annuity. Common time intervals for annuity payments are annual, semi-annual, quarterly, and monthly. In this chapter, we will discuss simple annuities, where uniform series of payments are made, and gradient series, in which the amounts of payments change throughout the course of a loan.

The focus of this chapter is to develop the engineering economics formula and apply the commonly used interest factors to determine the equivalences for cash flow streams that are essential to any economic decision making. Cash flows occur in many forms but the focus of this chapter is on annuities (uniform series), and gradient series that increase or decrease by constant amounts or constant percentages. This chapter also aims to enable practitioners to calculate the future value and the present value of streams of equal cash flows that occur over a given period of time. In addition, it intends to empower readers to determine the annuity amount of a single present amount or of a single future sum for a given period of time. Furthermore, this chapter assists students to determine the present value and future value of arithmetic gradient series, cash flow series that increase or decrease by constant amounts, and geometric gradient series, cash flow series that increase or decrease by constant percentages.

The following sections briefly address the equivalence calculations for a variety of cash flow series:

- ◆ Annuities or Uniform Series
 - Future Value of an Annuity
 - Annuity of a Future Value
 - Present Value of an Annuity
 - Annuity of a Present Value
- ◆ Amortization and Amortization Schedule
- ◆ Annuity Due
- ◆ Capitalized Cost
- ◆ Arithmetic Gradient Series
 - Present Value of an Arithmetic Gradient Series
 - Annuity of an Arithmetic Gradient Series
 - Future Value of an Arithmetic Gradient Series
- ◆ Geometric Gradient Series
 - Present Value of a Geometric Gradient Series
 - Annuity of a Geometric Gradient Series
 - Future Value of a Geometric Gradient Series
- ◆ Shifted Cash Flow Series
- ◆ Using Spreadsheets for Equivalences Computation
- ◆ Summary
- ◆ Discussion Questions and Problems

Annuities or Uniform Series

An annuity is defined as a uniform set of payments (A) that occur consecutively over a given period of time. Usually, the time intervals for the payments are annually, semi-annually, quarterly and

Aviation Snippets

Boeing is offering an option of a lump-sum payout or an annuity to 40,000 former employees who have not yet retired, the first time the company has offered such choices from its existing defined benefit plans.

Ron Taylor, Boeing, October 2014

monthly. There are many examples of annuity in practice. Common forms of annuities are seen in mortgages, pension funds, aircraft leases, property leases, car loans, insurance premiums, and retirement planning.

There are two types of annuities which differ based on the timing of annuity payments, ordinary annuity and annuity due. An ordinary annuity is when payment is made at the end of the period, while in annuity due, payment is made at the beginning of the period. Ordinary annuity is the type that is often used in engineering economics practice; therefore, in this book the word annuity refers to ordinary annuity with end-of-the-period convention.

Let us consider the case of aircraft leasing as an example of annuity. Considering the large investment needed to purchase an aircraft and noting the volatility of airline business, many airlines prefer to lease their fleet instead of owning them. Generally, there are three types of aircraft lease contracts: (1) operating lease, (2) capital lease, and (3) secure lease. In operating lease, an airline does not take the ownership risk of an aircraft but instead a financial company (e.g., banks, leasing companies, or even another airline), which owns the plane, accepts this risk, and the leasing airline pays for the right of using the asset as an expense. These types of leases are usually shorter (in comparison to the nominal life of an aircraft) and may have some restrictions and terms for operating the aircraft (e.g., restricting operations to a geographical region). These types of leases are very common in the airline industry and may be accompanied by other services (e.g., wet lease, where the lessor is generally another airline and provides an aircraft, complete crew, ground staff, maintenance, and insurance to the lessee; or a dry lease, where the lessor is generally a financing company and provides only the aircraft without crew, maintenance, and insurance). During the lease period, the leasing airline furnishes a set of monthly payments to the asset owner and at the end of the lease the aircraft is returned to the owner. On the other hand, under capital lease contract, the leasing airlines have the intent to own the leased aircraft at the end of the contract. This type of lease is usually longer and payments are expected to be higher in comparison to operating leases; however, at the end of the lease agreement, the leasing airlines have the option of taking the ownership of the aircraft at a discounted rate in comparison to fair market value of the asset at that time. Accordingly, this approach gives an airline an opportunity to gradually pay for the purchase of an aircraft while generating revenue operating it. One of the major advantages of this type of lease for the leasing airline is that it can claim depreciation on the aircraft that provides some tax benefit. We will discuss depreciation in detail in Chapter 11. Finally, in secure lending, the airline actually owns the aircraft but uses it as a security for borrowing capital from creditors. The details of this type of lease are outside the scope of our discussion; thus, we focus on the first two types for now.

As can be anticipated, the main questions for evaluating the lease contract from the perspective of the airlines or the aircraft leasing companies is to evaluate the worth of a contract. For instance, an airline may be interested in assessing the annuity payments required by a lease; this can be done by transforming the annuity payments to a present value equivalent. On the other side, an aircraft leasing company that is contemplating a capital lease contract may be interested in figuring out how much in total they would collect by the end of the lease; that is, in calculating the future value of an annuity. In this section, we will discuss the engineering economics tools needed to answer these questions.

There are four formulas related to annuities that compute future value of an annuity (F/A), annuity of a future single sum (A/F), present value of an annuity (P/A), and annuity of a present amount (A/P).

Future Value of an Annuity

The future value of an annuity is simply the sum of the future value of all payments at the end of n periods. We can express that in the following equation:

$$F = A(1+i)^{n-1} + A(1+i)^{n-2} + \dots + A(1+i)^3 + A(1+i)^2 + A(1+i)^1 + A$$

The above equation can be simplified as follows:

$$F = A \times \left[\frac{(1+i)^n - 1}{i} \right]$$

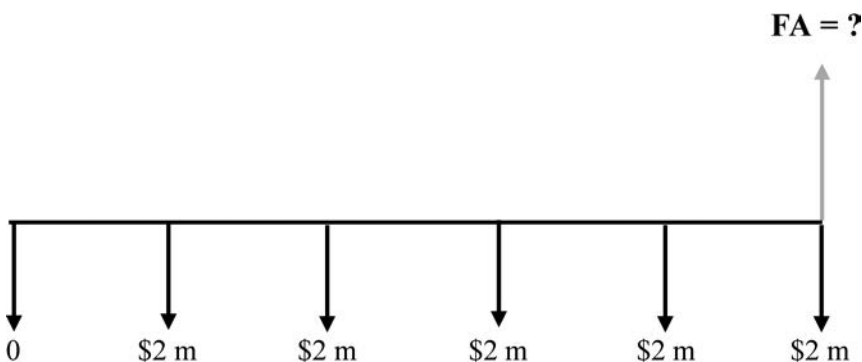
In aviation, it is customary to plan for major expected future expenditures. For instance, aircraft have to go through mandatory inspections based on their flight hours and cycle times. Typically, we refer to these inspections as A check, B check, C check, and D check. While A checks and B checks are lighter inspections, C and D checks are thorough, time consuming, and expensive. D check, which is performed about every six years, is particularly expensive as it is labor consuming and takes the plane off operation for months. In current practice of airlines, an aircraft may undergo D check multiple times. Sometimes due to the notable cost of performing the D check, airlines may even forego the plane if the remaining economic life of the asset and its current market value do not justify the cost of the inspection. The overall D check expenditure for airlines with larger fleets requires them to plan and collect capital to pay for the inspection when the time comes.

Example 3.1

AvJets is considering purchasing a property in six years to expand its operations and is planning to deposit \$2 million in an investment account at the end of each year for six years. If AvJets earns 5 percent return on its investment, how much will AvJets accumulate in this account to purchase the property in six years?

Figure 3.1 shows the cash flow diagram. Plugging the information into the equation above, we can compute the future value.

Figure 3.1 Annuity Cash Flow Diagram



$$F_A = \$2 \times \left[\frac{(1 + 0.05)^6 - 1}{0.05} \right]$$
$$F_A = \$2 \times \left[\frac{1.3401 - 1}{0.05} \right]$$
$$F_A = \$2 \times \left[\frac{0.3401}{0.05} \right]$$
$$F_A = \$2 \times [6.8019]$$
$$F_A = \$13.60 \text{ million}$$

In the equation denoting future value of an annuity, the term $\left[\frac{(1 + i)^n - 1}{i} \right]$ is a future value of an annuity interest factor and can be expressed as $(F/A, i, n)$. The pre-calculated values of these interest factors are tabulated in Appendix A. Therefore, the equation can also be expressed as follows:

$$F_A = A(F/A, i, n)$$

Using Example 3.1, the future value of annual investments made by AvJets can be calculated using the above equation as follows:

$$F_A = \$2(F/A, 5\%, 6)$$

From Table 3.1, the value of the interest factor $(F/A, 5\%, 6)$ is equal to 6.8019. We now plug in the value of the interest factor, 6.8019, in the above relationship and compute the future value of an annuity.

Table 3.1 Compound Interest Factors for 5 Percent

5%			5%					
	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	F/A	A/F	P/A	A/P	P/G	A/G
1	1.0500	0.9524	1.0000	1.0000	0.9524	1.0500	0	0
2	1.1025	0.9070	2.0500	0.4878	1.8594	0.5378	0.9070	0.4878
3	1.1576	0.8638	3.1525	0.3172	2.7232	0.3672	2.6347	0.9675
4	1.2155	0.8227	4.3101	0.2320	3.5460	0.2820	5.1028	1.4391
5	1.2763	0.7835	5.5256	0.1810	4.3295	0.2310	8.2369	1.9025
6	1.3401	0.7462	6.8019	0.1470	5.0757	0.1970	11.9680	2.3579
7	1.4071	0.7107	8.1420	0.1228	5.7864	0.1728	16.2321	2.8052
8	1.4775	0.6768	9.5491	0.1047	6.4632	0.1547	20.9700	3.2445
9	1.5513	0.6446	11.0266	0.0907	7.1078	0.1407	26.1268	3.6758
10	1.6289	0.6139	12.5779	0.0795	7.7217	0.1295	31.6520	4.0991

$$F_A = \$2 \times [6.8019]$$

$$F_A = \$13.60 \text{ million}$$

Annuity of a Future Value

To calculate the annuity of a future sum, we solve the future value of annuity equation for A as follows:

$$F = A \times \left[\frac{(1+i)^n - 1}{i} \right]$$

$$A = F \times \left[\frac{i}{(1+i)^n - 1} \right]$$

In the beginning of this chapter, we discussed the aircraft leasing practices. One of the approaches was secure lending or debt financing. In this approach, an airline issues bonds to collect needed capital and an aircraft is used as loan security. Technically the ownership of the aircraft is transferred to creditors. Issuing bonds is a common practice to acquire capital. Bonds provide a guaranty of annuity with a predetermined maturity where the principal will be paid back. Now, the airline, as bond issuing entity, has the option of issuing bonds with sinking fund provision. Under this arrangement, the airlines establish a fund and regularly deposit money in the fund which in turn is used to buy the issued bonds off the market. The motivation behind the sinking fund is to build up the needed capital to pay for the issued bonds at their maturity. In other words, in this example the airline knows the amount of money needed in the future and needs to figure out the annuity to get to that target at the time of bond maturity. The equation used to find the annuity is also known as the sinking fund equation.

Example 3.2

An employee of AvParts, a newly hired engineer, has determined that she will need \$2,400,000 when she retires in 40 years to live comfortably during the retirement years. Assuming she earns 10 percent on her investment, how much will she need to invest at the end of each year in equal amount for 40 years to accumulate \$2,400,000? To find the answer, we plug in the information in the annuity equation.

$$A = F \times \left[\frac{i}{(1+i)^n - 1} \right]$$

$$A = \$2,400,000 \times \left[\frac{0.10}{(1+0.10)^{40} - 1} \right]$$

$$A = \$2,400,000 \times \left[\frac{0.10}{45.2593 - 1} \right]$$

$$A = \$2,400,000 \times 0.00226$$

$$A = \$5,424$$

In the equation that calculates annuity, the term $\left[\frac{i}{(1+i)^n - 1} \right]$ is an annuity of a future amount interest factor and can be expressed as $(A/F, i, n)$. The pre-calculated values

Table 3.2 Compound Interest Factors for 10 Percent

10%			10%					
n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	F/A	A/F	P/A	A/P	P/G	A/G
31	19.1943	0.0521	181.944	0.00550	9.4790	0.10550	78.6395	8.2962
32	21.1138	0.0474	201.138	0.00497	9.5264	0.10497	80.1078	8.4091
33	23.2252	0.0431	222.252	0.00450	9.5694	0.10450	81.4856	8.5152
34	25.5477	0.0391	245.477	0.00407	9.6086	0.10407	82.7773	8.6149
35	28.1024	0.0356	271.025	0.00369	9.6442	0.10369	83.9872	8.7086
40	45.2593	0.0221	442.593	0.00226	9.7791	0.10226	88.9525	9.0960
45	72.8905	0.0137	718.905	0.00139	9.8628	0.10139	92.4544	9.3740
50	117.391	0.0085	1163.91	0.00086	9.9148	0.10086	94.8889	9.5704

of these interest factors are tabulated in Appendix A. Therefore, the annuity equation can also be articulated as follows:

$$A = F(A/F, i, n)$$

Using Example 3.2, we use the above equation to compute the amount that she needs to invest at the end of each year in equal amounts for 40 years to accumulate \$2,400,000.

$$A = F(A/F, i, n)$$

$$A = \$2,400,000 (A/F, 10\%, 40)$$

From Table 3.2, the value of the interest factor ($A/F, 10\%, 40$) is equal to 0.00226. The table is a portion of Table 16 in Appendix A. We now plug in the interest factor value of 0.00226 in the above relationship and compute the annuity of a future amount.

$$A = \$2,400,000 \times (0.00226)$$

$$A = \$5,424$$

The employee should invest approximately \$5,424 per year to achieve her goal of accumulating \$2,400,000 for her retirement. The difference between using formula and pre-calculated values is due to rounding and both answers are acceptable.

Present Value of an Annuity

The present value of an annuity is simply the sum of the present value of all payments at present time or ($t = 0$). We can express that concept in the following equation:

$$P = A \times \frac{1}{(1+i)^1} + A \times \frac{1}{(1+i)^2} + \cdots + A \times \frac{1}{(1+i)^{n-1}} + A \times \frac{1}{(1+i)^n}$$

Let's get the factor of A of the above equation:

$$P = A \times \left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n} \right]$$

The above equation can be further simplified as follows:

$$P = A \times \left[\frac{1}{i} + \frac{1}{i \times (1+i)^n} \right]$$

This equation can be articulated into the following equation:

$$P = A \times \left[\frac{[(1+i)^n - 1]}{i \times (1+i)^n} \right]$$

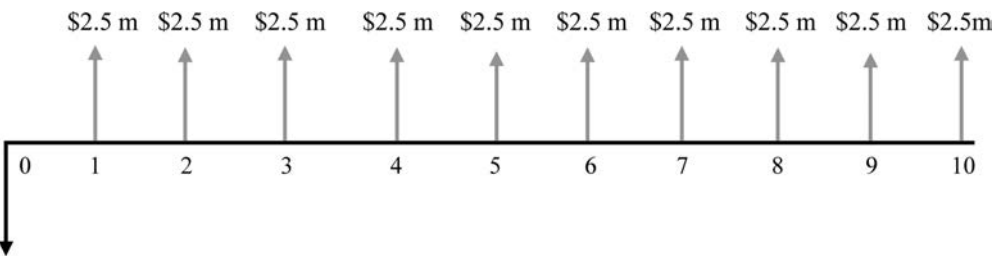
The final equation obtained can be used to calculate the present value of an annuity for uniform series.

Understanding the present worth of future annuity is a critical measure in evaluating an investment project that promises future revenue. Air cargo transportation often requires a location at the airport for receiving, processing, transshipping, and loading air cargo. Leasing cargo facilities is a major source of revenue for airports and existence of these facilities or extra services such as refrigerated warehouses for pharmaceutical and perishable products can act as a major attractor of airlines to a given airport. Indeed, airlines that operate out of an airport also provide other revenue for the airport in landing fees, parking fees, ramp access fees, and so forth. Airport authorities have few options for planning, development, construction, and marketing the cargo facilities. Airports can take charge of the development and do it in-house. Clearly, this option gives airports more control and flexibility and eventually better revenue. Additionally, airport-owned facilities tend to have lower rent which makes them more attractive for airlines and more marketable. This is due to the fact that airports often have access to cheaper capital through government funding or tax exempt bonds. On the other hand, using outside developers releases airports from the investment risk of building, maintaining, and marketing the facilities. In this approach, airports get into a long-term agreement with a third party to develop the facilities and provide the airport with a monthly revenue off leasing/using the facility. Indeed, a critical measure in helping airport authorities in making a decision about developing a cargo facility is an estimate of the present worth of the expected annuity to be collected from tenants.

Example 3.3

DirectJet is signing a lease agreement for a property to service aircraft. The lease payments are \$2.5 million per year for 10 years. If DirectJet can earn 6 percent on its investment, what is the present value of these payments today?

Figure 3.2 Annuity Cash Flow Diagram



PA = ?

Figure 3.2 shows the cash flow diagram for Example 3.3. Plugging the information into equation $P = A \times \left[\frac{[(1 + i)^n - 1]}{i \times (1 + i)^n} \right]$, we can compute the present value of the lease payments.

$$P = \$2,500,000 \times \left[\frac{[(1 + 0.06)^{10} - 1]}{0.06 \times (1 + 0.06)^{10}} \right]$$
$$P = \$2,500,000 \times [7.3601]$$
$$P = \$18,400,250$$

Table 3.3 shows the present value of each payment and sum of present value of all payments. The difference between the answer in Example 3.3 and the following table is due to rounding.

Table 3.3 Present Value of an Annuity Calculation

Time Period (n)	Lease Payment (A)	Interest Rate (i)	Present Value (P)
0	0	6%	0
1	\$2,500,000	6%	\$2,358,491
2	\$2,500,000	6%	\$2,224,499
3	\$2,500,000	6%	\$2,099,048
4	\$2,500,000	6%	\$1,980,234
5	\$2,500,000	6%	\$1,868,145
6	\$2,500,000	6%	\$1,762,401
7	\$2,500,000	6%	\$1,662,643
8	\$2,500,000	6%	\$1,568,531
9	\$2,500,000	6%	\$1,479,746
10	\$2,500,000	6%	\$1,395,987
Total	\$25,000,000		\$18,399,725

In the present value of annuity equation, the term $\left[\frac{[(1+i)^n - 1]}{i \times (1+i)^n} \right]$ is the present value of an annuity interest factor and can be expressed as $(P/A, i, n)$. The pre-calculated values of these interest factors are tabulated in Appendix A. Therefore, the equation can also be articulated to the following equation:

$$P = A(P/A, i, n)$$

Using Example 3.3, the present value of annual lease payments made by DirectJet can be calculated using the equation above as follows:

$$P = A(P/A, i, n)$$

$$P = \$2,500,000(P/A, 6\%, 10)$$

From Table 3.4, the present value interest factor $(P/A, 6\%, 10)$ is equal to 7.3601. We now plug in 7.3601 in the above relationship and compute the present value of all lease payments.

Table 3.4 Compound Interest Factors for 6 Percent

6%			6%					
	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/A	P/F	F/A	A/F	P/A	A/P	P/G	A/G
1	1.0600	0.9434	1.0000	1.0000	0.9434	1.0600	0	0
2	1.1236	0.8900	2.0600	0.4854	1.8334	0.5454	0.8900	0.4854
3	1.1910	0.8396	3.1836	0.3141	2.6730	0.3741	2.5692	0.9612
4	1.2625	0.7921	4.3746	0.2286	3.4651	0.2886	4.9455	01.4272
5	1.3382	0.7473	5.6371	0.1774	4.2124	0.2374	7.9345	1.8836
6	1.4185	0.7050	6.9753	0.1434	4.9173	0.2034	11.4594	2.3304
7	1.5036	0.6651	8.3938	0.1191	5.5824	0.1791	15.4497	2.7676
8	1.5938	0.6274	9.8975	0.1010	6.2098	0.1610	19.8416	3.1952
9	1.6895	0.5919	11.4913	0.0870	6.8017	0.1470	24.5768	3.6133
10	1.7908	0.5584	13.1808	0.0759	7.3601	0.1359	29.6023	4.0220
11	1.8983	0.5268	14.9716	0.0668	7.8869	0.1268	34.8702	4.4213
12	2.0122	0.4970	16.8699	0.0593	8.3838	0.1193	40.3369	4.8113
13	2.1329	0.4688	18.8821	0.0530	8.8527	0.1130	45.9629	5.1920
14	2.2609	0.4423	21.0151	0.0476	9.2950	0.1076	51.7128	5.5635
15	2.3966	0.4173	23.2760	0.0430	9.7122	0.1030	57.5546	5.9260

$$P = \$2,500,000 \times (7.3601)$$

$$P = \$18,400,250$$

Annuity of a Present Value

To calculate the annuity of a present value, we solve the present value of annuity equation for A as follows:

$$P = A \times \left[\frac{(1+i)^n - 1}{i \times (1+i)^n} \right]$$

$$A = P \times \left[\frac{i \times (1+i)^n}{(1+i)^n - 1} \right]$$

The above relationship is also known as loan payment and is used often for financial planning purposes. Individuals use this equation to calculate mortgage payments and car payments. Aviation companies use this formula to determine the uniform payments on their loans received from financial institutions. Airlines may use this equation to compute their lease payments on aircraft leasing.

Example 3.4

EZjET Airlines is expanding its operation and needs nine Boeing 737 Max. The current price of the aircraft is \$114 million. EZjET approaches International Financial Corporation (IFC) to finance the purchase. IFC agrees to grant a loan to EZjET to finance the entire purchase price at 8 percent compound interest for 20 years. What would be the annual loan payment for EZjET? To compute the annual loan payments, we use the annuity of a present value equation.

$$A = \$114,000,000 \times \left[\frac{0.08 \times (1 + 0.08)^{20}}{(1 + 0.08)^{20} - 1} \right]$$

$$A = \$114,000,000 \times \left[\frac{0.3729}{3.6610} \right]$$

$$A = \$114,000,000 \times (0.1019)$$

$$A = \$11,616,600$$

To use standard compound interest factor notation, the annuity of present value equation can be articulated as the following equation:

$$A = P(A/P, i, n)$$

Where the value of the interest factor ($A/P, i, n$) is pre-calculated and presented in Appendix A.

To compute the annual loan payment of the loan amount of \$114 million by EZjET, using standard compound factor notation, we plug in the data given in Example 3.4 into the following equation:

$$A = \$114,000,000 (A/P, 8\%, 20)$$

$$A = \$114,000,000 \times (0.1019)$$

$$A = \$11,616,600$$

Table 3.5 Amortization Schedule for EZjET (Numbers are in \$1,000)

Time Period	Payment Date	Beginning Balance	Schedule Payments	Total Payments	Principal	Interest	Ending Balance
1	7/1/2015	114,000.00	11,616.60	11,616.60	2,496.60	9,120.00	111,503.40
2	7/1/2016	111,503.40	11,616.60	11,616.60	2,696.33	8,920.27	108,807.07
3	7/1/2017	108,807.07	11,616.60	11,616.60	2,912.03	8,704.57	105,895.04
4	7/1/2018	105,895.04	11,616.60	11,616.60	3,145.00	8,471.60	102,750.04
5	7/1/2019	102,750.04	11,616.60	11,616.60	3,396.60	8,220.00	99,353.44
6	7/1/2020	99,353.44	11,616.60	11,616.60	3,668.32	7,948.28	95,685.12
7	7/1/2021	95,685.12	11,616.60	11,616.60	3,961.79	7,654.81	91,723.33
8	7/1/2022	91,723.33	11,616.60	11,616.60	4,278.73	7,337.87	87,444.60
9	7/1/2023	87,444.60	11,616.60	11,616.60	4,621.03	6,995.57	82,823.56
10	7/1/2024	82,823.56	11,616.60	11,616.60	4,990.71	6,625.89	77,832.85
11	7/1/2025	77,832.85	11,616.60	11,616.60	5,389.97	6,226.63	72,442.88
12	7/1/2026	72,442.88	11,616.60	11,616.60	5,821.17	5,795.43	66,621.71
13	7/1/2027	66,621.71	11,616.60	11,616.60	6,286.86	5,329.74	60,334.84
14	7/1/2028	60,334.84	11,616.60	11,616.60	6,789.81	4,826.79	53,545.03
15	7/1/2029	53,545.03	11,616.60	11,616.60	7,333.00	4,283.60	46,212.03
16	7/1/2030	46,212.03	11,616.60	11,616.60	7,919.64	3,696.96	38,292.39
17	7/1/2031	38,292.39	11,616.60	11,616.60	8,553.21	3,063.39	29,739.19
18	7/1/2032	29,739.19	11,616.60	11,616.60	9,237.47	2,379.13	20,501.72
19	7/1/2033	20,501.72	11,616.60	11,616.60	9,976.46	1,640.14	10,525.26
20	7/1/2034	10,525.26	11,616.60	10,525.26	9,683.24	842.02	0.00
Total					114,000.0		

Amortization and Amortization Schedule

In the previous section, we discussed loan payment as annuity payment of a present loan. Indeed, as we mentioned in Chapter 2, the payment consists of principal and compound interest for the privilege of having the capital during the course of the loan. We mentioned how to calculate the annuity payment for a given loan based on the interest rate and length

Amortizing means spreading out the cost of an asset over the duration of its lifespan. Note, amortization is not used for tangible assets. It is mainly used for intangible assets such as trademarks, patents on inventions, licenses, and goodwill.

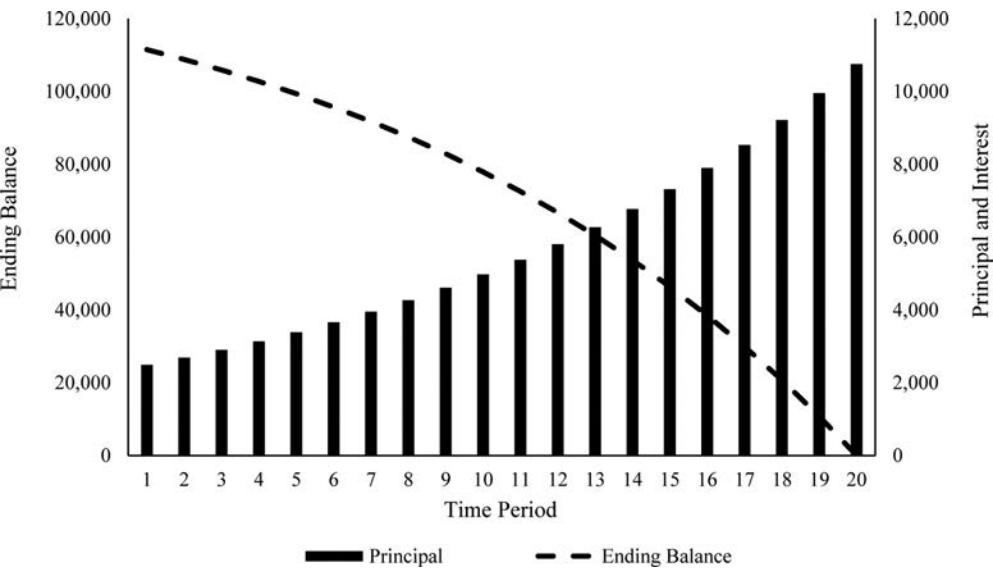
of the loan. In practice, however, investors and lenders often prefer to separate the payment into the interest and principal. So for each payment a part of the payment contributes toward interest and the remaining for retiring of the principal. Amortization is the distribution of a single cash flow into smaller cash flows. Amortization schedule is a loan payment schedule which dictates the payments contribution share toward interest and principal over the life of the loan. This schedule consists of a number of payments, the balance at the beginning of each period, total payments per period, dollar amount of principal and interest paid each period, and the balance at the end of the period. The amortized schedule is often set in a way that a larger portion of payment in the beginning of the loan payback goes to cover the interest (as high as 80 to 90 percent). As the loan gets closer to maturity, the share of principal payback increases as the interest portion is mostly paid back already. Disregarding the share of interest and principal, the payment instalments are often the same. There are several methods for calculating the share of interest vs. principal for payment.

Amortized schedule is very common in real estate mortgage, although it is used for other loan payments as well. Consider the following example.

Example 3.5

Principal Borrowed:	\$100,000.00	Annual Payments:	12
Regular Payment amount:	\$599.55	Total Payments:	360 (30.00 years)
Interest-only payment:	\$500.00	Annual interest rate:	6.00%
Total Repaid:	\$215,838.00	Periodic interest rate:	0.5000%
Total Interest Paid:	\$115,838.00	Debt Service Constant:	7.1946%
		Total interest paid as a percentage of Principal:	115.838%

Figure 3.3 Amortization Schedule Depiction



As can be seen in this example, the total loan is \$100,000 for 30 years. The interest rate is 6 percent, which is equivalent to a monthly rate of 0.5 percent. Using the annuity of present value equation, we can calculate the annuity payment as \$599.55. Now, Figure 3.3 shows how this payment is contributing to paying back interest and principal throughout the length of the loan.

Annuity Due

Annuity due is similar to ordinary annuity except that payments are made at the beginning of the period rather than the end of the period. Because payments are made at the beginning of each interval, both future and present values of annuity due will earn one extra period of interest. Therefore, to compute the future value of an annuity due, we multiply future value of ordinary annuity by $(1 + i)$.

$$F_{ordinary} = A \times \left[\frac{(1+i)^n - 1}{i} \right]$$

$$F_{due} = A \times \left[\frac{(1+i)^n - 1}{i} \right] \times (1+i)$$

$$F_{ordinary} = A(F/A, i, n)$$

$$F_{due} = A(F/A, i, n) \times (1+i)$$

Similarly, to compute the present value of an annuity due, we multiply the present value of ordinary annuity by $(1 + i)$.

$$P_{ordinary} = A \times \left[\frac{(1+i)^n - 1}{i \times (1+i)^n} \right]$$

$$P_{due} = A \times \left[\frac{(1+i)^n - 1}{i \times (1+i)^n} \right] \times (1+i)$$

$$P_{ordinary} = A(P/A, i, n)$$

$$P_{due} = A(P/A, i, n) \times (1+i)$$

In Example 3.3, if DirectJet Airline makes the lease payments of \$2.5 million per year at the beginning of each year for 10 years, the present value of the annuity due payments can be computed as follows:

$$P_{due} = A(P/A, i, n) \times (1+i)$$

$$P_{due} = \$2,500,000 (P/A, 6\%, 10) \times (1 + 6\%)$$

$$P_{due} = \$2,500,000 \times (7.3601) \times (1.06)$$

$$P_{due} = \$19,504,265$$

Capitalized Cost

Capitalized cost is a term used in engineering economics and it refers to the present worth of a project with an infinite life. In other words, capitalized cost is a lump sum

of money needed today ($t = 0$) to support an infinite life project simply on earned interest only. The concept of capitalized cost usually applies to public projects such as airports, bridges, dams, and long-term private projects such as hospitals and private airports. Since most present value interest factors are the same after 50 to 100 years depending on interest rates, the concept of perpetual annuity may be used to determine the present worth of infinite life projects as capitalized cost.

Since it is difficult to calculate the capitalized cost of a project with a stream of infinite cash flows when they vary from year to year or they occur irregularly, it is necessary first to convert those cash flows to a uniform series or annuity and then use the perpetual annuity concept to calculate the capitalized cost of that project.

Capitalized cost (CC) can also be defined as the present worth of an annuity that will last forever. To drive the CC equation, we use the present value of annuity formula, assuming n approaches infinity.

$$P = A \times \left[\frac{(1+i)^n - 1}{i \times (1+i)^n} \right]$$

By dividing both numerator and denominator of the above equation by $(1+i)^n$, it can be converted to another equation as follows:

$$P = A \left[\frac{1 - \frac{1}{(1+i)^n}}{i} \right]$$

In the above equation, if $n \rightarrow \infty$, the value of the numerator will be reduced to one and the equation will be simplified as follows:

$$P = CC = A \times \frac{1}{i}$$

$$CC = \frac{A}{i}$$

In other words, capitalized cost (CC) is equal to annuity payment divided by interest rate.

Example 3.6

An insurance agent approaches you and makes an offer of paying you \$1,000 per year forever. If you earn 8 percent on your savings account, what is the value of this offer to you? In other words, what is the maximum price you are willing to pay for this offer?

To find the maximum price for this offer, we compute the CC of this annuity at an interest rate of 8 percent.

$$CC = \frac{\$1,000}{0.08}$$

$$CC = \$12,500$$

The answer is \$12,500 because if you had \$12,500 and deposited it into your savings account, every year you would receive \$1,000 ($\$12,500 \times 0.08 = \$1,000$) in interest. So long as the money is deposited in your account and interest rate remains at 8 percent, you would receive \$1,000 per year forever.

Arithmetic Gradient Series

In engineering economics, not all cash flows are uniform series or annuity series but they may be graduating annuities. A graduating annuity is a stream of payments or cash flows that occur at the end of each period for a given time period similar to annuities, but payments are not constant and are changing in a uniform way. An arithmetic gradient series is a graduated annuity series in which cash flows uniformly increase or decrease by a constant amount. Figure 3.4 presents an arithmetic gradient cash flow series in which A is the base amount that occurs at the end of year 1 and G is the gradient amount; the increase in cash flows starts at the end of year 2 and continues through the end of the cash flow period.

Present Value of an Arithmetic Gradient Series

To drive an equation for computing the present value of an arithmetic gradient, we divide the cash flow series in Figure 3.4 into two series. The first series is an annuity of A amount per year for six years. The second series is the gradient amount G , a series of G amount in year 2 that increases by the same amount every year as shown in Figure 3.5.

To compute the present value of the first cash flow series, we use the equation,

$$P_1 = A \times \left[\frac{(1+i)^n - 1}{i \times (1+i)^n} \right]$$

To compute the present value of the second cash flow series, we use the equation shown below:

$$P_2 = \frac{G}{i} \times \left[\frac{(1+i)^n - 1}{i \times (1+i)^n} - \frac{n}{(1+i)^n} \right]$$

Figure 3.4 Arithmetic Gradient Series

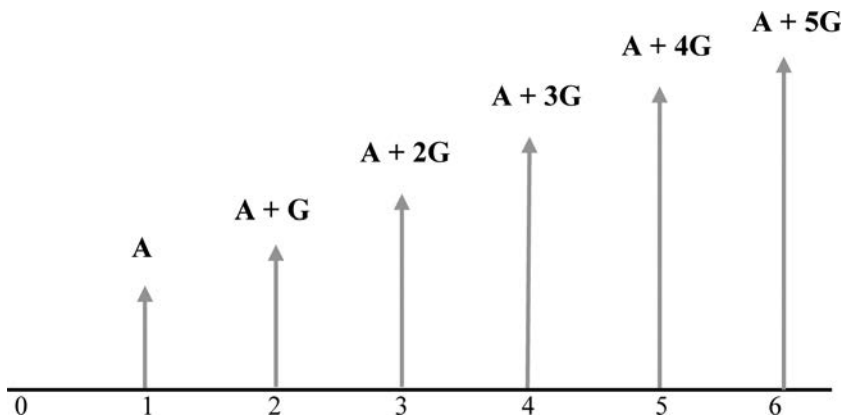
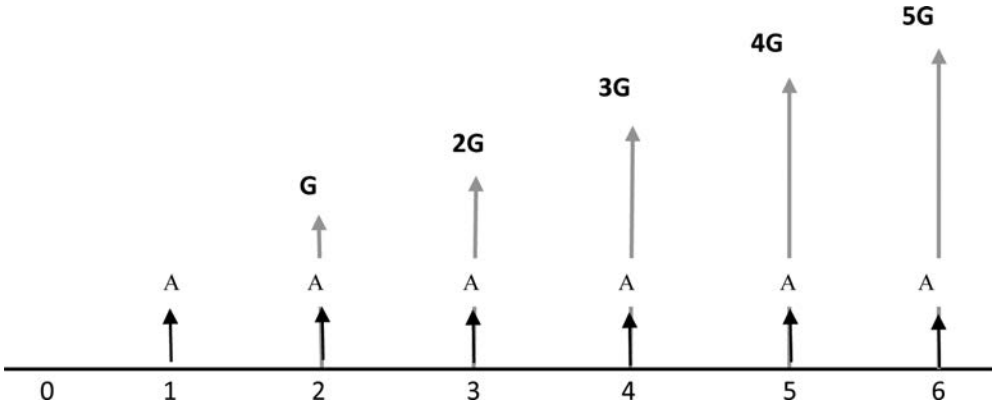


Figure 3.5 Arithmetic Gradient Series

The above equation is derived from the cash flows in Figure 3.5. The present value of all gradient amounts are calculated to present time value; add them all together, and simplify the mathematics to derive the equation.

To compute the present value of the whole arithmetic gradient series, we add the above two equations into one general equation as follows:

$$P_G = A \times \left[\frac{(1+i)^n - 1}{i \times (1+i)^n} \right] + \frac{G}{i} \times \left[\frac{(1+i)^n - 1}{i \times (1+i)^n} - \frac{n}{(1+i)^n} \right]$$

To use standard compound interest factor notation, the above equation can also be articulated to an equation as follows:

$$P_G = A (P/A, i, n) + G (P/G, i, n)$$

Where the values of the interest factors $(P/A, i, n)$ and $(P/G, i, n)$ are pre-calculated and presented in Appendix A.

Example 3.7

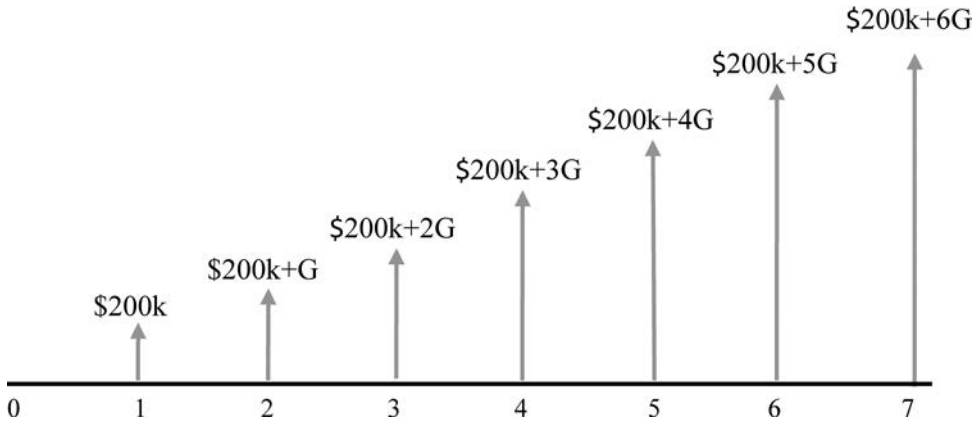
An aerospace engineer is signing up a consulting agreement with an airport in Solomon Islands. The length of the contract is seven years and consulting fees are \$200,000 at the end of year 1 increasing by \$25,000 per year starting at the end of the second year through the end of the contract. Assuming the engineer earns 8 percent return on his investment, what is the present value of his consulting contract?

Figure 3.6 presents the cash flow diagram for this contract. The \$200,000 is the base amount ($A = \$200,000$) and the \$25,000 increase per year is the gradient amount ($G = \$25,000$).

To calculate the present value of the arithmetic gradient series in Example 3.7, we apply the data given into the equation as follows:

$$P_G = \$200,000 (P/A, 8\%, 7) + \$25,000 (P/G, 8\%, 7)$$

From Table 3.6, the present value interest factor for the annuity $(P/A, 8\%, 7)$ is equal to 5.2064 and the present value interest factor of the gradient $(P/G, 8\%, 7)$ is equal to

Figure 3.6 Arithmetic Gradient Series**Table 3.6** Compound Interest Factors for 8 Percent

8%							8%	
n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	F/A	A/F	P/A	A/P	P/G	A/G
1	1.0800	0.9259	1.0000	1.0000	0.9259	1.0800	0	0
2	1.1664	0.8573	2.0800	0.4808	1.7833	0.5608	0.9573	0.4808
3	1.2597	0.7938	3.2464	0.3080	2.5771	0.3880	2.4450	0.9487
4	1.3605	0.7350	4.5061	0.2219	3.3121	0.3019	4.6501	1.4040
5	1.4693	0.6806	5.8666	0.1705	3.9927	0.2505	7.3724	1.8465
6	1.5869	0.6302	7.3359	0.1363	4.6229	0.2163	10.5233	2.2763
7	1.7138	0.5835	8.9228	0.1121	5.2064	0.1921	14.0242	2.6937
8	1.8509	0.5403	10.6366	0.0940	5.7466	0.1740	17.8061	3.0985
9	1.9990	0.5002	12.4876	0.0801	6.2469	0.1601	21.8081	3.4910
10	2.1589	0.4632	14.4866	0.0690	6.7101	0.1490	25.9768	3.8713

14.0242. We now plug in the value of the interest factors in the above relationship and compute the present value of this arithmetic gradient series.

$$P_G = \$200,000 (5.2064) + \$25,000 (14.0242)$$

$$P_G = \$1,041,280 + \$350,605$$

$$P_G = \$1,391,885$$

Annuity of an Arithmetic Gradient Series

To convert an arithmetic gradient series into an annuity or uniform series, we divide the cash flow series in Figure 3.7 into two series. First, we have the cash flow series of the base amount A_1 , which is a constant amount per year for six years. Second, we convert the cash flow series of the gradient amount G to a uniform series of A_2 per year using the following equation:

$$A_2 = G \times \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1} \right]$$

The total annuity or uniform series of an arithmetic gradient series (A_G) is simply the sum of the annuity series A_1 and annuity series A_2 .

$$A_G = A_1 + A_2$$

Plugging in the first equation into the above equation provides us with a formula to convert an arithmetic gradient series to an annuity series as shown in the following equation:

$$A_G = A_1 + G \times \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1} \right]$$

In Example 3.7, convert the engineer's consulting fees over the seven years into a uniform series.

We plug in the information in Example 3.7 into the above equation to compute the uniform amount.

$$A_G = \$200,000 + \$25,000 \times \left[\frac{1}{0.08} - \frac{7}{(1 + 0.08)^7 - 1} \right]$$

$$A_G = \$200,000 + \$25,000 \times [12.50 - 9.8063]$$

$$A_G = \$200,000 + \$25,000 \times [2.6937]$$

$$A_G = \$200,000 + \$67,325$$

$$A_G = \$267,325$$

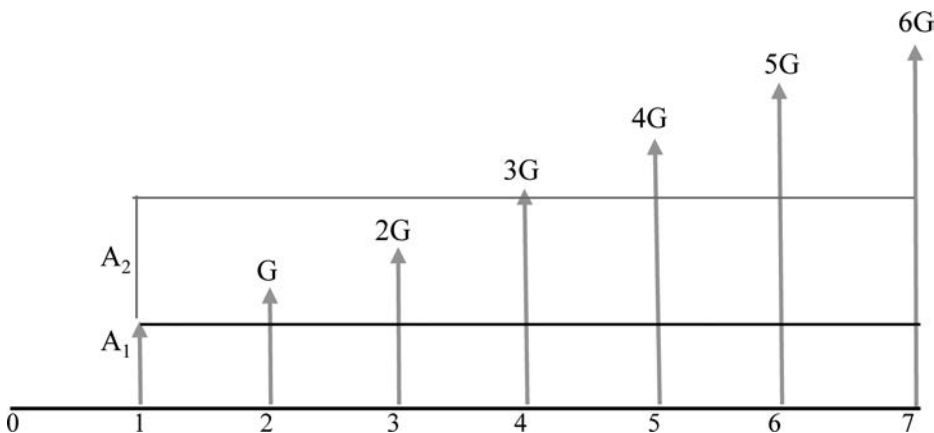


Figure 3.7 Arithmetic Gradient Series

To use standard compound interest factor notation, the equation can also be articulated as follows:

$$A_G = A_1 + G (A/G, i, n)$$

Where the value of the interest factor $(A/G, i, n)$ is pre-calculated and presented in Appendix A.

To compute the annuity of the arithmetic gradient series in Example 3.7, using standard compound factor notation, we apply the data given in the example into the equation as follows:

$$A_G = \$200,000 + \$25,000(A/G, 8\%, 7)$$

From Table A14 in Appendix A, for 8 percent interest rate, which is partially shown here as Table 3.7, the annuity of the gradient interest factor $(A/G, 8\%, 7)$ is equal to 2.6937. We now plug in the value of the interest factor in the above relationship and compute the annuity of this arithmetic gradient series.

$$A_G = \$200,000 + \$25,000(2.6937)$$

$$A_G = \$200,000 + \$67,325$$

$$A_G = \$200,000 + \$67,325$$

$$A_G = \$267,325$$

Table 3.7 Compound Interest Factors for 8 Percent

8%							8%	
n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	F/A	A/F	P/A	A/P	P/G	A/G
1	1.0800	0.9259	1.0000	1.0000	0.9259	1.0800	0	0
2	1.1664	0.8573	2.0800	0.4808	1.7833	0.5608	0.9573	0.4808
3	1.2597	0.7938	3.2464	0.3080	2.5771	0.3880	2.4450	0.9487
4	1.3605	0.7350	4.5061	0.2219	3.3121	0.3019	4.6501	1.4040
5	1.4693	0.6806	5.8666	0.1705	3.9927	0.2505	7.3724	1.8465
6	1.5869	0.6302	7.3359	0.1363	4.6229	0.2163	10.5233	2.2763
7	1.7138	0.5835	8.9228	0.1121	5.2064	0.1921	14.0242	2.6937
8	1.8509	0.5403	10.6366	0.0940	5.7466	0.1740	17.8061	3.0985
9	1.9990	0.5002	12.4876	0.0801	6.2469	0.1601	21.8081	3.4910
10	2.1589	0.4632	14.4866	0.0690	6.7101	0.1490	25.9768	3.8713

Future Value of an Arithmetic Gradient Series

To derive an equation for computing the future value of an arithmetic gradient, we divide the cash flow series in Figure 3.4 into two series. First, the cash flow series of the base amount A per year, an annuity of A amount per year for six years. Second, cash flow series of the gradient amount G , a series of G amount in year 2 and increase by the same amount per year as shown in Figure 3.5.

To compute the future value of the first cash flow series (the annuity series), we use the following equation:

$$F_1 = A \times \left[\frac{(1+i)^n - 1}{i} \right]$$

To compute the future value of the second cash flow series, we use the equation shown below:

$$F_2 = \frac{G}{i} \times \left[\frac{(1+i)^n - 1}{i} - n \right]$$

The above equation is derived from the cash flows in Figure 3.5. The future value of all gradient amounts are calculated at the end of the time period; add them all together, and simplify the mathematics to derive the equation.

To compute the future value of the whole arithmetic gradient series, we add the above two equations into one general equation as follows:

$$F_G = A \times \left[\frac{(1+i)^n - 1}{i} \right] + \frac{G}{i} \times \left[\frac{(1+i)^n - 1}{i} - n \right]$$

To use standard compound interest factor notation, the above equation can also be articulated as follows:

$$F_G = A (F/A, i, n) + \frac{G}{i} [(F/A, i, n) - n]$$

Where the value of the interest factors $(F/A, i, n)$ is pre-calculated and presented in Appendix A.

Example 3.8

A dispatcher who works for Frankfurt International Airport would like to retire in 20 years. He is planning to invest \$10,000 at the end of year 1 and increase that amount by a constant amount of \$1,000 per year for 20 years. Assuming that the dispatcher is earning 12 percent on his long-term investment, how much can he accumulate in his retirement account by the end of 20 years?

We simply plug the information into the future value of arithmetic gradient series equation.

$$F_G = \$10,000 (F/A, 12\%, 20) + \frac{\$1,000}{0.12} [(F/A, 12\%, 20) - 20]$$

From Table A18 in Appendix A, for 12 percent interest rate, we plug in the future value interest factor for the annuity $(F/A, i, n)$ in the above equation.

$$F_G = \$10,000 (72.0524) + \frac{\$1,000}{0.12} [72.0524 - 20]$$

$$F_G = \$720,524 + \$433,770$$

$$F_G = \$1,154,294$$

To compute the future value of an arithmetic gradient, it is also possible to calculate the present value of the series and then compute the future value of that present single amount at the end of the period.

Using Example 3.8, first we compute the present value of the arithmetic gradient and then compute the future value of that present amount.

First, use the present value of gradient series equation to compute the present value of the gradient series.

$$P_G = \$10,000 (P/A, 12\%, 20) + \$1,000 (P/G, 12\%, 20)$$

From Table A18 in Appendix A, for 12 percent interest rate, the present value interest factor for the annuity ($P/A, 12\%, 20$) is equal to 7.4694 and the present value interest factor of the gradient ($P/G, 12\%, 20$) is equal to 44.9676. We now plug in the value of the interest factors in the above relationship and compute the present value of this arithmetic gradient series.

$$P_G = \$10,000 (7.4694) + \$1,000 (44.9676)$$

$$P_G = \$74,694 + \$44,968$$

$$P_G = \$119,662$$

Next, we use the equation to compute the future value of the dispatcher contributions to the retirement account at the end of 20 years.

$$F_{20} = \$119,662 \times (1 + 0.12)^{20}$$

$$F_{20} = \$119,662 \times (9.6463)$$

$$F_{20} = \$1,154,296$$

Geometric Gradient Series

Geometric gradient series is similar to arithmetic gradient where the cash flows are increasing uniformly, except that the increase per period is not a constant amount. In geometric gradient series the cash flows grow at a constant percentage rate (g). Figure 3.8 presents a geometric gradient cash flow series in which A_1 is the base amount that occurs at the end of year 1 and g is the gradient rate. The constant percentage increase in cash flows which starts at the end of year 2 and continues through the end of the cash flow period.

Present Value of a Geometric Gradient Series

To compute the present value of a geometric gradient, we use the following equation:

$$P_g = A_1 \times \left[\frac{1 - \left[\frac{1+g}{(1+i)} \right]^n}{i - g} \right]$$

This equation is valid as long as $i \neq g$.

This equation is used often to determine the levels of activity where the change in the next period's cash flow is a percentage of the previous period's cash flow.

Example 3.9

Cirrus Aircraft is leasing a property for an aircraft hangar for 10 years. The first lease payment is \$20,000 and it is due at the end of the first year. Starting from the second year, the lease payments will increase by a constant 5 percent per year through the end of 10 years of the lease contract. If Cirrus can earn 10 percent return on its investment, how much should Cirrus deposit as a lump sum today in that investment account to be able to pay the lease payments every year for the next 10 years?

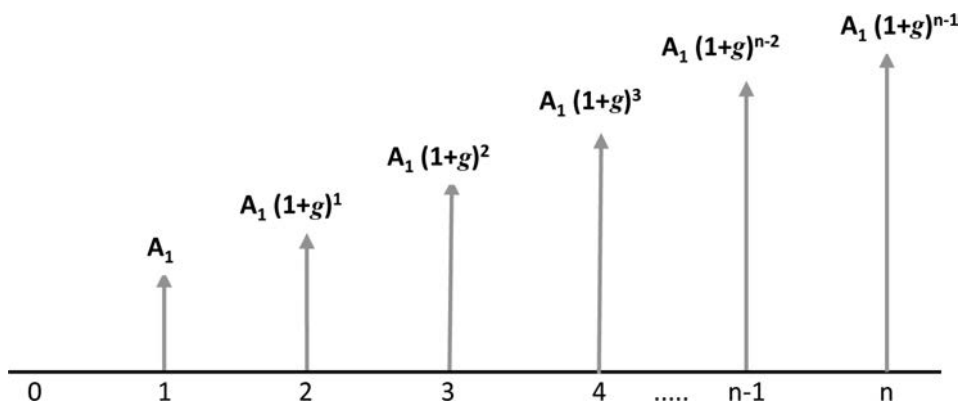
In this example, $A_1 = \$20,000$, $g = 5\%$, $i = 10\%$, $n = 10$ years, $P_g = ?$

To compute the present value of this geometric gradient series, we use the following equation:

$$P_g = A_1 \times \left[\frac{1 - \left[\frac{1+g}{(1+i)} \right]^n}{i - g} \right]$$

$$P_g = \$20,000 \times \left[\frac{1 - \left[\frac{1+0.05}{(1+0.10)} \right]^{10}}{0.10 - 0.05} \right]$$

Figure 3.8 Geometric Gradient Series



$$P_g = \$20,000 \times \left[\frac{1 - 0.6280}{0.05} \right]$$

$$P_g = \$20,000 \times (7.44)$$

$$P_g = \$148,800$$

If interest rate is equal to the growth rate, we cannot use the above equation; the present value of the geometric gradient can be calculated using the equation shown below:

$$P_g = \frac{n \times A_1}{1 + i}$$

Annuity of a Geometric Gradient Series

To convert a geometric gradient to an annuity or a uniform series, it is best to calculate the present value of the geometric gradient and then compute the annuity of that present single amount for a 10-year period.

Assume in Example 3.9, we were asked to compute the annuity of the lease payments over the 10-year period. First, we would have calculated the present value of the geometric gradient series as shown in Example 3.9. Next, we compute the annuity of the present value as follows:

$$P_g = \$148,800$$

$$A = P(A/P, i, n)$$

$$A = \$148,800(A/P, 10\%, 10)$$

From Table A16 in Appendix A, for a 10 percent interest rate, the annuity of the present value interest factor ($A/P, 10\%, 10$) is equal to 0.16275. We now plug in the value of the interest factors in the above relationship and compute the annuity of the present amount.

$$A = \$148,800 \times (0.16275)$$

$$A = \$24,217$$

In other words, Cirrus, instead of depositing a single sum of \$148,800, now may deposit a uniform series of \$24,217 at the end of each year for 10 years to be able to make the lease payments.

Future Value of a Geometric Gradient Series

To compute the future value of a geometric gradient, similar to arithmetic gradient, it is best to calculate the present value of the series and then compute the future value of that present single amount at the end of the period.

Example 3.10

Unite Technologies offers retirement benefits to its employees and matches up to 5 percent of the employees' contributions. An engineer who was just hired by Unite Technologies started with a salary of \$100,000 in the first year with an increase of

4 percent per year for 30 years. The employee who wants to take advantage of matching by employer will contribute 5 percent of his salary every year. If the employee can earn 10 percent return on his retirement investment, how much will he accumulate in his retirement account at the end of 30 years?

For geometric gradient series, first we compute the present value of the series and then compute the future value of that present amount.

Total contributions by employee and employer in the first year = \$100,000 (0.05 + 0.05) = \$10,000. This amount will increase by the same percentage increase of employee's salary or by 4 percent.

First use the following equation to compute present value of the geometric gradient.

$$P_g = \$10,000 \times \left[\frac{1 - \left[\frac{1 + 0.04}{(1 + 0.10)} \right]^{30}}{0.10 - 0.04} \right]$$

$$P_g = \$10,000 \times \left[\frac{1 - 0.1859}{0.06} \right]$$

$$P_g = \$10,000 \times (13.5683)$$

$$P_g = \$135,683$$

Second, calculate the future value of the present amount at the end of 30 years.

$$F_{30} = \$135,683 \times (1 + 0.10)^{30}$$

$$F_{30} = \$135,683 \times (17.4494)$$

$$F_{30} = \$2,367,587$$

Shifted Cash Flow Series

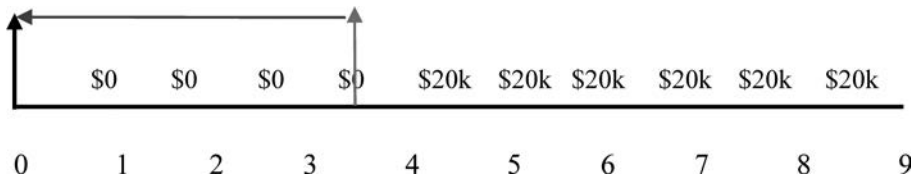
Not all cash flow series start at the end of the first year. In that case, we have to calculate the present value of the cash flow stream at the beginning of the year when the payment starts and then shift it as a single amount to time zero ($t = 0$).

Example 3.11

You are considering an investment opportunity that provides no revenues in the first four years, but starting at the end of year 5, revenue is expected to be \$20,000 per year for five years. If you earn 8 percent on your investment, what is the maximum price that you are willing to pay for this investment opportunity?

The maximum price that you are willing to pay for this investment is the present value of these cash flows discounted at 8 percent to the time zero. Figure 3.9 presents the cash flow diagram.

To calculate the present value of cash flows at the time zero, we first compute the present value of the cash flow at the beginning of year 5 or end of year 4 (P_4) and then shift the P_4 to the time zero to compute P_0 .

Figure 3.9 Shifted Cash Flow Series

$$P_4 = \$20,000 (P/A, 8\%, 6)$$

From Table A14 in Appendix A, for 8 percent interest rate, the present value of annuity interest factor ($P/A, 8\%, 6$) is equal to 4.6229. We now plug in the value of the interest factor in the above relationship and compute the present value.

$$P_4 = \$20,000 \times (4.6229)$$

$$P_4 = \$92,458$$

Next, we compute the present value of P_4 at the time zero as follows:

$$PV_0 = P_4 (P/F, 8\%, 4)$$

From Table A14 in Appendix A, for 8 percent interest rate, the present value interest factor ($P/F, 8\%, 4$) is equal to 0.7350. We now plug in the value of the interest factor in the above relationship and compute the present value.

$$P_0 = \$92,458 \times (0.7350)$$

$$P_0 = \$67,957$$

By the same reasoning, not all cash flow series continue to the end of a given time period. If we need to calculate the future value of cash flows at the end of the period, first we calculate the future value of cash flows at the end of the year when cash flow stops and then shift that future value to the end of the time period.

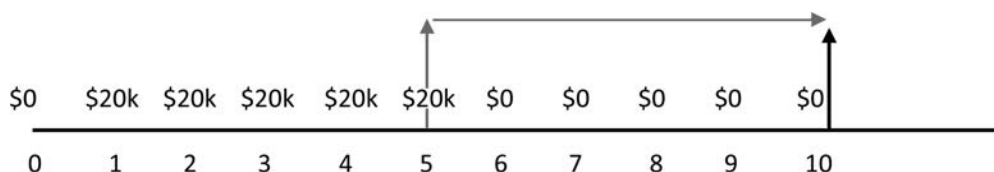
Example 3.12

Booz Allen Jackson is planning to replace equipment in 10 years. The company is investing \$20,000 at the end of each year for the first five years with no additional investment. If Booz Allen can earn 9 percent on its investment, how much can it accumulate at the end of 10 years? Figure 3.10 presents the cash flow diagram.

To calculate the future value of cash flows at the end of 10 years, we first compute the future value of the cash flow at the end of year 5 (F_5) where the last deposit occurs and then shift the F_5 to the end of year 10 to compute F_{10} .

$$F_5 = \$20,000 (F/A, 9\%, 5)$$

From Table A15 in Appendix A, for 9 percent interest rate, the future value of annuity interest factor ($F/A, 9\%, 5$) is equal to 5.9847. We now plug in the value of the

Figure 3.10 Shifted Cash Flow Series

interest factor in the above relationship and compute the future value at the end of five years (F_5).

$$F_5 = \$20,000 (5.9847)$$

$$F_5 = \$119,694$$

Next, we compute the future value of F_5 at the end of year 10 as follows:

$$F_{10} = F_5 (F/P, 9\%, 5)$$

From Table A15 in Appendix A, for 9 percent interest rate, the future value interest factor ($F/P, 9\%, 5$) is equal to 1.5386. We now plug in the value of the interest factor in the above relationship and compute the future value (F_{10}).

$$F_{10} = \$119,694 \times (1.5386)$$

$$F_{10} = \$184,161$$

Using Spreadsheets for Equivalences Computation

This section presents how to run the financial equations referenced in this chapter using Microsoft Excel functions. To compute future value of an annuity in Excel, first we open the future value function. This formula is listed under the financial formulas option. Once this step is accomplished, the user can answer the formula questions and then Excel will solve this function.

Using Example 3.1, compute the future value of \$2 million at the end of each year for six years at 5 percent interest. As shown in Figure 3.11, enter 5 percent or 0.05 in the Rate line for interest rate, 6 in the Nper line for time period, and -2,000,000 in the Pmt line as an annuity amount, and zero for Pv. Excel will compute the solution for the future value of an annuity which in this case is \$13,603,826. Note that the Pmt value (annuity amount) is entered as a negative number to get a positive solution for future value of the annuity.

Alternatively, one can simply enter the following function in the Excel cell and press the enter key to get the result in that cell:

$$=FV(i\%,n,-A,0)$$

Using Example 3.1, enter the function $=FV(5\%,6,-2000000,0)$ in an Excel cell and press return: the resulting future value of annuity of \$13,603,826 appears in that cell. We use a negative value for annuity payments to get a positive future value.

Figure 3.11 Calculating the Future Value Using Excel Spreadsheets

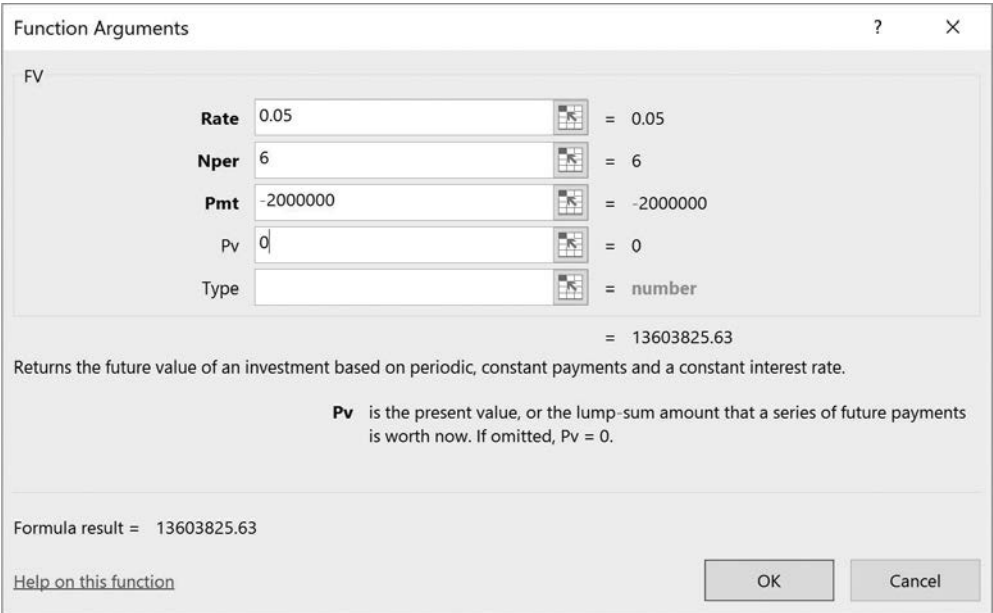


Figure 3.12 Calculating the Future Value Using Excel Spreadsheets

	A	B
1	=FV(5%,6,-2000000,0)	
2	FV(rate, nper, pmt, [pv], [type])	

	A	B
1	\$13,603,826	
2		

The present value of an annuity function in Excel is presented in the equation below:

=PV(i%,n,-A,0)

In Example 3.3, if we plug in the equation =PV(6%,10,-2500000,0) in an Excel cell and press return the answer \$18,400,217 appears in that cell, which is the present value of an annuity of \$2.5 million for 10 years at 6 percent interest rate. Similarly, we use a negative value for the annuity payment to get a positive present value.

Figure 3.13 Calculating the Present Value Using Excel Spreadsheets

	A	B
1	=PV(6%,10,-2500000,0)	
2	PV(rate, nper, pmt, [fv], [type])	

	A	B
1	\$18,400,218	
2		

Figure 3.14 Calculating the Annuity of a Future Amount Using Excel Spreadsheets

	A	B
1	=PMT(5%,6,0,-13603826)	
2	PMT(rate, nper, pv, [fv], [type])	

	A	B
1	\$2,000,000	
2		

Figure 3.15 Calculating the Annuity of a Present Amount Using Excel Spreadsheets

	B	C
1	=PMT(6%,10,-18400218,0)	
2	PMT(rate, nper, pv, [fv], [type])	

	B	C
1	\$2,500,000	
2		

The Excel functions to compute the annuity of a future amount and the annuity of a present amount are presented in the following equations. The notation PMT is the payment or the annuity amount.

Annuity of a future amount =PMT(i%,n,0,-F)

Annuity of a present amount =PMT(i%,n,0,-P)

The Excel functions to compute time period and the interest rate in an annuity series are presented below. The notation NPER is the time period and the notation RATE is the interest rate.

Time Period =NPER(i%,A,P,F)

Interest Rate =RATE(n,A,P,F)

In the above equations, one of the P or F has 0 value.

Figure 3.16 Calculating the Time Period Using Excel Spreadsheets

	A	B	C
1	=NPER(6%,2500000,-18400218,0)		
2	NPER(rate, pmt, pv, [fv], [type])		

	A	B	C
1	10		
2			

Figure 3.17 Calculating the Interest Rate Using Excel Spreadsheets

	A	B	C	D
1	=RATE(10,2500000,-18400218,0)			
2	RATE(nper, pmt, pv, [fv], [type], [guess])			

	A	B	C	D
1	6%			
2				

To use Excel to calculate the present value of an arithmetic gradient or geometric gradient, we use the following equation, where the notation NPV is present value or present worth:

$$=NPV(i\%, \text{Cell [1]:Cell [n]})$$

Example 3.13

If you have an arithmetic gradient series with the base amount of \$10,000 in year 1 and gradient amount of \$1,000 per year starting the second year through year 5, what is the present value of this series using Excel functions? Assume 10 percent interest rate. The spreadsheet in Figure 3.18 presents the cash flows in cell A1 through A5, the Excel function (left) in cell A6, and the result (right), also in cell A6.

Figure 3.18 Calculating the Present Value of an Arithmetic Gradient Series Using Excel Spreadsheets

	A	B	C			A	B
1	\$10,000				1	\$10,000	
2	\$11,000				2	\$11,000	
3	\$12,000				3	\$12,000	
4	\$13,000				4	\$13,000	
5	\$14,000				5	\$14,000	
6	=NPV(10%,A1:A5)				6	\$44,769.67	
7	NPV(rate, value1, [value2], [value3], ...)				7		

Summary

In this chapter, we define annuity as a uniform set of payments that occur consecutively over a given time period. The most common payment intervals are yearly, semi-annually, quarterly, and monthly. In addition, the most common form of annuities is seen in mortgages, pension funds, aircraft leases, property leases, car loans, insurance premiums, and retirement planning. There are two types of annuities which differ based on the timing of annuity payments, ordinary annuity and annuity due. An ordinary annuity is when payments are made at the end of each period, while in annuity due payments are made at the beginning of the period. Ordinary annuity is commonly used in engineering economics practices, and therefore in this book the word annuity refers to ordinary annuity with end-of-the-period convention.

This chapter derives and presents the economic equations to compute the future value of an annuity and to calculate an annuity of a future sum known as a sinking fund which is used often for financial planning purposes. Individuals and aviation companies have short- and long-term financial goals to achieve and they use this formula to determine the uniform series needed to be invested in an interest-bearing account to accumulate a future sum to finance their future goals. It also

presents economic equations to compute the present value of an annuity series and to calculate an annuity of a present amount known as amortization which is often used in mortgages, and in personal and commercial loans. Individuals and businesses often buy capital equipment and machineries on credit. They can use this formula to compute their annual or monthly payments and to determine whether the payments fit with in their budget.

In addition, this chapter presents the economic equations to compute the present value of an arithmetic gradient series, to convert an arithmetic gradient series to a uniform series, to compute the future value of an arithmetic gradient series, and to calculate the present value of a geometric gradient series.

Furthermore, the computation of present values and future values of shifted cash flow series are presented. Shifted series are streams of cash flows that do not start at the end of year 1 or do not end at the end of the designated year. To compute the present value of a shifted series, the present value of cash flow series is calculated at the beginning of the year where payments start and that amount is shifted to time zero by multiplying it with the present value of future amount (P/F) interest factor. To compute the future value of a shifted series, the future value of the cash flow series is calculated at the end of the year where payments stop and that amount is shifted to the end of the designated year by multiplying it with the future value of a present amount (F/P) interest factor. In conclusion, the application of all the derived formulae using Excel functions is presented.

Discussion Questions and Problems

- 3.1 A leasing company is considering investing \$500,000 at the end of each year for five years to purchase equipment that increases its efficiency. The investment account pays 6 percent interest compounded annually. How much will the company accumulate in this account to purchase the equipment in five years?
- 3.2 You have a newborn daughter and are planning to accumulate \$400,000 for your daughter's college education in 18 years. Assuming you earn 8 percent on your investment, how much will you need to invest at the end of each year in equal amounts for 18 years to accumulate \$400,000?
- 3.3 An airport in Japan is planning to purchase a parcel of land for building additional executive hangars in five years. The price of land currently is \$2,000,000. The average inflation rate is 5 percent in the next five years. How much should the airport invest in a uniform amount per year for five years in an account that pays 8 percent interest annually to be able to purchase the land in five years?
- 3.4 Your son just graduated from high school and he has been accepted for an undergraduate program at an Ivy League university. The cost of attending the university is \$40,000 per year for tuition and room and board, due at the end of each year for four years. How much do you have to deposit in a lump sum today in a bank account that pays 7 percent interest per year to be able to pay for your son's college education for the next four years?
- 3.5 United Engine Leasing (UEL) is planning to lease an executive hangar for \$100,000 per year for 10 years. The company could build a 3,200 square

feet hangar at the cost of \$960,000 with \$5,000 maintenance cost per year. Assume UEL can borrow at 6 percent interest rate and will be able to sell the hangar at the end of 10 years for \$500,000. Using present value analysis, should UEL lease or build the hangar?

- 3.6 Travelers flying in and out of an international airport should soon get free wireless speeds that would be four times faster than the current service in the terminals (5 megabits per second). The airport has recently awarded a \$5.9 million contract to Bingo Wireless Inc. to upgrade, manage, and operate the Wi-Fi network at the airport for 10 years. What would be the annual cost of offering free Wi-Fi to airport travelers? Assume a MARR of 8 percent for the airport.
- 3.7 In Problem 3.4, assuming all relevant information, if the cost of attending the university is \$40,000 per year due at the beginning instead of the end of each year for four years, how much do you have to deposit in a lump sum to be able to pay for your son's college education for the next four years?
- 3.8 A newly graduated electrical engineer is signing a five-year consulting agreement with Dubai International Airport to improve the efficiency of the airport energy consumption. The length of the contract is five years and consulting fees are \$100,000 at the end of year 1 increasing by \$20,000 per year starting at the end of the second year through the end of the contract. Assuming the engineer earns 10 percent return on his investment, what is the present value of his consulting contract?
- 3.9 A new airline in Mexico has started a new non-stop flight from Mexico City to Boston in the United States. The airline requires an average sales revenue of at least \$10,000,000 per year for four years to break even. If sales are \$5,000,000 in year 1, how much should the sales increase by a constant amount per year in order for the airline to reach its break-even target in four years? The airline uses a MARR of 8 percent.
- 3.10 Huntington has budgeted \$2,000,000 per year for an expansion over the next eight years. Huntington spent \$1,000,000 at the end of the first year for this expansion. What should be the constant amount of annual increase in spending so that Huntington uses its total budgeted amount by the end of eight years? Huntington uses a MARR of 9 percent.
- 3.11 A newly hired engineer would like to retire in 25 years. She is planning to invest \$12,000 at the end of year 1 and increase that amount by a constant amount of \$1,200 per year for 25 years. Assuming that the engineer is earning 10 percent on her long-term investment, how much can she accumulate in her retirement account at the end of 25 years?
- 3.12 Spirit Mechanic is estimating a total payroll expense of \$1,000,000 at the end of the first year. Starting from the second year, the payroll expenses will increase by a constant 6 percent per year through the end of the 10-year study period. How much should Spirit deposit in a lump sum today in an investment account that pays 8 percent interest to be able to pay the payroll expenses every year for the next 10 years?
- 3.13 In Problem 3.12, assuming all relevant information, what is the annuity (uniform series) of payroll expenses per year over the next 10 years?
- 3.14 Star Technology offers retirement benefits to its employees and matches up to 4 percent of the employee's contribution. An engineer who was just

hired by Star Technology started with a salary of \$120,000 in the first year with increase of 3 percent per year for the next 30 years. The employee, who wants to take advantage of matching by employer, will contribute 4 percent of his salary every year. If the employee can earn 10 percent return on his retirement investment, how much will he accumulate in his retirement account at the end of 30 years?

- 3.15 Rockwell Machine is considering building a manufacturing plant that takes three years to complete. The plant is expected to generate a revenue of \$500,000 per year at the end of each year of operation for 10 years (year 4 through 13). If Rockwell earns 8 percent on its investment, what is the maximum price that Rockwell is willing to pay for this investment opportunity?
- 3.16 An aviation company is planning to replace equipment in 10 years with an expected cost of \$1,500,000. The company is planning to invest an equal amount of money at the end of each year only for the first five years with no further investment through year 10. If the company can earn 12 percent on its investment, how large must the size of uniform deposit be in the first five years to accumulate \$1,500,000 at the end of the 10 years?
- 3.17 Precision Machine is signing a 10-year lease contract with a lease payment schedule of \$40,000 at the end of each year for the first five years, \$60,000 at the end of year 6, and increases of \$20,000 at the end of each year through year 10. If Precision could earn 8 percent on its investment, how much does Precision need to deposit today to make all scheduled lease payments in the next 10 years?
- 3.18 Legacy Airport, which anticipates a significant increase in the number of passengers, is planning to construct a five-story parking garage in 10 years. The current cost of parking construction is about \$20,000,000. The average annual rate of inflation in the next 10 years is 4 percent. How much does Legacy have to deposit in equal amounts at the end of each year to be able to build the parking garage in 10 years? Assume an interest rate of 7 percent.
- 3.19 An airport manager is planning to pay \$40,000 per year for 10 years for weather monitoring and reporting services. The cost of buying weather monitoring equipment is \$200,000 with \$3,000 maintenance cost per year. Assuming the life of the equipment is 10 years and the airport can borrow at 8 percent, should the airport manager buy the equipment?
- 3.20 DirectJet has started a new route from Dubai Airport in the United Arab Emirates (UAE) to Islamabad Airport in Pakistan. DirectJet requires an accumulated sales revenue of at least \$30,000,000 at the end of year 5 to break even. DirectJet is earning 10 percent on its investment. If sales are \$4,000,000 in year 1, how much should the sales increase by a constant amount per year in order for DirectJet to reach its break-even target in five years?
- 3.21 Golden Aerospace Company has budgeted \$500,000 per year for an expansion over the next 10 years. Golden earns 8 percent return on its investment and spent \$200,000 at the end of the first year for this expansion. What should be the constant annual increase in spending so that Golden uses its total budgeted amount by the end of 10 years?

- 3.22** Precision Machine maintenance payment schedule division B for the next 10 years is given in the following table. If Precision could earn 6 percent on its investment, how much does it need to deposit today to make all scheduled payments for the next 10 years?

Precision's Payments Schedule	
End of the Year	Payments
1	\$50,000
2	\$50,000
3	\$50,000
4	\$50,000
5	\$50,000
6	\$100,000
7	\$120,000
8	\$140,000
9	\$160,000
10	\$180,000

4

Nominal and Effective Interest Rates

In regards to the price of commodities, the rise of wages operates as simple interest does, the rise of profit operates like compound interest. Our merchants and masters complain much of the bad effects of high wages in raising the price and lessening the sale of goods. They say nothing concerning the bad effects of high profits. They are silent with regard to the pernicious effects of their own gains. They complain only of those of other people.

Adam Smith



In the past two chapters, the interest rates were compounded annually. The majority of investment projects evaluated by professional engineers, and almost all financial institutions, use interest rates that are compounded more frequently than once a year. Common interest compounding time frames are daily, monthly, semi-annually, annually, and continuously compounding. More frequent compounding means that the investors receive a higher return, whereas less frequent compounding results in receiving a lower return. For instance, most credit card companies compound interest daily; that results in effective annual rates that exceed the annual stated interest rates.

The focus of this chapter is to enable practitioners and professional engineers to make a distinction between nominal and effective interest rates as applied to the engineering projects and financial activities of aviation and aerospace companies. Nominal and effective interest rates are common in business practices, financial management, and engineering economy. This chapter presents the process of converting nominal interest rates to effective interest rates.

The following sections briefly address the calculation of effective interest rates and computation of equivalences for single amounts and cash flow series where payment periods are equal to, exceeding, or less than compounding periods.

- ◆ Nominal Versus Effective Interest Rates
- ◆ Calculating Effective Interest Rate
 - Continuous Compounding
 - Effective Rate to Nominal Rate
- ◆ Comparing Compounding Periods and Payment Periods
 - Computing Equivalences Involving Only Single-Amount Factors
 - Matching the Compounding Period to Payment Period
 - Matching the Payment Period to Compounding Period
 - Computing Equivalences when Payment Period Equals Compounding Period
 - Computing Equivalences when Payment Period Exceeds Compounding Period
 - Computing Equivalences when Payment Period is Less than Compounding Period
- ◆ Using Spreadsheets for Effective Rate and Equivalences Computation
- ◆ Summary
- ◆ Discussion Questions and Problems

Nominal Versus Effective Interest Rates

In the past two chapters, the interest was compounded annually, but for the majority of projects evaluated by professional engineers in aviation, aerospace, and financial institutions, the interest is compounded more frequently than once a year.

In other words, interest may be compounded semi-annually, quarterly, monthly, daily, and potentially continuously. The effective interest rate, or annual equivalent rate (AER), is the actual rate of interest earned or paid. If interest is given per year (stated interest rate or nominal rate) but compounded more frequently than once a year, then effective

interest rate per year is different and it is greater than the nominal interest rate per year. Every nominal interest rate must be converted into an effective rate before it can be used for financial decision-making purposes. Each type of interest rate must be analyzed in order to solve various cases where interests are expressed in different manners.

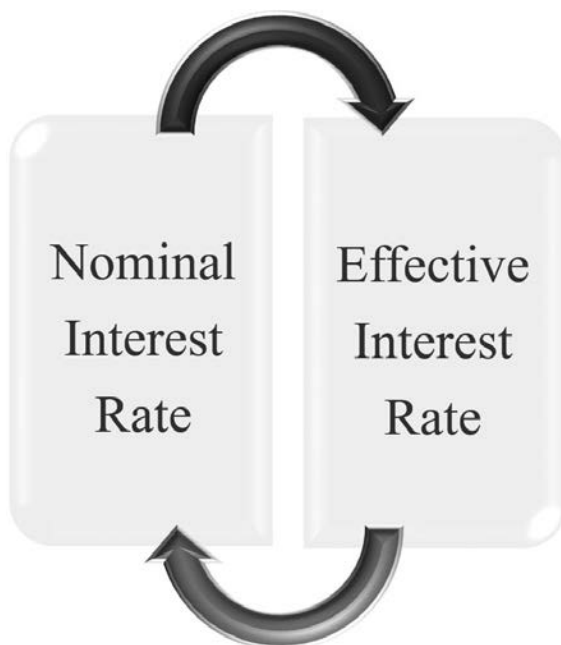
For the airlines industry, the single largest capital expenditure is the cost of aircraft. Therefore, many airline companies use either commercial bank loans or leasing companies to finance their capital needs. In 2015, airlines and leasing companies took deliveries of Boeing commercial aircraft worth approximately \$122 billion. With the strong increase in demand for new and fuel-efficient commercial aircraft for

In the U.S., 12 million people borrow nearly \$50 billion a year through payday loans. The rates charged on payday loans can be up to 35 times those charged on credit card loans and 80 times the rates charged on home mortgages and auto loans. The estimated annual percentage rate on payday loans in the U.S. ranges from a low of 196% to a high of 574%.

Business Insider

the foreseeable future, the aircraft finance markets play an important role in offering a broad and balanced range of financing options at competitive prices to support the airline industry.

Airlines pay explicit interest on commercial loans and implicit interest rates on the leasing arrangement. Although the interest rates on commercial loans and lease agreements are coded in annual percentage rates, the loan and lease payments are often made more frequently than annual bases. Therefore, the effective interest rates are higher than the stated interest rates on the loans and lease agreements. The following section presents the computation of the effective interest rates.



Calculating Effective Interest Rate

The effective interest rate can be calculated using the relationship between nominal rate and effective rate as follows:

$$i_{\text{effective}} = \left(1 + \frac{r}{m}\right)^m - 1$$

Where: i = Effective rate per some stated period
 r = Nominal rate for the same time period
 m = frequency of compounding per the same time period

Example 4.1

For a nominal rate of 12 percent per year, compute the annual effective rate if interest is compounded a) semi-annually, b) quarterly, c) monthly, and d) daily. We apply the equation above to compute the effective rates.

$$\text{a. } i_{\text{year}} = \left(1 + \frac{0.12}{2}\right)^2 - 1$$

$$i_{\text{year}} = 0.1236 \text{ or } 12.36\%$$

$$\text{b. } i_{\text{year}} = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$$i_{\text{year}} = 0.1255 \text{ or } 12.55\%$$

$$\text{c. } i_{\text{year}} = \left(1 + \frac{0.12}{12}\right)^{12} - 1$$

$$i_{\text{year}} = 0.1268 \text{ or } 12.68\%$$

$$\text{d. } i_{\text{year}} = \left(1 + \frac{0.12}{365}\right)^{365} - 1$$

$$i_{\text{year}} = 0.1275 \text{ or } 12.75\%$$

As you notice, the higher the compounding frequency, the greater the effective rate is.

We can also calculate the effective semi-annual rate and effective quarterly rate when interest is compounded monthly and the interest rate is 1 percent per month. In order to calculate effective rate semi-annually and quarterly, we need to calculate the semi-annual and quarterly nominal rates. A nominal interest rate can be computed for any time period that is greater than the compounding period as follows:

$$r_{\text{semi-annually}} = 1\% \times 6 \text{ month} = 6\%$$

$$r_{\text{quarterly}} = 1\% \times 3 \text{ month} = 3\%$$

Now, we can compute the effective rate semi-annually and quarterly where interest is compounding monthly. In other words, frequency of compounding is six times semi-annually (there are six months in a semi-annual period) and frequency of compounding is three times quarterly (there are three months in a quarter period).

$$i_{\text{semi-annually}} = \left(1 + \frac{0.06}{6}\right)^6 - 1$$

$$i_{\text{semi-annually}} = 6.15\%$$

$$i_{\text{quarterly}} = \left(1 + \frac{0.03}{3}\right)^3 - 1$$

$$i_{\text{quarterly}} = 3.03\%$$

Table 4.1 presents the effective interest rates per year when interest is compounded annually, semi-annually, quarterly, monthly, and daily with interest rates of 4, 6, 12, 18, 24, and 30 percent per year.

Figure 4.1 provides a visual illustration of the increasing amounts of effective rates as the frequency of compounding keeps increasing and the nominal rates are changed. Since at smaller nominal rates, the effective rates are close to the values of the nominal rates, nominal rates from 25 to 70 percent are used to illustrate the changes in effective interest rates.

Continuous Compounding

If compounding is agreed to happen more and more frequently, the compounding period becomes shorter and shorter. Then m , the number of compounding periods, increases.

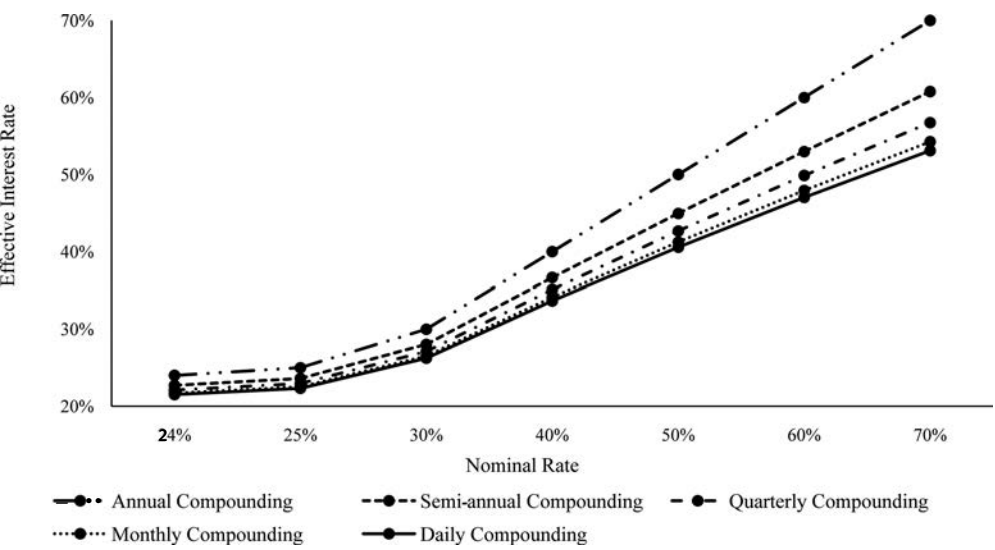
If interest is compounded continuously, then the frequency of compounding (m) in the equation is approaching infinity ($m \rightarrow \infty$).

$$i_{\text{effective}} = \left(1 + \frac{r}{m}\right)^{m \times n} - 1$$

Table 4.1 Effective Annual Interest Rates for Selected Nominal Rates

Nominal Rate	Annual Compounding	Semi-annual Compounding	Quarterly Compounding	Monthly Compounding	Daily Compounding
4%	4%	4.04%	4.06%	4.07%	4.08%
5%	5%	5.06%	5.09%	5.12%	5.13%
6%	6%	6.09%	6.14%	6.17%	6.18%
7%	7%	7.12%	7.19%	7.23%	7.25%
8%	8%	8.16%	8.24%	8.30%	8.33%
9%	9%	9.20%	9.31%	9.38%	9.42%
10%	10%	10.25%	10.38%	10.47%	10.52%
12%	12%	12.36%	12.55%	12.68%	12.75%
15%	15%	15.56%	15.87%	16.08%	16.18%
18%	18%	18.81%	19.25%	19.56%	19.72%
20%	20%	21.00%	21.55%	21.94%	22.13%
24%	24%	25.44%	26.25%	26.82%	27.11%
25%	25%	26.56%	27.44%	28.07%	28.39%
30%	30%	32.25%	33.55%	34.49%	34.97%

Figure 4.1 Effective Interest Rates for Various Nominal Rates



An important theory in calculus is:

$$\lim_{m \rightarrow \infty} \left(1 + \frac{1}{m}\right)^m = \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e = 2.7182$$

As ' m ' approaches infinity, x becomes 0, and then this equation will be reduced to:

$$i_{\text{effective}} = e^r - 1$$

So, if an investor puts \$1,000,000 in an account paying 10 percent interest, under continuous compounding the effective rate is:

$$\begin{aligned} i_{\text{effective}} &= e^{0.10} - 1 \\ i_{\text{effective}} &= 1.1049 - 1 \\ i_{\text{effective}} &= 10.49\% \end{aligned}$$

Example 4.2

Shelden Financial charges a stated 12 percent interest per year on its loans. The interest is, however, compounded continuously. What is the annual effective rate charged by this financial company?

$$\begin{aligned} i_{\text{effective}} &= e^{0.12} - 1 \\ i_{\text{effective}} &= 1.1275 - 1 \\ i_{\text{effective}} &= 0.1275 \\ i_{\text{effective}} &= 12.75\% \end{aligned}$$

Effective Rate to Nominal Rate

Sometimes the effective rate may be given and we are asked to compute the nominal rate. We use both of the equations mentioned above to calculate the nominal rate.

Table 4.2 presents the nominal interest rates per year when interest is compounded annually, semi-annually, quarterly, monthly and daily with effective interest rates of 4, 6, 12, 18, 24, and 30 percent per year.

The representation of nominal rates computed for various effective rates, using different frequencies of compounding is presented in Figure 4.2. It is the complete opposite to the earlier depiction of effective rate computed from nominal rates. The nominal rates keep decreasing as the compounding periods are increased and this can be clearly seen from the graph in this figure.

Example 4.3

Wilshire Financial charges an effective rate of 17 percent per year. If interest is compounded quarterly, what is the nominal rate per year?

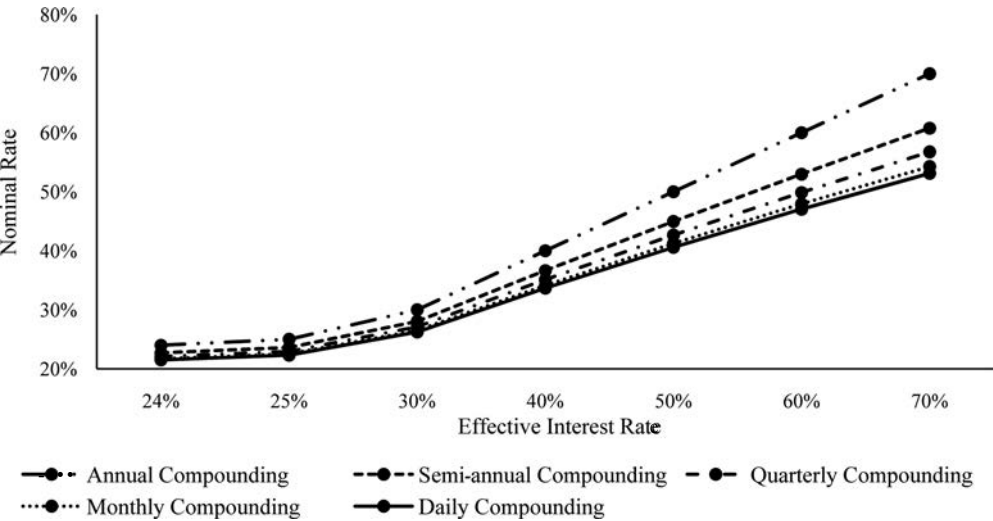
Plug in the given information in the equation and solve for nominal rate of r .

$$\begin{aligned} 0.17 &= \left(1 + \frac{r}{4}\right)^4 - 1 \\ 1.17 &= \left(1 + \frac{r}{4}\right)^4 \end{aligned}$$

Table 4.2 Nominal Annual Interest Rates for Selected Effective Annual Rates

Effective Interest Rate	Annual Compounding	Semi-annual Compounding	Quarterly Compounding	Monthly Compounding	Daily Compounding
4%	4%	3.96%	3.94%	3.93%	3.92%
5%	5%	4.94%	4.91%	4.89%	4.88%
6%	6%	5.91%	5.87%	5.84%	5.83%
7%	7%	6.88%	6.82%	6.78%	6.77%
8%	8%	7.85%	7.77%	7.72%	7.70%
9%	9%	8.81%	8.71%	8.65%	8.62%
10%	10%	9.76%	9.65%	9.57%	9.53%
12%	12%	11.66%	11.49%	11.39%	11.33%
15%	15%	14.48%	14.22%	14.06%	13.98%
18%	18%	17.26%	16.90%	16.67%	16.56%
20%	20%	19.09%	18.65%	18.37%	18.24%
24%	24%	22.71%	22.10%	21.71%	21.52%
25%	25%	23.61%	22.95%	22.52%	22.32%
30%	30%	28.04%	27.12%	26.53%	26.25%

Figure 4.2 Nominal Rates for Various Effective Interest Rates



Get the power of $\frac{1}{4}$,

$$(1.17)^{\frac{1}{4}} = \left(1 + \frac{r}{4}\right)$$

$$1.04 = 1 + \frac{r}{4}$$

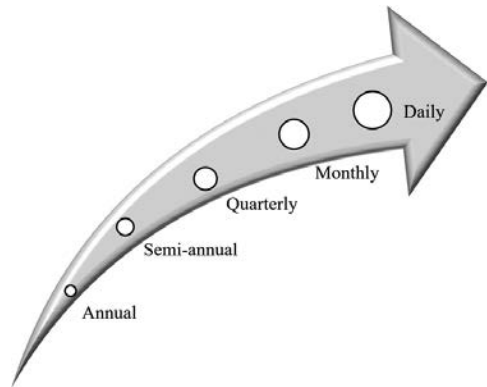
$$0.04 = \frac{r}{4}$$

$$r = 0.16$$

$$r = 16\%$$

Comparing Compounding Periods and Payment Periods

In this section, it is important to differentiate between payment periods and compounding periods and to be able to reconcile them. In many projects analyzed by engineers, the payment periods (PP) or frequency of making payments may not be the same as compounding periods (CP) or frequency of compounding interest. For example, consider a situation where you are depositing \$1,000 per month in a savings account that pays 6 percent interest per year compounded quarterly. In this case the PP is monthly and CP is quarterly and therefore $PP < CP$. Alternatively, consider a situation where you are depositing \$1,000 every quarter in a savings account that pays 6 percent interest per year compounded monthly. In this case the PP is quarterly and CP is monthly and therefore $PP > CP$. Therefore, it is essential to recognize the PP and CP for single amounts, uniform series, and gradient series before calculating the equivalences. Ultimately, PP has to equal CP ($PP = CP$), before we attempt to compute equivalences. In the following sections, we are differentiating between PP and CP for single amounts, for uniform series, and for gradient series, and then we decide how to reconcile those to compute equivalences.



Computing Equivalences Involving Only Single-Amount Factors

We have two alternative methods to compute the present and future value of a single amount when interest is compounded more frequently than once a year.

Matching the Compounding Period to Payment Period

Under this method, we first calculate the effective interest rate to match the payment period and plug that into the formula for either present value or future value of a single amount to compute the equivalences.

Example 4.4

Westwood Company has borrowed \$100 million at 6 percent per year compounded semi-annually. The principal and interest are due in 10 years. Calculate the equivalence future value of this loan in 10 years.

First we calculate the effective rate per year.

$$\text{Effective rate} = \left(1 + \frac{0.06}{2}\right)^2 - 1$$

$$\text{Effective rate} = 6.09\%$$

Now we plug in the loan amount and the effective interest rate in the equation $F = P \times (1 + i)^n$ to calculate the future amount.

$$F_{10} = \$100 \times (1 + 0.0609)^{10}$$

$$F_{10} = \$100 \times (1.8061)$$

$$F_{10} = \$180.61 \text{ million}$$

Matching the Payment Period to Compounding Period

Under this method, we convert the time period from annually to semi-annually from 10 years to 20 semi-annuals. The interest rate per year is 6 percent but because it is compounded semi-annually, it is 3 percent per semi-annual. Now we have time period semi-annual and interest rate compounded semi-annually, which is matching PP to CP, and therefore we are ready to calculate the future value.

For annual compounding, the F can be calculated as follows:

$$F = P \times (1 + i)^n$$

For m frequency of compounding per year, the F is calculated using the equation below, where the interest rate is divided by the frequency of compounding and the time period is multiplied by the frequency of compounding.

$$F = P \times \left(1 + \frac{i}{m}\right)^{n \times m}$$

To use standard compound interest factor notation, this equation can be articulated to the equation as follows:

$$F = P(F/P, \frac{i}{m}, n \times m)$$

In Example 4.4, we divided interest rate by frequency of compounding per year (2) and multiplied the time period in years by frequency of compounding per year (2) as follows:

$$F = P \times \left(1 + \frac{i}{2}\right)^{2n}$$

We now plug in the information from Example 4.4 in the above equation to compute future value equivalence. Keep in mind that the time period is expressed in semi-annuals.

$$F = \$100 \times \left(1 + \frac{0.06}{2}\right)^{2 \times 10}$$

$$F = \$100 \times (1 + 0.03)^{20}$$

$$F = \$100 \times (1.8061)$$

$$F = \$180.61 \text{ millions}$$

Computing Equivalences when Payment Period Equals Compounding Period

In annuities or gradient series when payment period is the same as compounding period, we simply adjust our equations to reflect the payment period which is the same as the compounding period. Let's look at Example 4.5.

Example 4.5

Western is purchasing aircraft structural components, fairings, and wing flaps for its aircraft from a middle-east manufacturer. Western is planning to purchase \$2,000,000 worth of aircraft components at the end of the first quarter, and increase that amount by \$250,000 per quarter for five years. How much should Western set aside today earning 12 percent per year compounded quarterly to be able to pay for purchases in the next five years?

In this example, we are asked to calculate the present value of this arithmetic gradient series. As you notice, the PP is quarterly and CP is also quarterly therefore,

$$PP = CP = \text{Quarterly}$$

Because the PP = CP, there is no need to calculate the annual effective rate as it would no longer match the timing of the purchases which is on a quarterly basis. We use the equation below and divide annual interest rate by 4 and multiply time period by 4 to convert them to quarterly bases.

$$P_G = A (P/A, \frac{i}{4}, 4n) + G (P/G, \frac{i}{4}, 4n)$$

$$P_G = \$2,000,000 (P/A, \frac{12\%}{4}, 4 \times 5) + \$250,000 (P/G, \frac{12\%}{4}, 4 \times 5)$$

$$P_G = \$2,000,000 (P/A, 3\%, 20) + \$250,000 (P/G, 3\%, 20)$$

From Table A9 in Appendix A, for 3 percent interest rate, the present value of annuity interest factor ($P/A, 3\%, 20$) and the present value of gradient interest factor ($P/G, 3\%, 20$) are plugged into the above equation and the present value of the arithmetic gradient is computed.

$$\begin{aligned}
 P_G &= \$2,000,000 \times (14.8775) + \$250,000 \times (126.7987) \\
 P_G &= \$29,755,000 + \$31,699,675 \\
 P_G &= \$61,454,675
 \end{aligned}$$

Computing Equivalences when Payment Period Exceeds Compounding Period

This is the only situation where effective rate has to be calculated because compounding period is less than payment period. By computing effective rate to match payment period we have $PP = CP$, and we can compute the present value or the future value of an annuity series or gradient series.

Example 4.6

Dontae Inc. is planning to deposit \$500,000 per year for five years for purchasing new equipment. The interest rate is 8 percent per year compounded quarterly. How much can Dontae accumulate at the end of five years?

In this example, payments are made annually and interest is compounded quarterly and therefore:

$$PP > CP$$

In order to achieve the relationship of $PP = CP$, we have to calculate the annual effective rate.

$$\begin{aligned}
 i_{\text{year}} &= \left(1 + \frac{0.08}{4}\right)^4 - 1 \\
 i_{\text{year}} &= 8.24\%
 \end{aligned}$$

Using annual effective rate, the time period of payment and compounding are the same $PP = CP$. Now, we can plug in information from Example 4.6 in the equation below:

$$\begin{aligned}
 F &= A \times \left[\frac{(1+i)^n - 1}{i} \right] \\
 F &= \$500,000 \times \left[\frac{(1 + 0.0824)^5 - 1}{0.0824} \right] \\
 F &= \$500,000 \times \frac{0.4857}{0.0824} \\
 F &= \$500,000 \times 5.8944 \\
 F &= \$2,947,200
 \end{aligned}$$

Example 4.7

Aircraft Competence Training (ACT) is a program developed by Mesa Aircraft manufacturer focusing on the “learning by doing” concept. Monarch Airlines, in order to improve their dispatch reliability and in-house capability, as well as ensure the safety of their operations, is sending a group of employees for ACT training

Figure 4.3 Annuity Series Cash Flows

during the next year. Monarch Airlines is planning to recover the training cost through maintenance cost savings in the next four years following the training period. How much can Monarch Airlines save on maintenance costs per year in order to recover the training cost in four years? Interest rate is 12 percent per year compounded monthly.

There are two steps to come up with the answer.

1. Compute the future value of training cost at the end of year 1 as shown in Figure 4.3.

Because PP = CP = Quarterly, we use the above equation to calculate the future value at the end of the first year.

$$F = A \left(F/A, \frac{i\%}{4}, n \times 4 \right)$$

$$F = \$50,000 \left(F/A, \frac{12\%}{4}, 1 \times 4 \right)$$

$$F = \$50,000 (F/A, 3\%, 4)$$

From Table A9 in Appendix A, for 3 percent interest rate, the future value of annuity interest factor is selected and plugged into the above equation.

$$F = \$50,000 (4.1836)$$

$$F = \$209,180$$

2. The future value in step 1 becomes the present value of the maintenance cost savings for the next four years. We need to compute the annuity of that present amount. In step 2, PP is annually and CP is quarterly, therefore PP < CP. We need to calculate effective rate per year to match the annual payment period.

$$i_{\text{year}} = \left(1 + \frac{0.12}{4} \right)^4 - 1$$

$$i_{\text{year}} = 12.55\%$$

Now, we plug in the above information in the equation below to solve for annuity amount.

$$A = P \times \left[\frac{i \times (1+i)^n}{(1+i)^n - 1} \right]$$

$$A = \$209,180 \times \left[\frac{0.1255 \times (1+0.1255)^4}{(1+0.1255)^4 - 1} \right]$$

$$A = \$209,180 \times \frac{0.2014}{0.6047}$$

$$A = \$209,180 \times 0.3331$$

$$A = \$69,678$$

Computing Equivalences when Payment Period is Less than Compounding Period

When the payment period is less than the compounding period, then inter-period payments do not earn interest. To compute equivalences, we move payments to match the compounding period. For example, if compounding period is quarterly and payments are made monthly, then $PP < CP$.

The payment made in the first month does not earn interest, the payment made in the second month also does not earn interest, until the third payment is made. Now the sum of all three payments earns interest because interest is compounded quarterly. By the same reasoning, the payments in months 4 and 5 do not earn interest until the sixth payment is made. Now the sum of all payments and accrued interest in the first quarter earn interest. In this example, we add the monthly payments together for three months to get the quarterly payment to match the compounding period.

Example 4.8

Great Lake Aircraft Production is planning to deposit \$100,000 per month in an account for five years to accumulate enough money to build a new aircraft hangar. If the account pays 8 percent interest per year compounded quarterly, how much would Great Lake accumulate at the end of five years?

In this example $PP < CP$, and we need to calculate quarterly payments to match the compounding period.

$$\text{Quarterly Payment} = \$100,000 \times 3 = \$300,000$$

Now, we can re-write the problem as Great Lake is depositing \$300,000 per quarter for five years. At interest rate of 8 percent per year compounded quarterly, what is the future value of all payments at the end of five years? Now $PP = CP =$ quarterly and we can use the equation below to compute the accumulated amount.

$$F = A \left(F/A, \frac{i\%}{4}, n \times 4 \right)$$

$$F = \$300,000 \left(F/A, \frac{8\%}{4}, 5 \times 4 \right)$$

$$F = \$300,000(F/A, 2\% 20)$$

From Table A8 in Appendix A, for 2 percent interest rate, the future value of the annuity interest factor is retrieved and plugged into the above equation to compute the total accumulated amount.

$$F = \$300,000 \times 24.2974$$

$$F = \$7,289,220$$

Using Spreadsheets for Effective Rate and Equivalences Computation

This section presents a short review of how to run the financial equations using Microsoft Excel functions. In Excel, two functions are used to convert between nominal and effective interest rates:

- ◆ EFFECT(nominal rate, compounding frequency)
- ◆ NOMINAL(effective rate, compounding frequency per year)

To find effective rate, insert a function, and execute the following steps:

First, select a cell. Then, click the Insert Function button. To compute effective rate in Excel, first we click the Formulas tab on the top row, then open the financial functions and click EFFECT. A window appears on the screen asking for nominal rate per year and frequency of compounding per year. Once we plug in those numbers, the answer appears on the next line. Alternatively, we may type `=EFFECT(r, m)` in a cell in an Excel spreadsheet and Excel will solve this function and provide the answer in that cell.

Example 4.9

For a nominal rate of 12 percent per year, compute the annual effective rate if interest is compounded

- a) semi-annually
- b) quarterly
- c) monthly
- d) daily

Using the Microsoft Excel function, we can answer this problem. Plug in the relevant information in the following formulas in the Excel spreadsheet and the answer to the function will appear in those cells.

- a. Figures 4.4 to 4.7 show examples of the Excel EFFECT function used to calculate the effective annual interest rate for four different compounding periods. Since in this example effective semi-annual rate is requested, locate and click on the function titled EFFECT, enter the nominal rate per quarter of 12 percent in cell B2 and enter $m = 2$, since semi-annually occurs two times in a year, and then click the OK button.

$$=EFFECT(12\%,2)$$

$$\text{Answer: } 0.1236$$

$$12.36\%$$

Figure 4.4 Calculating Semi-annual Effective Rate Using Excel Spreadsheets

	A	B			A	B
1	Nominal Rate (r)	12%		1	Nominal Rate (r)	12%
2	Compounding Periods (m)	2		2	Compounding Periods (m)	2
3				3		
4	Semi-Annual			4	Semi-Annual	
5	Effective Rate	=EFFECT(B1,B2)		5	Effective Rate	12.36%

- b. In this part effective quarterly rate is requested, so enter the nominal rate per year (12 percent) to get an effective rate per quarter, and enter $m = 4$, since quarterly occurs four times in a year.

=EFFECT(12%,4) Answer: 0.1255 12.55%

Figure 4.5 Calculating Quarterly Effective Rate Using Excel Spreadsheets

	A	B			A	B
1	Nominal Rate (r)	12%		1	Nominal Rate (r)	12%
2	Compounding Periods (m)	4		2	Compounding Periods (m)	4
3				3		
4	Quarterly			4	Quarterly	
5	Effective Rate	=EFFECT(B1,B2)		5	Effective Rate	12.55%

- c. Similarly, the m entered for this section must equal the number of times interest is compounded annually. Since monthly compounding occurs 12 times in a year, $m = 12$ and the formula entered will be:

=EFFECT(12%,12) Answer : 0.1268 12.68%

Figure 4.6 Calculating Monthly Effective Rate Using Excel Spreadsheets

	A	B			A	B
1	Nominal Rate (r)	12%		1	Nominal Rate (r)	12%
2	Compounding Periods (m)	12		2	Compounding Periods (m)	12
3				3		
4	Monthly			4	Monthly	
5	Effective Rate	=EFFECT(B1,B2)		5	Effective Rate	12.68%

- d. To compute the effective interest rate for daily compounding, the m entered must be 365.

=EFFECT(12%,365) Answer : 0.1275 12.75%

Figure 4.7 Calculating Daily Effective Rate Using Excel Spreadsheets

	A	B		A	B
1	Nominal Rate (r)	12%	1	Nominal Rate (r)	12%
2	Compounding Periods (m)	365	2	Compounding Periods (m)	365
3			3		
4	Daily		4	Daily	
5	Effective Rate	=EFFECT(B1,B2)	5	Effective Rate	12.75%

Conversely, you can also use Excel functions to calculate the nominal rate if an effective rate and the frequency of compounding are given. We can type **=NOMINAL(i, m)** in a cell in an Excel spreadsheet and Excel will solve the function and provide the answer in that cell.

Example 4.10

For an effective rate of 12 percent where interest is compounded

- semi-annually
- quarterly
- monthly
- daily

calculate the nominal rates.

- To determine the nominal rate per year when the effective rate is 12 percent and the compounded period is semi-annual, we use the **NOMINAL** function and enter the effective rate (r) as 12 percent and $m = 2$.

=NOMINAL(12%,2) Answer : 0.1166 11.66%

Figure 4.8 Calculating Semi-annual Nominal Rate Using Excel Spreadsheets

	A	B	C		A	B
1	Effective Rate (i)	12%		1	Effective Rate (i)	12%
2	Compounding Periods (m)	2		2	Compounding Periods (m)	2
3				3		
4	Semi-Annual			4	Semi-Annual	
5	Nominal Rate	=NOMINAL(B1,B2)		5	Nominal Rate	11.66%

- In this case, the compounded period is quarterly and so, the value of m must be equal to 4 and the function is entered as follows:

=NOMINAL(12%,4) Answer : 0.1149 11.49%

Figure 4.9 Calculating Quarterly Nominal Rate Using Excel Spreadsheets

	A	B	C		A	B
1	Effective Rate (i)	12%		1	Effective Rate (i)	12%
2	Compounding Periods (m)	4		2	Compounding Periods (m)	4
3				3		
4	Quarterly			4	Quarterly	
5	Nominal Rate	=NOMINAL(B1,B2)		5	Nominal Rate	11.49%

- c. When the compounding period is monthly ($m = 12$) and the effective rate is 12 percent, the formula to find the nominal rate is entered in the cell as follows:

=NOMINAL(12%,12) Answer : 0.1139 11.39%

Figure 4.10 Calculating Monthly Nominal Rate Using Excel Spreadsheets

	A	B	C		A	B
1	Effective Rate (i)	12%		1	Effective Rate (i)	12%
2	Compounding Periods (m)	12		2	Compounding Periods (m)	12
3				3		
4	Monthly			4	Monthly	
5	Nominal Rate	=NOMINAL(B1,B2)		5	Nominal Rate	11.39%

- d. For a daily period of compounding, the value of m must be entered as 365 and the following results are obtained in Excel when the formula is entered.

=NOMINAL(12%,365) Answer : 0.1133 11.33%

Figure 4.11 Calculating Daily Nominal Rate Using Excel Spreadsheets

	A	B	C		A	B
1	Effective Rate (i)	12%		1	Effective Rate (i)	12%
2	Compounding Periods (m)	365		2	Compounding Periods (m)	365
3				3		
4	Daily			4	Daily	
5	Nominal Rate	=NOMINAL(B1,B2)		5	Nominal Rate	11.33%

Summary

For the majority of projects evaluated by professional engineers, almost all financial institutions use interest rates that are compounded more frequently than once a year, including semi-annually, quarterly, monthly, and daily. With the high frequency of compounding interest comes a greater rate of return on the investment and that is an incentive for lenders to compound interest very frequently. Credit card companies

generally compound interest on credit card balances on a daily basis. The nominal interest rate is usually given per year and to calculate nominal rates per quarter or monthly, we simply divide the annual rate by four for quarterly rate and by twelve for monthly rate. To calculate the effective rate per quarter or per month, we have to compute the effective rates for quarterly and monthly compounding. The resulting effective rates are greater than their nominal rates for the same time period. Therefore, every nominal interest rate must be converted into an effective rate before it can be used for the financial decision-making process.

If the frequency of compounding period approaches infinity ($m \rightarrow \infty$), a continuous compounding will occur and the effective interest rate can be computed.

It is also important to differentiate between payment periods and compounding periods. In many projects analyzed by engineers, the payment periods (PP) may or may not be the same as compounding periods (CP). If $PP = CP$, then there is no need to calculate the effective rate. We adjust interest rate and time period to conform with the payment period. If PP is either greater than or smaller than CP, then we need to reconcile them. In case of $PP > CP$, we have to calculate the effective rate to correspond to payment period. If $PP < CP$, then inter-period payments would not earn interest and we need to sum up payments to conform to the compounding period.

The last section of this chapter discusses the use of spreadsheets for the effective rate and nominal rate computation.

Discussion Questions and Problems

- 4.1 For a nominal rate of 8 percent per year compounded monthly, compute the following:
 - a. Effective rate per quarter
 - b. Semi-annual effective rate
 - c. Annual effective rate
- 4.2 Reward Credit Card Company charges a stated 18 percent annual interest on credit card balances. The interest is compounded daily. Assume 360 days per year, compute the following:
 - a. Effective rate per month
 - b. Effective rate per quarter
 - c. Effective rate per year
- 4.3 West Coast Financial Company charges a stated 10 percent per year on its loans. The interest is compounded continuously. What is the annual effective rate charged by this financial company?
- 4.4 Sapphire Financial Company charges a stated 18 percent interest rate on credit card balances. The interest is compounded continuously. Compute the following:
 - a. Nominal rate per month
 - b. Effective rate per month
 - c. Nominal rate per quarter
 - d. Effective rate per quarter
 - e. Nominal rate per semi-annual
 - f. Effective rate per semi-annual
 - g. Effective rate per year

- 4.5 Mesa Bank charges an effective rate of 19.56 percent per year. If interest is compounded monthly, what is the nominal rate per year?
- 4.6 General Financial Company charges an effective rate of 12.75 percent per year. What is the nominal rate per year if interest is compounded continuously?
- 4.7 Major Aviation has borrowed \$120 million at 8 percent per year compounded semi-annually. The principal and interest are due in 10 years. How much does Major Aviation have to pay in 10 years to pay off the loan?
- 4.8 Global Aviation Group is planning to accumulate \$5,000,000 in six years for future expansion. How much does Global Aviation have to set aside today earning 8 percent interest rate per year compounded quarterly to accumulate \$5,000,000 in six years?
- 4.9 Sky Aviation is purchasing aircraft structural components, fairings, and wing flaps for its signature aircraft from Group Aerospace Manufacturing. Sky Aviation is planning to spend \$1,000,000 at the end of the first quarter, and increase that amount by \$200,000 per quarter for five years. How much should Sky Aviation set aside today earning 8 percent per year compounded quarterly to be able to pay for purchases in the next five years?
- 4.10 Gold Coast Aviation Competence Training (GCACT) is a “learning by doing” concept approved by the airworthiness authorities. Atlantic Airlines in order to improve their dispatch reliability and in-house capability, as well as ensure the safety of their operations, is sending a group of employees for GCACT training during the next year. The cost of training is \$50,000 per month for one year. Atlantic Airlines is planning to recover the training cost through maintenance cost savings in the next five years following the training period. How much can Atlantic Airlines save on maintenance costs per year in order to recover the training cost in five years? Interest rate is 12 percent per year compounded monthly.
- 4.11 Kazak Airline is planning to deposit \$200,000 per month in an account for four years to accumulate enough money to build a new aircraft hangar. If the account pays 8 percent interest per year compounded quarterly, how much would Kazak Airline accumulate at the end of four years?
- 4.12 Pacific Airlines is planning to accumulate \$500,000 to fund a scheduled engine overhaul in three years. How much should Pacific deposit every month in an account that pays 10 percent interest per year compounded semi-annually to accumulate the cost of engine overhaul in three years?
- 4.13 Reward Credit Card Company charges a stated 24 percent annual interest on credit card balances. The interest is compounded daily. What is the effective annual interest rate charged on this credit card balance? Assume 360 days in a year.
- 4.14 Reward Financial Company charges a stated 12 percent interest rate on credit card balances. The interest is compounded continuously. Find the following:
- Nominal rate per month
 - Effective rate per month
 - Nominal rate per quarter

- d. Effective rate per quarter
 - e. Nominal rate per semi-annual
 - f. Effective rate per semi-annual
 - g. Effective rate per year
- 4.15** General Financial charges an effective rate of 19.72 percent per year. What is the nominal rate per year if interest is compounded continuously?
- 4.16** Eastern Aviation Group (EAG) is planning to accumulate \$1,000,000 in five years for future expansion. How much does EAG have to set aside today earning 8 percent interest per year compounded quarterly to accumulate \$1,000,000 in five years?

5

Analysis of Present Worth

The F-35 Joint Strike Fighter is the most expensive, and possibly the most error ridden, project in the history of the United States military. But DOD has sunk so much money into the F-35 which is expected to cost \$1.5 trillion over the 55-year life of the program.

David Francis, CNBC



Present worth analysis is a very popular technique used by professional engineers to evaluate single projects or compare alternative projects that are mutually exclusive or independent from each other. To evaluate a single project, the present worth analysis consists of calculating the equivalence of all the project's cash flows, both inflows and outflows, using a given minimum acceptable rate of return, to the present time value ($t = 0$). Based on the value of the equivalence, one may accept or reject the project. Present worth analysis requires equivalent service lives for competing, mutually exclusive alternatives.

The focus of this chapter is to enable students and practitioners to use present worth analysis to evaluate single projects, and compare mutually exclusive alternatives and independent projects. This chapter also presents the concept of a capitalized cost that is equivalent to present worth analysis for projects with infinite lives.

The following sections briefly address the present worth calculation of single projects, the criteria for accepting or rejecting projects, and discuss the comparison of and selection among mutually exclusive alternatives and independent projects.

- ◆ Measuring Project Cash Flows
 - First Cost
 - Net Annual Cash Flow (NACF)
 - Terminal Cash Flow (TCF)

- ◆ Evaluation of Single Projects
- ◆ Bond Valuation
- ◆ Types of Project Alternatives
 - Mutually Exclusive (ME) Projects
 - Cost-based Projects
 - Revenue-based Projects
 - Independent Projects (IP)
 - Do-nothing Alternative
- ◆ Evaluation of Mutually Exclusive Equal-life Alternatives
- ◆ Evaluation of Mutually Exclusive Different-life Alternatives
 - Study Period Approach
 - Least Common Multiple or Replacement Chain Approach
 - Capitalized Cost or Equivalent Annual Annuity Approach
- ◆ Evaluation of Independent Projects
- ◆ Using Spreadsheets for Present Worth Computation
- ◆ Life Cycle Costing (LCC)
- ◆ Summary
- ◆ Discussion Questions and Problems

Measuring Project Cash Flows

In engineering economics, we are interested in measuring the incremental after-tax cash flows resulting from an investment proposal. Economic cash flows generate both inflow (revenue) and outflow (expense) streams. In general, there are three major sources of cash flows associated with a new project, including:

- ◆ First cost (initial outlay)
- ◆ Annual cash flows (cash inflows less cash outflows per year)
- ◆ Terminal cash flow (or salvage value)



First Cost (P)

Initial investment in engineering economics is called first cost and denoted by P. First cost includes all cash flows that are necessary to get the project in running order. These cash flows include the following:

- ◆ Cost of the new equipment
- ◆ Design

- ♦ Testing
- ♦ New buildings for manufacturing
- ♦ Shipping, handling, and installation cost
- ♦ Any cost items, such as training, necessary for the operation of proposed equipment
- ♦ Any other non-expense cash outlays required, such as working capital requirement
- ♦ Research and development

In July 2004, Bombardier announced the development of the CSeries family of airliners capable of carrying 110 to 130 passengers. The initial cost of the Bombardier CSeries jet program is now estimated at about \$5.4 billion.¹

Net Annual Cash Flow (NACF)

Net annual cash flows are the sum of incremental after-tax cash inflows and outflows per year over the project's life and are a measure of a company's financial health. NACF is the cash receipts minus cash payments over a given period of time (usually a year); furthermore, the amounts charged off for depreciation, depletion, and amortization must be included.

The annual cash flows may include the following:

- ♦ Cash receipts from the sale of goods and services (inflows)
- ♦ Cash receipts relating to future contracts (inflows)
- ♦ Cash receipts from royalties, patents, and other revenue (inflows)
- ♦ Cash payments to suppliers for goods and services (outflows)
- ♦ Cash payments to employees (outflows)
- ♦ Payments of refunds or tax payments (outflows)

Note: We should not include the sunk cost and financial cost of the project.

$$NACF = [(R - AOC - D) \times (1 - T)] + D$$

Where:

$NACF$ = Net Annual Cash Flow

R = Annual Revenue

AOC = Annual Operating Cost

D = Annual Depreciation

T = Marginal Tax Rate

In the above equation, we add back depreciation to NACF at the end because depreciation is an expense that does not require an outlay of cash.

Terminal Cash Flow (TCF)

Terminal cash flow (salvage value) includes any incremental cash flows resolved at the termination of the project. The salvage value is an important part of any project valuation. A manager needs to understand how terminal cash flows are estimated

and what factors impact these values. Retiring an aircraft, or selling a piece of asset generate terminal cash flows. For almost all projects, it is very difficult to estimate the terminal cash flow; but it is critical to forecast it as accurately as possible. These cash flows include the following:

- ◆ Net of tax project's salvage value
- ◆ Recovery of any non-expense cash outlay such as recovery of working capital associated with the project
- ◆ Any terminal cash flow needed, including disposal of obsolete equipment

Example 5.1

Majestic Aircraft Company (MAC) is a manufacturer of single engine, high performance turboprop aircraft. The company specializes in the manufacturing of advanced composite structures for military and commercial aviation customers, space systems, and other agencies. MAC is considering purchasing a new machine with a purchase price of \$75,000. The shipping and handling cost is \$1,000, installation cost is \$2,500, the training cost of an employee is \$5,000 to operate the machine, and it requires an increase in working capital of \$6,000. Expected annual revenue from this machine is \$25,000 and expected annual operating cost is \$4,000. The machine will be depreciated based on a Straight Line method over a five-year life toward a zero salvage value. Assuming a tax rate of 40 percent and \$7,000 value of the machine at the end of five years, determine the following:

- a) First cost
 - b) NACF
 - c) Terminal cash flow
- a. $P = \$75,000 + \$1,000 + \$2,500 + \$5,000 + \$6,000$
 $P = \$89,500$
 - b. To calculate NACF, first we need to calculate depreciation per year using the Straight Line method. The annual depreciation is calculated by dividing the sum of the purchase price of the machine plus shipping, handling, and installation costs over the useful life of the machine.

$$D = \frac{(\$75,000 + \$1,000 + \$2,500)}{5}$$

$$D = \$15,700$$

$$NACF = [(\$25,000 - \$4,000 - \$15,700) \times (1 - 0.40)] + \$15,700$$

$$NACF = [(\$5,300) \times (1 - 0.40)] + \$15,700$$

$$NACF = \$18,880$$

- c. $TCF = [\$7,000 \times (1 - 0.40)] + \$6,000$
 $TCF = \$10,200$

In this section, we learned how to calculate the net of tax annual cash flows associated with a given project, and that taxes, like inflation, must be considered to

determine the real payoff of a project. In most engineering problems, however, taxes will affect alternatives similarly and that allows us to compare alternatives accurately without considering tax effects.

Evaluation of Single Projects

Present worth analysis of a single project is a decision-making process to evaluate a project to determine whether or not it is economically justified. In other words, whether the project or investment is profitable. In the airline industry, most capital expenditure decisions are about acquisition of aircraft that facilitate the airline's growth, profitability, sustainability, and efficient business operation. Present worth analysis is probably the most popular and the most accurate technique available to engineers for evaluation of a single project or multiple projects that are independent from each other or are mutually exclusive.

There are numerous examples of projects in the aviation industry requiring present worth analysis. For example, when an aircraft manufacturer decides to build a new aircraft, it goes through an evaluation process such as present worth analysis to determine whether or not the investment is worthwhile or economically justified. To determine that, the aircraft manufacturer needs to consider all cash flows, both inflows and outflows associated with the investment.

Present worth (PW) analysis is simply equal to the present value of net cash flows minus the first cost. An appropriate Minimum Acceptable Rate of Return (MARR) for each particular project is needed to compute the PW. Note that in engineering economics:

- ♦ P now represents the first cost and present value is called present worth (PW).
- ♦ All costs have a negative sign.
- ♦ Net cash flows are calculated per period.

Aviation Snippets

Was the Launch of Ozjet Airline Economically Justified?

Ozjet airline was launched in 2004 by a businessman, Paul Stoddart, a former head of the Minardi Formula One racing team, to provide air transportation services to business travelers in Australia. Ozjet started its services between Sydney and Melbourne, adopting a low-cost strategy, and planned to expand its operations to other major cities.

Paul Stoddart wanted to take advantage of the stable economic growth and growing demand by business travelers in Australia. His strategy was to capture a small portion of the business traveler market share. But the fierce competition from major airlines in Australia such as Qantas, Jetstar, and Virgin Blue forced Ozjet to cease its operation within two years with about \$10 million loss.

This is a present worth analysis decision in which Ozjet underestimated the fierce competition in the business travelers market that resulted in its failure.

Strategic Analysis of Ozjet Airlines Company, UK Essay, March 2013

Table 5.1 Present Worth Analysis Accept and Reject Criteria

Present Worth	Selection Criteria
$PW > 0$	Accept the project, because it is economically justified. In other words it produces a return that is greater than MARR.
$PW = 0$	Accept the project, because it is economically justified. In other words it produces a return that is equal to MARR.
$PW < 0$	Reject the project, because it is <u>not</u> economically justified. In other words it produces a return that is less than MARR.

$$PW = -P + \text{Sum of Present values of all net annual cash flows}$$

Or

$$PW = -P + \sum_{t=1}^n \left(\frac{NACF_t}{(1+i)^t} \right)$$

$$NACF = \text{Annual cash inflow} - \text{Annual cash outflow}$$

$$PW = -P + NACF_1 (P/E, i\%, 1) + NACF_2 (P/E, i\%, 2) \dots + NACF_n (P/E, i\%, n)$$

If net annual cash flows have a format of an annuity series, then PW is calculated as follows:

$$PW = -P + A (P/E, i\%, n)$$

The criteria to select or to reject a project are summarized in Table 5.1.

Example 5.2

Majestic Aircraft Company (MAC) is a manufacturer of single engine, high performance turboprop aircraft. MAC is considering purchasing composite wing fixtures for the assembly of its signature aircraft. The cost of the assembly system is \$4 million with life expectancy of 20 years and annual operating cost of \$250,000. MAC anticipates a stream of \$1,000,000 in revenue per year for 20 years. Using present worth analysis and assuming a MARR of 10 percent, is this investment economically justified?

The initial investment or first cost is \$4 million. The net cash flow per year is calculated as follows:

$$\text{Net cash flow per year} = \$1,000,000 - \$250,000$$

$$\text{Net cash flow per year} = \$750,000$$

$$PW = -P + A (P/A, i\%, n)$$

$$PW = -\$4,000,000 + \$750,000 \times (P/A, 10\%, 20)$$

From Table A16 in Appendix A for 10 percent interest rate, we plug in the value of the interest factor in the above relationship and compute the present worth.

$$PW = -\$4,000,000 + \$750,000 \times 8.5136)$$

$$PW = -\$4,000,000 + 6,385,200$$

$$PW = \$2,385,200$$

Because PW is positive, this investment is economically justified and it is a good project to undertake.

Example 5.3

Consider Boeing's decision to manufacture the new B787, Dreamliner. The cash flows for B787 involve expenditures at the beginning of the project including design and development cost, production and operation cost, and testing and support cost where these costs are called cash outflows. Cash inflows include receiving revenue over a number of years from customers' purchase deposits, full aircraft payments, and spare part revenue. Both cash inflows and outflows are based on the forecast of market demand which depends on many factors including economic cycles, production level, and all of the other host of events that impact air transportation domestically and internationally. The Boeing 787 Dreamliner is the most advanced airplane with a body made of 50 percent composites and 20 percent aluminum.² The lower weight of composites relative to aluminum is the main contributor to the fuel efficiency of the aircraft, which is known to be 20 percent more efficient, emits 20 percent less pollutants, and has a 30 percent smaller maintenance cost than similar size airplanes.

To evaluate this major investment, Boeing needed to estimate the cash outflows including the initial investment and the cost of production per unit. It also needed to forecast the cash inflows including deliveries per year and price per unit. Boeing's development and production costs had added up to about \$32 billion dollars by 2011 when the first airplane was delivered, of which half was development cost and the other half was production cost (Gates, 2011). The retail prices for the 787 class of airplanes vary from \$220 million to \$280 million with an average of approximately \$250 million (The Boeing Company, 2015).³

To determine the production cost of 787 class airplanes, although it was difficult to assign a total cost to each airplane due to the early stage of production and not reaching the break-even point yet, the report indicated that the manufacturing cost of the 787 class would be between \$160 million to \$190 million with an average of approximately \$175 million per airplane for the 235 units delivered by January of 2015.⁴ Boeing has anticipated that the total number of orders for this aircraft will be approximately 3,500 units for 20 years between 2010 and 2030. The average cost of production will be expected to decrease starting in 2015 to about \$120 million per airplane due to economies of scale.

Table 5.2 summarizes the cash flows for the development of the B787 Dreamliner. We assume the following:

- a. Cash flows occurred at the end of each year.
- b. The \$16 billion development cost occurred at the end of 2005.
- c. The \$16 billion production cost occurred evenly from the end of 2006 through 2010.
- d. The production cost and the revenue per aircraft remained the same from 2011 through 2014 at \$175 million and \$250 million respectively.

- e. No deposit is made when the orders are placed and revenue is received when the delivery is made.
- f. The production cost and the revenue per aircraft remain the same from 2015 through 2030 at \$120 million and \$250 million respectively.

Because of the risky nature of this project, we assume a MARR of 15 percent to evaluate the development of the B787 Dreamliner. Cash inflows, outflows, and net cash flows are summarized in Table 5.3.

Table 5.2 Forecast of Costs and Revenues for Development of B787 Dreamliner

Year	Cost	Revenue
2005 Development cost. Assume development cost occurred at the end of 2005	\$16 billion	
2006–2010 Production cost	Total cost: \$16 billion Average cost per year: \$3.2 billion	
2011 Delivery: 32 units	Estimated cost per unit: \$175 million	Estimated revenue per unit: \$250 million
2012 Delivery: 93 units	Estimated cost per unit: \$175 million	Estimated revenue per unit: \$250 million
2013 Delivery: 69 units	Estimated cost per unit: \$175 million	Estimated revenue per unit: \$250 million
2014 Delivery: 41 units	Estimated cost per unit: \$175 million	Estimated revenue per unit: \$250 million
2015–2030 Delivery of average 210 units per year, based on the Boeing forecast of 3,500 delivery for 20 years between 2011 to 2030	Average cost per unit: \$120 million	Estimated revenue per unit: \$250 million

Table 5.3 Net Cash Flows per Year for B787 Development (in USD billions)

Year	Cash Outflows	Cash Inflows	Net Cash Flows
2005	−\$16	0	−\$16
2006–2010 per year	−\$3.2	0	−\$3.2
2011	−\$5.6	\$8	\$2.4
2012	−\$16.26	\$23.25	\$6.99
2013	−\$12.08	\$17.25	\$5.17
2014	−\$7.18	\$10.25	\$3.07
2015–2030 per year	−\$25.20	\$52.50	\$27.3

We are using formulas from Chapter 2, present value of future amount, present value of annuities, and present value of shifted series, to compute the PW of this investment as follows:

$$PW = -\$16 - \$3.2 (P/A, 15\%, 5) + \$2.4 (P/F, 15\%, 6) + \$6.99 (P/F, 15\%, 7) \\ + \$5.17 (P/F, 15\%, 8) + \$3.07 (P/F, 15\%, 9) + \$27.3 (P/A, 15\%, 16) \\ \times (P/F, 15\%, 9)$$

From Table A21 in Appendix A for 15 percent interest rate, we plug in the value of the interest factors in the above relationship and compute the present worth.

$$PW = -16 - \$3.2 (3.3522) + \$2.4 (0.4323) + \$6.99 (0.3759) + \$5.17 (0.3269) + \$3.07 \\ (0.2843) + \$27.3 (5.9542) \times (0.2843) \\ PW = -\$16 - \$10.73 + \$1.04 + \$2.63 + \$1.69 + \$0.87 + \$46.21 \\ PW = \$25.71 \text{ billion}$$

The investment in the 787 Dreamliner over a 25-year period is profitable and therefore it is economically justified. In other words, the PW of \$25.71 billion means that at 2005's dollar value, the investment will net Boeing \$25.71 billion.

Bond Valuation

A bond is a debt instrument issued by major corporations and federal and local government agencies to raise large sums of money to fund their projects. Bonds are usually sold at discount prices. When you purchase a bond you will receive a bond certificate that contains the following information:

- ♦ Face value, par value, or maturity value (V), which is usually \$1,000. This amount is received at bond maturity date.
- ♦ Coupon rate (c), the rate used to determine the amount of payment you receive periodically.
- ♦ Interest rate or required yield (i), the required return which would motivate an investor to buy the bond.
- ♦ Maturity date, the year in which the investors receive the face value of the bond. The number of periods to maturity is normally denoted as " n ." For a bond making annual payments, the number of periods to maturity is " n " years. If the bond makes semi-annual payments, the number of periods to maturity is " $n \times 2$ " periods. Another notation for this is " f ," which denotes frequency of payments; $f = 1$ implies annual payments, and $f = 2$ implies semi-annual payments.

The dollar amount of interest payment (I) received per period is determined by the following equation:

$$I + \frac{V \times c}{f}$$

PW of bond or bond price, making payments annually is calculated using the following equation:

$$PW_{\text{Bond}} = I (P/A, i, n) + V (P/F, i, n)$$

If the bond makes semi-annual payments, the equation will change to

$$PW_{\text{Bond}} = I \left(P/A, \frac{i}{2}, n \times 2 \right) + V \left(P/F, \frac{i}{2}, n \times 2 \right)$$

The simplified mathematical equation to determine the value of a bond, or bond price, is represented by the following equation:

$$\text{Bond price} = I \times \frac{1}{i} \left[1 - \left[\frac{1}{(1+i)^n} \right] \right] + \frac{V}{(1+i)^n}$$

Example 5.4

A bond with a face value of \$1,000 and coupon rate of 6 percent, makes annual payments and has a time to maturity of 20 years. Analyze the change in bond price over the duration toward maturity for two scenarios, one with the required yield of 8 percent and another with the required yield of 4 percent.

Scenario 1: Required yield of 8 percent

When time to maturity $n = 20$ years, the bond value is as follows:

$$\text{Bond price} = \$60 \times \frac{1}{0.08} \left[1 - \left[\frac{1}{(1+0.08)^{20}} \right] \right] + \frac{\$1000}{(1+0.08)^{20}}$$

$$\text{Bond price} = \$803.64$$

When time to maturity $n = 10$ years, the bond value is as follows:

$$\text{Bond price} = \$60 \times \frac{1}{0.08} \left[1 - \left[\frac{1}{(1+0.08)^{10}} \right] \right] + \frac{\$1000}{(1+0.08)^{10}}$$

$$\text{Bond price} = \$865.80$$

When time to maturity $n = 0$ years, i.e., at the time of maturity, the bond value is:

$$\text{Bond price} = \$60 \times \frac{1}{0.08} \left[1 - \left[\frac{1}{(1+0.08)^0} \right] \right] + \frac{\$1000}{(1+0.08)^0}$$

$$\text{Bond price} = \$1,000$$

Scenario 2: Required yield of 4 percent

When time to maturity $n = 20$ years, the bond value is calculated as follows:

$$\text{Bond price} = \$60 \times \frac{1}{0.04} \left[1 - \left[\frac{1}{(1 + 0.04)^{20}} \right] \right] + \frac{\$1000}{(1 + 0.04)^{20}}$$

$$\text{Bond price} = \$1,271.81$$

When time to maturity $n = 10$ years, the bond value is as follows:

$$\text{Bond price} = \$60 \times \frac{1}{0.04} \left[1 - \left[\frac{1}{(1 + 0.04)^{10}} \right] \right] + \frac{\$1000}{(1 + 0.04)^{10}}$$

$$\text{Bond price} = \$1,162.22$$

When time to maturity $n = 0$ years, i.e., at the time of maturity, the bond value is:

$$\text{Bond price} = \$60 \times \frac{1}{0.04} \left[1 - \left[\frac{1}{(1 + 0.04)^0} \right] \right] + \frac{\$1000}{(1 + 0.04)^0}$$

$$\text{Bond price} = 1,000$$

The change in value over the 20 years is presented in Table 5.4.

Using Spreadsheets for Bond Value Computation

The value of a bond can be calculated using Excel functions as presented in the figures that follow. The present value function normally returns a negative value, owing to the fact that the purchase of anything is a cash outflow. Hence, there is the use of a negative sign before the Excel function to ensure the value returned is positive.

To calculate the present value of a bond, the following formula is used:

$$= -\text{PV}(\text{rate}, \text{nper}, \text{pmt}, \text{fv})$$

Where:

rate = the required yield for the bond

nper = the time to maturity on the bond (represented as number of periods)

pmt = the coupon payment made in each period

fv = the future value or face value of the bond

To find the present value for a bond with a face value of \$1,000 and time to maturity of 20 years with annual coupon payments of \$60 and required yield of 8 percent, the formula to be entered must be:

$$= -\text{PV}(8\%, 20, 60, 1000)$$

And the result obtained would be \$803.64, which is represented in Figure 5.2. The coupon payment can also be entered as a product of the annual coupon rate and the face value ($6\% \times \$1000$).

Table 5.4 Bond Valuation

Scenario 1: Required Yield of 8%		Scenario 2: Required Yield of 4%	
Time to Maturity	Price of Bond	Time to Maturity	Price of Bond
20	\$803.64	20	\$1,271.81
19	\$807.93	19	\$1,262.68
18	\$812.56	18	\$1,253.19
17	\$817.57	17	\$1,243.31
16	\$822.97	16	\$1,233.05
15	\$828.81	15	\$1,222.37
14	\$835.12	14	\$1,211.26
13	\$841.92	13	\$1,199.71
12	\$849.28	12	\$1,187.70
11	\$857.22	11	\$1,175.21
10	\$865.80	10	\$1,162.22
9	\$875.06	9	\$1,148.71
8	\$885.07	8	\$1,134.65
7	\$895.87	7	\$1,120.04
6	\$907.54	6	\$1,104.84
5	\$920.15	5	\$1,089.04
4	\$933.76	4	\$1,072.60
3	\$948.46	3	\$1,055.50
2	\$964.33	2	\$1,037.72
1	\$981.48	1	\$1,019.23
0	\$1,000.00	0	\$1,000.00

A step by step approach to Excel calculation is presented below.

Step 1: In cell B1, enter the face value of \$1,000

Step 2: In cell B2, enter the coupon rate of 6%

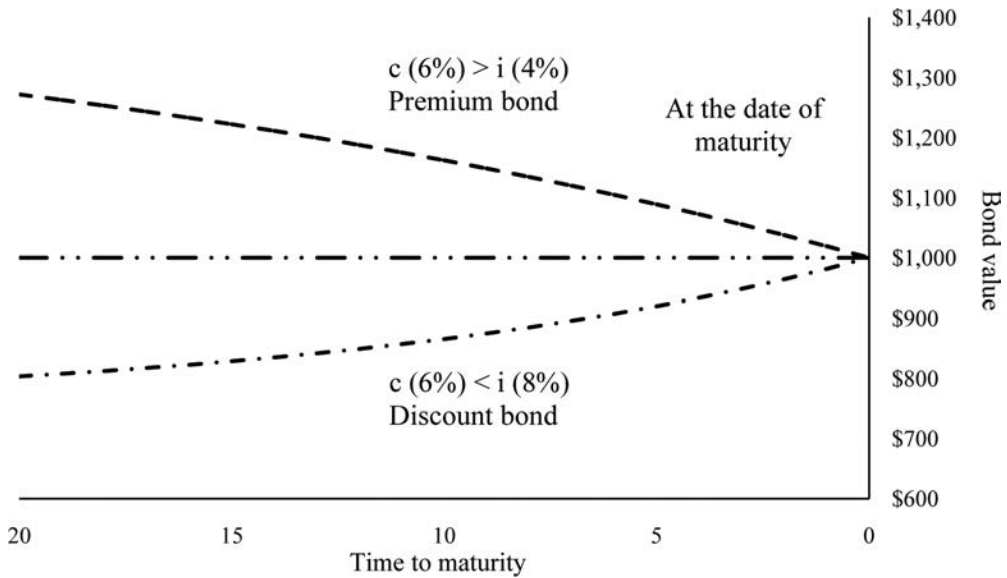
Step 3: In cell B6, enter the required yield or YTM on the bonds as 8%

Step 4: Create a list for times to maturity (number of periods) from cells A9 to A12 and enter the values 20, 19, 18 and 17.

Step 5: In cell B9, the formula for present value of the bond (price of the bond) is entered as follows:

$$= -PV(\$B\$6,A9,(\$B\$2*\$B\$1),\$B\$1)$$

Figure 5.1 Bond Valuation



Step 6: Drag down the formula till cell B12; that generates the results presented in the figures below.

If the required yield is 4 percent, the same process as mentioned above is followed and the required yield in the formula is changed to 4 percent as opposed to 8 percent in the previous case. The formula in this scenario will be as follows:

$$=-PV(4\%,20,60,1000)$$

And the result obtained will be \$1,271.81 (see Figure 5.3).

Figure 5.2 Calculating the Value of the Bond Using Excel Spreadsheets

	A	B	C	D		A	B	C
1	Face value	\$1,000			1	Face value	\$1,000	
2	Coupon rate	6%			2	Coupon rate	6%	
3	Time to maturity	20 years			3	Time to maturity	20 years	
4					4			
5	Scenario 1				5	Scenario 1		
6	Interest rate or YTM	8%			6	Interest rate or YTM	8%	
7					7			
8	Time to maturity	Price of Bond			8	Time to maturity	Price of Bond	
9	20	=-PV(\$B\$6,A9,(\$B\$2*\$B\$1),\$B\$1)			9	20	\$803.64	
10	19	PV(rate, nper, pmt, [fv], [type])			10	19	\$807.93	
11	18	\$812.56			11	18	\$812.56	
12	17	\$817.57			12	17	\$817.57	

Figure 5.3 Calculating the Value of the Bond Using Excel Spreadsheets

	A	B	C	D		A	B	C
1	Face value	\$1,000			1	Face value	\$1,000	
2	Coupon rate	6%			2	Coupon rate	6%	
3	Time to maturity	20 years			3	Time to maturity	20 years	
4					4			
5	Scenario 2				5	Scenario 2		
6	Interest rate or YTM	4%			6	Interest rate or YTM	4%	
7					7			
8	Time to maturity	Price of Bond			8	Time to maturity	Price of Bond	
9	20	=PV(\$B\$6,A9,(\$B\$2*\$B\$1),\$B\$1)			9	20	\$1,271.81	
10	19	PV(rate, nper, pmt, [fv], [type])			10	19	\$1,262.68	
11	18	\$1,253.19			11	18	\$1,253.19	
12	17	\$1,243.31			12	17	\$1,243.31	

One of the applications of PW analysis is to determine whether or not an investment in a bond issue is economically justified. In addition, PW analysis is used to calculate the value of bonds and decide whether or not you make at least a required rate of return if you purchase a bond at a given price.

Example 5.5

DirectJet Airline is a major carrier in the US and has an extensive network of operation. DirectJet has decided to pursue the issuance of bonds with face value of \$1,000, coupon rate of 8 percent, with 10 years to maturity. The interest is paid semi-annually and the bond is currently priced at \$920. If you require a minimum rate of return of 10 percent compounded semi-annually, is this a good investment for you?

To answer this question, we compute the present worth of this bond at 10 percent MARR, i.e., $i = 10$ percent. If PW is positive or equal to zero, it means you earn at least your minimum rate of return and it is a good investment. If PW is negative, it means you are not making the MARR on this investment and therefore it is not a viable investment.

First, the semi-annual interest payment can be calculated using the following equation:

$$I = \frac{V \times c}{f}$$

$$I = \frac{\$1,000 \times 0.08}{2}$$

$$I = \$40$$

In this example, the interest is paid semi-annually and the discount rate is 10 percent compounded semi-annually and therefore similar to payment; so the required rate of return and the number of periods are converted to semi-annual quantities. The actual present worth of the bond is found using the following equation:

$$PW_{\text{Bond}} = I \left(P/A, \frac{i}{2}, n \times 2 \right) + V \left(P/F, \frac{i}{2}, n \times 2 \right)$$

But, to determine if the purchase of the bond is efficient, we add the above present worth of the bond to the negative cash outflow given to the purchase of the bond.

$$PW_{\text{Bond}} = -\$920 + \$40 \left(P/A, \frac{10\%}{2}, 10 \times 2 \right) + \$1,000 \left(P/F, \frac{10\%}{2}, 10 \times 2 \right)$$

$$PW_{\text{Bond}} = -\$920 + \$40 (P/A, 5\%, 20) + \$1,000 (P/F, 5\%, 20)$$

From Table A11 in Appendix A for 5 percent interest rate, the value of the interest factors can be plugged into the above relationship to compute the present worth as shown below:

$$PW_{\text{Bond}} = -\$920 + \$40 (12.4622) + \$1,000 (0.3769)$$

$$PW_{\text{Bond}} = -\$920 + \$498.49 + \$376.90$$

$$PW_{\text{Bond}} = -\$920 + \$875.39$$

$$PW_{\text{Bond}} = -\$44.61$$

Because $PW < 0$, purchasing this bond is not economically justified and therefore it is not a good investment.

The airline industry often issues billions of dollars of bonds to fund aircraft purchases or expansion through mergers and acquisition. Airlines also use Enhanced Equipment Trust Certificates (EETCs) which are basically corporate secured debt instruments that rely on the credit worthiness of the airline and are secured by the aircraft as collateral. The EETC debt instruments are often used by foreign airline companies to obtain fairly inexpensive capital from US investors who may not otherwise be seeking investment opportunities in foreign companies.

The EETCs are also used by aircraft leasing companies to purchase a large fleet of aircraft and lease them to several airlines. Doric Nimrod Air France Alpha Limited⁵ issued two tranches of EETCs in 2012 with a total amount of \$587.5 million to fund the acquisition of four Airbus A380 aircraft and, in turn, lease them to Emirates Airlines.

Aviation Snippets

United Airlines launches a \$949 million Enhanced Equipment Trust Certificate (EETC)

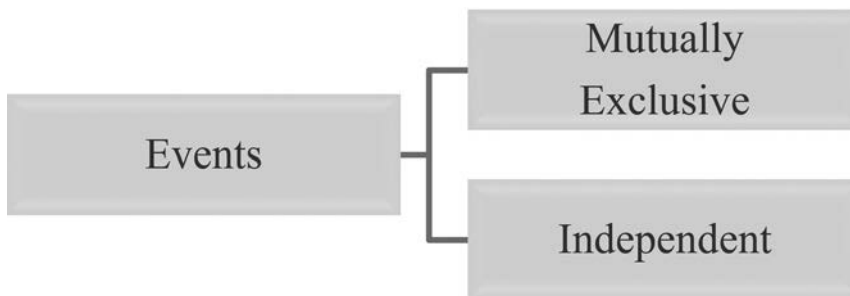
United Airlines (United Continental Holding), a leading passenger airline company in North America with a market capitalization of \$15.9 billion, issued a \$949.5 million enhanced equipment trust certificate (or EETC) last week. This was the company's first aircraft-backed public bond of 2014.

An equipment trust certificate (or EETC) is a financial security used in aircraft finance, most commonly to take advantage of tax benefits in North America. In a typical EETC transaction, a "trust certificate" is sold to investors in order to finance the purchase of an aircraft by a trust managed on the investor's behalf. The trust then leases the aircraft to the airline, and the trustee routes payments through the trust to investors. Upon the maturity of the note, the airline receives the title to the aircraft.

Sandra Nathanson,
Marketrealist.com, April 8, 2014

Types of Project Alternatives

When multiple projects are under consideration, one has to determine whether the projects are mutually exclusive or independent from each other. Depending on the relationship, each project is evaluated differently.



Mutually Exclusive (ME) Projects

Two or more projects are considered mutually exclusive if selection of one project results in the rejection of the rest of the projects even though they might be profitable projects. In mutually exclusive projects, a firm will evaluate all alternatives and select the best project that contributes the most to the value of the firm. An example of mutually exclusive projects for an airline is when the airline needs a new aircraft with capacity between 200 to 300 passenger seats.

The alternatives are B767, B777, B787, A330, and A350. The airline may evaluate these five aircraft with respect to price, efficiency, maintenance cost, consistency with the rest of its fleet, noise, and pollution and select the one best alternative that contributes the most to the profitability of the airline. There are two types of mutually exclusive alternatives.

Cost-based Alternatives

When a set of projects that are being evaluated provide the same service and produce the same amount of revenue in the same time period, they are called mutually exclusive cost-based alternatives. The focus of PW analysis is to determine the most cost-efficient project. The selection criteria are to choose the one project that numerically has the highest PW value. In cost-based alternatives, usually the revenue is not provided.

Revenue-based Alternatives

When a group of mutually exclusive projects are being evaluated that produce different amounts of revenue in the same time period, they are called mutually exclusive revenue-based alternatives. The PW analysis is performed to determine the most profitable alternative. The selection criteria are the same as for cost-based alternatives, to choose the one project that numerically has the highest PW value.

Independent Projects (IP)

Projects are considered to be independent from each other if, when we accept or reject a project, it does not have any impact on the rest of the projects. Independent projects evaluation does not require equal service lives. Examples of independent projects for an airport may include:

- ♦ Construction of a new terminal
- ♦ Construction of a new runway
- ♦ Building a new weather monitoring station
- ♦ Building new concession area
- ♦ Building a new parking garage

If an airport is considering all of the above projects, assuming all projects are economically justified and there is no budget limitation, then the airport may undertake all projects because all of them contribute to the growth or value of the airport.

Do-nothing Alternative

This is a scenario in which none of the projects is being selected. In mutually exclusive revenue alternatives and independent projects, there may be a scenario where none of the projects is economically justified and therefore do-nothing or status quo is selected. Do-nothing will leave the value of the firm unchanged.

Evaluation of Mutually Exclusive Equal-life Alternatives

The focus of engineering economics is primarily on mutually exclusive cost-based alternatives. However, all engineering economics techniques can be used to evaluate mutually exclusive and independent projects that are cost or revenue based.

Many manufacturing companies are contemplating the use of alternative energy in their manufacturing process. Many airline companies are considering retiring inefficient aircraft that are still operable for many years with new and more efficient aircraft which would require significant cash flows at the time of replacement. One of the basic techniques that assists decision makers to choose the best alternative is Present Worth (PW) analysis. We can see that in the following example:

Example 5.6

Majestic Aircraft Company (MAC) is a manufacturer of single engine, high performance turboprop aircraft. Majestic Aircraft Company (MAC) engineers are introducing a new process for their new engine design. Either a robotics or a non-robotics testing system will ensure that their new signature program does indeed work for every engine produced. Table 5.5 presents the cash flows associated with both alternatives. Assuming a MARR of 10 percent, which of the following two mutually exclusive testing systems should be selected?

To select the best alternative, we compute the PW of both systems.

$$PW = -P + \sum_{t=1}^n \left(\frac{NACF_t}{(1+i)^t} \right)$$

Table 5.5 Cash Flows for Both Robotics and Non-robotics

	Robotics (R)	Non-robotics (NR)
Testing system and support equipment (first cost)	−\$1,500,000	−\$2,250,000
Annual operating cost (AOC)	−\$700,000	−\$500,000
Rebuild cost in year 3	−\$200,000	\$0
Salvage value	\$50,000	\$100,000
Estimated life, years	8	8

For Robotics,

$$PW_R = -\$1,500,000 - \$700,000 \times (P/A, 10\%, 8) - \$200,000 \times (P/F, 10\%, 3) + \$50,000 \times (P/F, 10\%, 8)$$

From Table A16 in Appendix A for 10 percent interest rate, we plug in the value of the interest factors in the above relationship and compute the present worth for the Robotics system.

$$\begin{aligned} PW_R &= -\$1,500,000 - [\$700,000 \times (5.3349)] - [\$200,000 \times (0.7513)] + [\$50,000 \times (0.4665)] \\ PW_R &= -\$1,500,000 - \$3,734,430 - \$150,260 + \$23,325 \\ PW_R &= -\$5,361,365 \end{aligned}$$

Similarly, for Non-robotics,

$$PW_{NR} = -\$2,250,000 - \$500,000 (P/A, 10\%, 8) + \$100,000 (P/F, 10\%, 8)$$

From Table A16 in Appendix A for 10 percent interest rate, we plug in the value of the interest factors in the above relationship and compute the present worth for the Non-robotics system.

$$\begin{aligned} PW_{NR} &= -\$2,250,000 - [\$500,000 \times (5.3349)] + (\$100,000 \times (0.4665)] \\ PW_{NR} &= -\$2,250,000 - \$2,667,450 + \$46,650 \\ PW_{NR} &= -\$4,870,800 \end{aligned}$$

Because the Non-robotics system has numerically the highest PW value, it will be the selected alternative.

Airports are also looking for the most cost-efficient alternative source of energy to light the airport. In the following example, solar cells and power lines are being evaluated to determine which method is cost efficient.

Example 5.7

Legacy Regional Airport (LRA) is conveniently located in the center of the City of Ho Chi Minh, Vietnam. The airport offers commercial service to Paris and regular scheduled charter flights to different destinations. Since the new management

assumed responsibility for the airport, passenger traffic has increased more than 15 percent. To accommodate the new traffic by opening night flights, LRA airport has the following two options to light the taxiways and runways: solar cells and a new power line.

1. Solar cells (SC) will have a first cost of \$24,000 including the installation cost. Annual operating costs for inspection and cleaning are expected to be \$2,500. The solar cells will have a useful life of 10 years with an expected salvage value of 20 percent of the first cost.
2. A new power line (PL) will have a first cost of \$18,000. The annual operation cost is expected to be \$3,000 in year 1 and to increase by a constant amount of \$500 per year starting from the second year through the end of 10 years of life. The power line will stay in place and therefore has zero salvage value.

If LRA uses a 12 percent MARR for these projects, which option should be selected to light the runways and taxiways at LRA?

Present worth analysis is used to evaluate each alternative. From Table A18, Appendix A for 12 percent interest rate, the values of interest factors are plugged in the following equations.

$$PW_{SC} = -\$24,000 - \$2,500 (P/A, 12\%, 10) + (0.20) \times (\$24,000) \times (P/F, 12\%, 10)$$

$$PW_{SC} = -\$24,000 - \$2,500 (5.6502) + (0.20) \times (\$24,000) \times (0.3220)$$

$$PW_{SC} = -\$24,000 - \$14,126 + \$1,546$$

$$PW_{SC} = -\$36,580$$

$$PW_{PL} = -\$18,000 - \$3,000(P/A, 12\%, 10) - \$500 (P/G, 12\%, 10)$$

$$PW_{PL} = -\$18,000 - \$3,000 (5.6502) - \$500 (20.2541)$$

$$PW_{PL} = -\$18,000 - \$16,951 - \$10,127$$

$$PW_{PL} = -\$45,078$$

The solar cells system has a numerically higher PW ; thus it is more cost efficient, and therefore should be selected.

Evaluation of Mutually Exclusive Different-life Alternatives

In present worth analysis, an equal life service comparison is a requirement. Projects must be evaluated over the same time period. For example, consider two projects: one with a five-year life and the other with a 10-year life. If we evaluate them over a five-year period, we are ignoring the rest of the cash flows of the longer-term project. If we evaluate them over a 10-year period, we are ignoring the return on the cash flows of the shorter life project for the last five years. This section presents the following three approaches to compare projects with different lives:

- ♦ Study period approach
- ♦ Least common multiple or replacement approach
- ♦ Capitalized cost approach or equivalent annual annuity approach

Study Period Approach

This approach is used when a company needs, for example, equipment for only a fixed period of time and has no need after that. The assumptions of this approach are as follows:

- ◆ Once the time period is established, only cash flows within that time period will be evaluated and cash flows outside of the time frame of the study period are disregarded.
- ◆ If the life of the project is greater than the study period, the market value of the project at the end of the study period is used as the salvage value of the project.
- ◆ If the life of the project is less than the study period, the cash flows for the time period through the end of the study period must be estimated and if there is a salvage value, that also needs to be adjusted to reflect the market value at the end of the study period.

In the examples and problems in this book, the study period would not be greater than the life of any projects because we do not have information to estimate the cash flows beyond the project's life. In addition, when the life of a project is greater than the study period, the project's salvage value moves to the end of the study period because we do not have information to determine the market value of the equipment at the end of the study period.

Example 5.8

DirectJet Airline is considering improving customer service. DirectJet is looking into offering one of the following two free services in Table 5.6 to customers:

- a. Free live television services onboard its aircraft, with a first cost of \$50 million and annual operating cost of \$10 million for 10 years and no salvage value.
- b. Free high speed internet services on board its aircraft, with the first cost of \$30 million and annual operating cost of \$8 million for five years with no salvage value.

DirectJet's MARR is 10 percent. Using a study period of five years, which alternative is cost efficient?

Using a five-year study period, and Table A16 in Appendix A for 10 percent interest rate, we calculate PW for both alternatives as follows:

$$PW_{LT} = -\$50,000,000 - \$10,000,000 (P/A, 10\%, 5)$$

$$PW_{LT} = -\$50,000,000 - \$10,000,000 (3.7908)$$

$$PW_{LT} = -\$50,000,000 - \$37,908,000$$

$$PW_{LT} = -\$87,908,000$$

$$PW_{HSI} = -\$30,000,000 - \$8,000,000 (P/A, 10\%, 5)$$

$$PW_{HSI} = -\$30,000,000 - \$8,000,000 (3.7908)$$

$$PW_{HSI} = -\$30,000,000 - \$30,326,400$$

$$PW_{HSI} = -\$60,326,400$$

Based on a study period of five years, the high speed internet service has numerically the highest PW value and therefore is the most cost-efficient alternative.

Table 5.6 Cash Flows for Both Alternatives

Year	Live Television (LT)	High Speed Internet (HSI)
0	−\$50,000,000	−\$30,000,000
1	−\$10,000,000	−\$8,000,000
2	−\$10,000,000	−\$8,000,000
3	−\$10,000,000	−\$8,000,000
4	−\$10,000,000	−\$8,000,000
5	−\$10,000,000	−\$8,000,000
6	−\$10,000,000	
7	−\$10,000,000	
8	−\$10,000,000	
9	−\$10,000,000	
10	−\$10,000,000	

Least Common Multiple or Replacement Chain Approach

The second option to compare projects with different lives is the least common multiple (LCM) or replacement chain approach. Under this option, the analysis period is the least common multiple of the projects' lives. For example, if one project has four years of life and the alternative project has six years of life, the analysis period is the least common multiple of four and six, which is 12 years. In other words, we replace a six-year life project once by itself and replace the project with a four-year life twice by itself to have 12 years of analysis period.

The assumptions of this approach are as follows. The third assumption may not be realistic as cash flows for second and third cycles may change due to inflation or deflation.

- ♦ The service is needed for LCM analysis period
- ♦ Projects are replaced by themselves to reach LCM year
- ♦ Cash flows would not change during the replacement periods

In Example 5.8, the LCM period for live television and high speed internet alternatives is 10 years. Therefore, we are going to compare one life cycle of live television with two cycles of high speed internet as shown in Table 5.7.

Using LCM of 10 years, and Table A16 in Appendix A for 10 percent interest rate, we calculate PW of both projects as follows:

$$PW_{LT} = -50,000,000 - \$10,000,000 (P/A, 10\%, 10)$$

$$PW_{LT} = -\$50,000,000 - \$10,000,000 (6.1446)$$

$$PW_{LT} = -\$50,000,000 - \$61,446,000$$

$$PW_{LT} = -\$111,446,000$$

Table 5.7 Cash Flows for Both Alternatives under Replacement Chain Approach

Year	Live Television (LT)	High Speed Internet (HSI)	
		First Cycle	Second Cycle
0	−\$50,000,000	−\$30,000,000	
1	−\$10,000,000	−\$8,000,000	
2	−\$10,000,000	−\$8,000,000	
3	−\$10,000,000	−\$8,000,000	
4	−\$10,000,000	−\$8,000,000	
5	−\$10,000,000	−\$8,000,000	−\$30,000,000
6	−\$10,000,000		−\$8,000,000
7	−\$10,000,000		−\$8,000,000
8	−\$10,000,000		−\$8,000,000
9	−\$10,000,000		−\$8,000,000
10	−\$10,000,000		−\$8,000,000

$$PW_{HSI} = -\$30,000,000 - \$30,000,000 (P/F, 10\%, 5) - \$8,000,000 (P/A, 10\%, 10)$$

$$PW_{HSI} = -\$30,000,000 - [\$30,000,000 \times (0.6209)] - [\$8,000,000 \times (6.1446)]$$

$$PW_{HSI} = -\$30,000,000 - \$18,627,000 - \$49,156,800$$

$$PW_{HSI} = -\$97,783,800$$

Based on an LCM period of 10 years, using the replacement chain approach, the high speed internet service has numerically the highest PW value and therefore, is the most cost-efficient alternative.

Assuming passengers place the same value for both free services of live television and high speed internet, DirectJet is better off offering high speed internet, because it is the most cost-efficient alternative.

Capitalized Cost or Equivalent Annual Annuity Approach

The third option to compare projects with different lives is called capitalized cost or equivalent annual annuity approach.

This approach assumes infinite lives for alternative projects. In other words, this approach assumes that projects are replaced by themselves indefinitely and therefore $LCM = \infty$. We use the concept of capitalized cost to compare the alternatives. This approach is particularly useful when life of at least one of the projects is a prime number and therefore LCM is the

Capitalized cost is the present worth of a project that will last forever.

Facilities such as airport terminals, runways, shipyards, subway systems, and railroads fall into this category, since they have useful lives of 30 or more years.

product of their lives and possibly a very large number. For example, if the life of project A is 9 years and the life of project B is 13 years then the LCM is 9×13 or 117 years and it would be very time consuming to use the LCM approach. Therefore, it is best to use capitalized cost approach.

Under this approach, first we calculate the PW of the projects over their original lives. Second, we compute the annuity of their PW over their original lives. Third, we assume that projects are replaced by themselves infinitely, and therefore using capitalized cost equation we compute the PW of each alternative over an infinite time period. Finally, we apply PW selection criteria to choose the best project.

The assumptions of this approach are similar to the LCM approach. They are as follows:

- ♦ The service is needed for an infinite time period
- ♦ Projects are replaced by themselves indefinitely
- ♦ Cash flows would not change during the infinite replacement periods

Using Example 5.8, compare the two mutually exclusive alternatives, live television and high speed internet, for DirectJet with the capitalized cost approach.

First, we compute PW of each alternative over their original lives.

$$PW_{LT} = -\$50,000,000 - \$10,000,000 (P/A, 10\%, 10)$$

$$PW_{LT} = -\$50,000,000 - \$10,000,000 (6.1446)$$

$$PW_{LT} = -\$50,000,000 - \$61,446,000$$

$$PW_{LT} = -\$111,446,000$$

$$PW_{HSI} = -\$30,000,000 - \$8,000,000 (P/A, 10\%, 5)$$

$$PW_{HSI} = -\$30,000,000 - \$8,000,000 (3.7908)$$

$$PW_{HSI} = -\$30,000,000 - \$30,326,400$$

$$PW_{HSI} = -\$60,326,400$$

Second, we compute the annuity of their PW over their original lives. The values of interest factors are taken from Table A16 in Appendix A for 10 percent interest rate.

$$A_{LT} = -\$111,446,000 (A/P, 10\%, 10)$$

$$A_{LT} = -\$111,446,000 (0.16275)$$

$$A_{LT} = -\$18,137,837$$

$$A_{HSI} = -\$60,326,400 (A/P, 10\%, 5)$$

$$A_{HSI} = -\$60,326,400 (0.26380)$$

$$A_{HSI} = -\$15,914,104$$

Third, as projects are replaced by themselves infinitely, the annuities A_{LT} and A_{HSI} continue to infinity and we compute the PW over the infinite life period.

$$PW_{LT \& \infty} = - \frac{\$18,137,837}{0.10}$$

$$PW_{LT \& \infty} = -\$181,378,370$$

$$PW_{HSI \& \infty} = - \frac{\$15,914,104}{0.10}$$

$$PW_{HSI \& \infty} = -\$159,141,040$$

Based on capitalized cost approach, assuming infinite replacement of projects by themselves, the high speed internet service has the numerically highest PW value and therefore is the most cost-efficient alternative. DirectJet is better off offering high speed internet.

Evaluation of Independent Projects

As we described earlier, independent projects (IP) exist where when we accept or reject one project, it does not have any impact on the rest of the projects. Therefore, if an airline company has a portfolio of independent projects that are all economically justified and it has no budget constraints, then it should undertake the entire portfolio of projects.

In certain cases, only limited funds may be available for investment and, at least in the short-run, no additional capital may be obtainable from external sources. This situation is referred to as capital rationing. Capital rationing might occur if the decision unit is not an entire firm but a division with a specific capital allocation for investment determined by the parent company. Capital constraints also characterize the situation confronted by government agencies that must allocate fixed appropriations among competing alternatives.

The present worth approach to evaluating the capital rationing problem is to invest the funds in the one set or bundle of projects that provides the highest present worth or contributes the most to the value of the firm. We follow the procedures below to select the optimal set of projects.

1. Compute the PW of all projects at company's MARR.
2. Eliminate all projects that have negative PW.
3. Determine all sets or bundles of projects with total investment of equal to or lesser than the given budget.
4. Compute total present worth of all bundles or sets.
5. Select the one set that has the highest total present worth value.
6. The projects in the selected set are the acceptable projects.

Example 5.9

DirectJet Airline is a major carrier in the US and has an extensive network of operation. DirectJet is considering the following four independent projects:

- a. Installing 10 electronic kiosks at its Chicago hub. The kiosks expedite passengers to check in, view seat assignments, make seat changes, review itineraries, and print boarding passes. The total cost of kiosks, annual operating cost, and annual revenue are provided in Table 5.8.
- b. Offering Wi-Fi onboard its airplanes with cash flows presented in Table 5.8.

Table 5.8 DirectJet Airlines Independent Projects Cash Flows

	Electronic Kiosks (EK)	Wi-Fi (WF)	Live Television (LT)	Embraer 170 (EM)
Investment (Lease deposit)	-\$500,000	-\$1,000,000	-\$2,500,000	-\$15,000,000
Annual operating cost	-\$50,000	-\$10,000	-\$20,000	-\$1,000,000
Annual revenue	\$200,000	\$50,000	\$500,000	\$4,000,000
Salvage value	\$10,000	0	\$50,000	
(Deposit return)				\$2,000,000
Life in years	10	25	10	20

- c. Providing live television service onboard its airplanes with cash flows summarized in Table 5.8.
- d. DirectJet is planning to lease an additional Embraer 170 to add it to its fleet. The cash flows associated with the lease of the new aircraft are shown in Table 5.8.

DirectJet Airlines has a limited budget of \$17 million with a MARR of 8 percent. Use PW analysis to determine which alternatives, if any, DirectJet should undertake to maximize its profits.

First, we compute the PW of each alternative using Table A14 in Appendix A for 8 percent interest rate.

$$PW_{EK} = -\$500,000 + [(\$200,000 - \$50,000) \times (P/A, 8\%, 10)] + \$10,000 (P/F, 8\%, 10)$$

$$PW_{EK} = -\$500,000 + [(\$150,000) \times (6.7101)] + \$10,000 (0.4632)$$

$$PW_{EK} = -\$500,000 + \$1,006,515 + \$4,632$$

$$PW_{EK} = \$511,147 > 0 \text{ economically justified}$$

$$PW_{WF} = -\$1,000,000 + (\$50,000 - \$10,000) \times (P/A, 8\%, 25)$$

$$PW_{WF} = -\$1,000,000 + (\$40,000) \times (10.6784)$$

$$PW_{WF} = -\$573,008 < 0 \text{ eliminate WF, because it is not economically justified}$$

$$PW_{LT} = -\$2,500,000 + (\$500,000 - \$20,000) \times (P/A, 8\%, 10) + \$50,000 (P/F, 8\%, 10)$$

$$PW_{LT} = -\$2,500,000 + (\$480,000) \times (6.7101) + \$50,000 (0.4632)$$

$$PW_{LT} = -\$2,500,000 + \$3,220,848 + \$23,160$$

$$PW_{LT} = \$774,008 > 0 \text{ economically justified}$$

$$PW_{EM} = -\$15,000,000 + (\$4,000,000 - \$1,000,000) \times (P/A, 8\%, 20) + \$2,000,000 (P/F, 8\%, 20)$$

Table 5.9 DirectJet Airlines Possible Bundles, Total Investments, and PWs

Bundle	Total Investments (Must be $\leq \$17,000,000$)	Total Present Worth
EK	−\$500,000	\$511,147
LT	\$2,500,000	\$744,008
EM	−\$15,000,000	\$14,883,300
EK & LT	−\$3,000,000	\$1,255,155
EK & EM	−\$15,500,000	\$15,394,447

$$PW_{EM} = -\$15,000,000 + (\$3,000,000) \times (9.8181) + \$2,000,000 (0.2145)$$

$$PW_{EM} = -\$15,000,000 + \$29,454,300 + \$429,000$$

$$PW_{EM} = -\$14,883,300 > 0 \text{ economically justified}$$

Out of four alternatives, the Wi-Fi project is not profitable and therefore is eliminated. Table 5.9 shows the possible project bundles of the remaining projects that fit within the budget, their total investment, and the total PW for each bundle.

Among the above bundles, the set of electronic kiosks and Embraer 170 have the highest PW and therefore, those two will be selected.

Using Spreadsheets for Present Worth Computation

This section explains how to run the financial equation (PW) using Microsoft Excel functions. The PW equation in Excel is called Net Present Value and denoted as NPV. To calculate PW, we plug the following function into an Excel spreadsheet cell unit.

$$=NPV(\text{rate}, \text{Cell [1]:Cell [n]}) + \text{Cell [0]}$$

Where rate is the MARR, (Cell [1]: Cell [n]) is the range of cash flows from period 1 through period n, Cell [1] is the net annual cash flow (NACF) in Year 1, Cell [n] is the NACF in year n (the last year), and Cell [0] is the first cost or initial investment that occurs at the present time (year 0) and has a negative value.

Example 5.10

An investment of \$100,000 results in net annual cash flows presented in Table 5.10 for eight years. Compute the PW of this investment using the Excel financial equation and a MARR of 10 percent.

The cash flow in year 0 is the first cost and NACFs are in years 1 through 8. The Excel function NPV can be used to determine the present worth of the investment. A depiction of the calculation in Excel is presented in Figure 5.4.

Table 5.10 Net Annual Cash Flows

Year	NACF
1	\$25,000
2	\$20,000
3	\$15,000
4	\$14,000
5	\$18,000
6	\$20,000
7	\$20,000
8	\$20,000

Figure 5.4 Calculating NPV Using Excel Spreadsheets

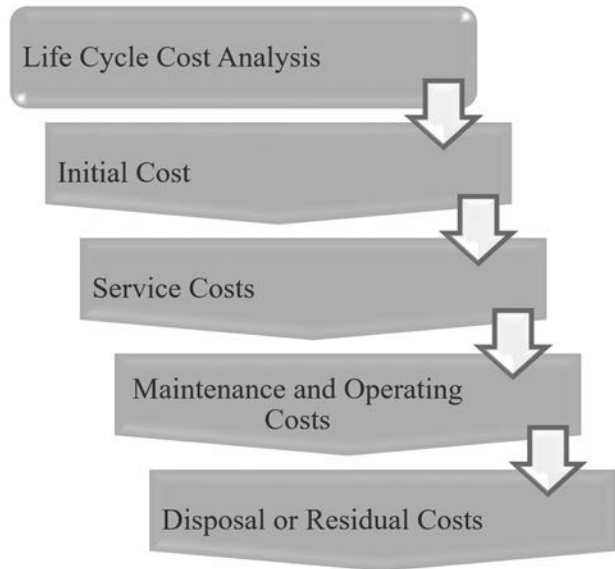
	A	B	C	D
1	MARR	10%		
2				
3	Year	NACF		
4		0 (\$100,000)		
5		1 \$25,000		
6		2 \$20,000		
7		3 \$15,000		
8		4 \$14,000		
9		5 \$18,000		
10		6 \$20,000		
11		7 \$20,000		
12		8 \$20,000		
13				
14	NPV	=NPV(B1,B5:B12)+B4		
15		NPV(rate, value1, [value2], [value3], ...)		

	A	B
1	MARR	10%
2		
3	Year	NACF
4		0 (\$100,000)
5		1 \$25,000
6		2 \$20,000
7		3 \$15,000
8		4 \$14,000
9		5 \$18,000
10		6 \$20,000
11		7 \$20,000
12		8 \$20,000
13		
14	NPV	\$2,147.48
15		

Life Cycle Costing (LCC)

At the beginning of any project, it is very imperative to understand and forecast the costs associated with the project. Life cycle costs refer to costs associated with various phases of a product life cycle, and life cycle costing refers to evaluation of the entire life cycle from inception to disposal of equipment or asset. LCC is a tool to determine the most cost-effective option among different competing alternatives to manufacture products from design stage to phase-out stage or to purchase, own,

operate, maintain, and, finally, dispose of products. Many times redundant equipment may be needed to avoid expected costly failures and to reduce risk. General aviation aircraft as well as commercial aircraft have many redundant parts to minimize the LCC of the aircraft. For example, Cirrus SR22 aircraft have two alternators, two batteries, two GPSs, and extra flight instruments. In the case of prevention, cost management is to avoid adverse outcomes. The LCC framework requires the analysis of criteria and principles by which the costs of alternative design strategies are compared and contrasted, such as:



- ◆ The time period over which the project is evaluated
- ◆ The development of alternative strategies
- ◆ Initial and future cash flows associated with the project
- ◆ The development of an appropriate methodology, as defined by cost factors, economic analysis techniques, analysis period, risk analysis techniques, and processes for accurately comparing different projects
- ◆ The analysis of results

The LCC of an aviation project examines all of the factors that affect an investment over its lifetime from design stage to retirement stage including the following:

- ◆ Aircraft range
- ◆ Noise (cabin and community)
- ◆ Development cost
- ◆ Marketing cost
- ◆ Maintenance cost
- ◆ Operating cost
- ◆ Payload capacity (aircraft)
- ◆ Phase out cost
- ◆ Production cost
- ◆ Schedule reliability
- ◆ Warranty commitment cost
- ◆ Weight

For example, to make the Boeing 787 Dreamliner more efficient, in the early development stage, Boeing designed a greater use of composite materials into its airframe

Table 5.11 Life Cycle Costing

Years	A	B	C
0	\$200,000	\$250,000	\$300,000
1	\$90,000	\$50,000	\$15,000
2	\$40,000	\$20,000	\$15,000
3	\$20,000	\$20,000	\$15,000
4	\$10,000	\$20,000	\$15,000
NPV	\$336,732.46	\$340,670.04	\$347,547.98

than any previous Boeing commercial aircraft. The airframe consisted of 50 percent composites in B787, versus 20 percent in B777, resulting in a weight reduction of about 20 percent relative to conventional aluminum design and increasing its efficiency by about 20 percent.

Present worth analysis can be used to determine the equivalence of all costs to present day value for all stages of all competing alternative products to select the most cost-efficient or optimal alternative.

Example 5.11

Majestic Aircraft Company (MAC) is a manufacturer of single engine, high performance turboprop aircraft. MAC is considering choosing between developing three separate models (A, B, and C) of aircraft scheduled for training purposes. The expected costs of the three models are presented in Table 5.11. Determine the most cost-efficient model which MAC can choose to develop. Use a MARR of 10 percent. The costs in year 0 are the initial costs of the projects and from then on, the costs provided are the estimated total costs, including service, maintenance, and operating costs for the following four years.

NPV is found using the Excel function, as discussed in the previous section. Since these are all costs, the NPV with the least value is the most efficient model. Based on that, Model A has the least NPV and hence, MAC must choose to develop Model A aircraft.

Summary

This chapter presents the computation of present worth, a technique used by professional engineers, to evaluate a single project to determine whether or not it is economically justified. It is also used to compare alternative projects that are mutually exclusive or are independent from each other. The present worth analysis consists of calculating the equivalence of all cash flows, both inflows and outflows, associated with a project, discounted at MARR to the present time value ($t = 0$). If PW is equal to or greater than zero, the project is economically justified and accepting it increases the value of the firm. If PW is negative, it should be rejected as it negatively affects the value of the firm.

When multiple projects are under consideration, we have to determine whether the projects are mutually exclusive or independent from each other. Two or more projects are considered mutually exclusive if selection of one project results in the rejection of the rest of the projects even though they might be profitable projects. There are two types of mutually exclusive alternatives: cost-based alternatives and revenue-based alternatives. When a set of projects that are being evaluated provide the same service and produce the same amount of revenue in the same time period, they are called mutually exclusive cost-based alternatives. The focus of PW analysis is to determine the most cost-efficient project. The selection criteria are to choose the one project that numerically has the highest PW value. In cost-based alternatives, usually the revenue is not provided.

Present worth analysis requires equivalent service lives for comparison of mutually exclusive alternatives. A replacement chain or least common multiple (LCM) technique is used to determine the common lives for both alternatives. If one or both projects have infinite life, then $LCM = \infty$ and capitalized cost approach is used to evaluate those projects.

Two or more projects are considered to be independent when a decision to accept or to reject a project does not have any impact on the rest of the projects. Independent projects evaluation does not require common lives and therefore LCM is not needed.

At the beginning of any project, it is imperative to understand and forecast the costs associated with the project. The last section of the chapter discusses the life cycle costing (LCC) that refers to evaluation of the entire life cycle from inception to disposal of equipment or asset. LCC is a tool to determine the most cost-effective option among different competing alternatives to manufacture products from design stage to phase-out stage or to purchase, own, operate, maintain, and, finally, dispose of products. The LCC of an aviation project examines all of the factors that affect an investment over its lifetime from design stage to retirement stage, including aircraft range, noise (cabin and community), development cost, marketing cost, maintenance cost, operating cost, payload capacity (aircraft), phase-out cost, production cost, schedule reliability, warranty commitment cost, and weight. Present worth analysis can be used to determine the equivalence of all costs to present day value for all stages of all competing alternative products to select the most cost-efficient or optimal alternative.

Discussion Questions and Problems

- 5.1 Majestic Aircraft Company (MAC) is considering purchasing new equipment with a purchase price of \$200,000, shipping and handling of \$4,000, and installation cost of \$6,000. MAC is sending an employee for training to operate the equipment at a cost of \$12,000. A working capital of \$15,000 is needed to get the equipment up and running. The expected annual revenue from this machine is \$50,000 and expected annual operating cost is \$10,000. The equipment will be depreciated based on a Straight Line method over a seven-year life toward a zero salvage value. MAC is expecting to sell the equipment at \$8,000 at the end of the seven years. Assuming tax rate of 40 percent calculate the following:
 - a. First cost (P)

- b. Net annual cash flow (NACF)
 - c. Terminal cash flow (TCF)
- 5.2 Samsung Aviation is considering purchasing equipment to manufacture a part of the wing fixture for an aircraft. The cost of the system is \$5 million with life expectancy of 10 years and annual operating cost of \$200,000. Samsung is using a MARR of 12 percent, expecting an annual revenue of \$1.5 million for 10 years, and a terminal cash flow of \$25,000. Using present worth analysis, is this investment economically justified?
- 5.3 How much would an investor pay to purchase a bond today, which is redeemable in four years for its par value or face value of \$1,000 and pays an annual coupon of 5 percent on the par value? Assume, the required rate of return for a bond in this risk class is 4 percent.
- 5.4 DirectJet (DJ) has a bond issue with face value of \$1,000, coupon rate of 7 percent, with 10 years to maturity. The interest is paid semi-annually and the bond is currently selling at 10 percent discount. Assuming your MARR is 8% compounded semi-annually, is this a good investment?
- 5.5 EZjET Airline has a new bond issue with face value of \$1,000, coupon rate of 8 percent, with 10 years to maturity, and interest is paid semi-annually. What is the maximum price you are willing to pay for this bond if your required rate of return is 6 percent compounded semi-annually?
- 5.6 An airline has a new bond issue with face value of \$1,000, coupon rate of 6 percent, 10 years to maturity, and interest is paid semi-annually. What is the maximum price you are willing to pay for this bond if your required rate of return is 8 percent compounded semi-annually?
Hint: In order to earn exactly 8 percent return on this bond, the PW of this bond should be equal to zero at MARR of 8 percent. Therefore, set the $PW = 0$ and solve for the price which is the same as the first cost (P).
- 5.7 Legacy Regional Airport (LRA) has the following two options to light taxiways and runways:
 - ◆ Solar cells (SC) with the first cost of \$250,000 including the installation cost. Annual operating costs for inspection and cleaning are expected to be \$15,000. The solar cells will have a useful life of 10 years with an expected salvage value of 15 percent of the first cost.
 - ◆ A new power line (PL) with the first cost of \$150,000. The annual operation costs are expected to be \$10,000 in year 1 and increase with a constant amount of \$2,000 per year starting the second year through the end of its 10 year life. The power line will stay in place and therefore has zero salvage value.

If LRA uses a MARR of 10 percent for these projects, which option should be selected to light the runways and taxiways at LRA?
- 5.8 MyJet Airlines is planning to expand its operations to India. It needs a hangar in Bangalore for its new non-stop route from Frankfurt to Bangalore. MyJet Airlines has the following two leasing options:
 - ◆ Location A with first cost of \$20,000,000, annual lease payment of \$4,000,000 for four years, and a deposit return of \$1,000,000.
 - ◆ Location B with first cost of \$25,000,000, annual lease payment of \$3,000,000 for six years, and a deposit return of \$2,000,000.

MyJet Airlines has a MARR of 8 percent. Using present worth analysis, which location is cost efficient?

- 5.9** DirectJet Airlines is considering improving customer service. DirectJet is looking into offering one of the following two free services to customers:

- ◆ Free live television services onboard its aircraft with the first cost of \$70 million and annual operating cost of \$5 million for 10 years with no salvage value.
- ◆ Free high speed internet services onboard its aircraft with the first cost of \$25 million and annual operating cost of \$15 million for five years.

DirectJet's MARR is 9 percent. Using a study period of five years, which alternative is cost efficient?

The following table presents the cash flows for both alternatives.

Year	Live Television (LT)	High Speed Internet (HSI)
0	−\$70,000,000	−\$25,000,000
1	−\$5,000,000	−\$15,000,000
2	−\$5,000,000	−\$15,000,000
3	−\$5,000,000	−\$15,000,000
4	−\$5,000,000	−\$15,000,000
5	−\$5,000,000	−\$15,000,000
6	−\$5,000,000	
7	−\$5,000,000	
8	−\$5,000,000	
9	−\$5,000,000	
10	−\$5,000,000	

- 5.10** Using the information in Problem 5.9, compare the two mutually exclusive alternatives, live television and high speed internet, for DirectJet Airlines based on the LCM approach.
- 5.11** Using the information in Problem 5.9, compare the two mutually exclusive alternatives, live television and high speed internet, for DirectJet Airlines based on capitalized cost approach.
- 5.12** A bond has a face value of \$1,000 and was issued five years ago at a coupon rate of 10 percent. The bond had a maturity period of five years. If the current market interest rate is 14 percent, what should be the price of the bond today?
- 5.13** Two machines with the following cost estimates are under consideration to manufacture computer drives to play HD movie disc formats. Assuming a MARR of 8 percent, which alternative should be selected using PW analysis?

	Machine A	Machine B
Equipment (first cost)	−\$200,000	−\$300,000
Annual operating cost (AOC) per year	−\$60,000	−\$40,000
Salvage value	\$20,000	\$40,000
Estimated life, years	4	6

5.14 MyJet Airways is considering the following three mutually exclusive projects with different lives.

- ♦ Project A with first cost of \$100 million and operating cost of \$1.2 million per quarter. This project is considered permanent or has an infinite life.
- ♦ Project B with first cost of \$25 million and operating cost of \$3 million per quarter with 10-year life.
- ♦ Project C with first cost of \$10 million and operating cost of \$0.8 million per quarter with four-year life.

Using PW analysis, assuming a MARR of 12 percent per year compounded quarterly, which project should be selected by MyJet Airways?

5.15 DirectJet Airlines is considering the following four independent projects:

The cash flows for the four alternatives are presented below. DirectJet has a limited budget of \$10 million with a MARR of 6 percent. Using PW analysis, determine which alternatives, if any, should DirectJet undertake to maximize its profits?

	Project A	Project B	Project C	Project D	Project E
Net investment (P)	−\$2,000,000	−\$3,000,000	−\$5,000,000	−\$10,000,000	−\$15,000,000
Annual operating cost	−\$50,000	−\$100,000	−\$200,000	−\$500,000	−\$1,000,000
Annual revenue	\$600,000	\$600,000	\$650,000	\$1,500,000	\$2,250,000
Salvage value	\$100,000	200,000	\$300,000	\$500,000	\$800,000
Life in years	5	10	15	20	25

5.16 Determine the capitalized cost of \$2,000,000 at time 0, \$500,000 in years 1 through 5, and \$1,000,000 per year from year 6 on. Use an interest rate of 6 percent per year.

5.17 DirectJet is planning to invest in a new electronic ticket system. The investment is expected to produce profit from its rental of \$300,000 the first year it is in service. The profit is expected to decrease by \$50,000 each year thereafter. At the end of six years assume the salvage value is zero. Calculate the present worth at 12 percent interest rate.

- 5.18** Calculate the capitalized cost of an infinite series of annual payments equal to \$100,000, using an interest rate of 5 percent compounded annually.

Notes

- 1 Bombardier CSeries Cost Estimate Rises to \$5.4 billion. Reuters, February 12, 2015.
- 2 Boeing Celebrates the Premiere of the 787 Dreamliner (Press release). Boeing. July 8, 2007.
- 3 Final Assembly Begins on Another Boeing 787 Dreamliner (Press release). Boeing. February 15, 2008.
- 4 *Forbes*, May 21, 2013.
- 5 EETC: The Next Generation of Debt Financing for Airlines around the Globe? By Mehtap Cevher Conti, Marketrealist.com, October 25, 2013.

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6

Analysis of Annual Worth

Running an airline is like having a baby: fun to conceive, but hell to deliver.

C. E. Woolman, principal founder of Delta Air Lines



Annual worth (AW) analysis is a simple technique used by professional engineers to evaluate single, multiple, and competing alternative projects. To evaluate a single project, the annual worth analysis consists of calculating the sum of annual equivalent amount (annuity amount) of all cash flows, both inflows and outflows, associated with a project. Annual worth analysis is generally preferred over a present worth analysis due to its simplicity in calculating the worth and its not requiring equal service lives for comparison of mutually exclusive alternatives.

The focus of this chapter is to enable readers to use annual worth analysis to evaluate single projects and to compare mutually exclusive alternatives and independent projects. This chapter also presents the concept of a capital recovery. Capital recovery of an investment in equipment is the annual revenue needed to recover the cost of the equipment (first cost), any non-recurring costs, and the salvage value of equipment at a given interest rate over the life of the project.

The following sections briefly address the calculation of annual worth for single projects, the criteria for accepting or rejecting projects, and the comparison and selection of mutually exclusive alternatives and independent projects.

- ◆ Motivation for Using Annual Worth Analysis
- ◆ Evaluation of Single Projects
 - Capital Recovery
 - Annuity of Net Annual Cash Flows
- ◆ Evaluation of Mutually Exclusive Projects

- Annual Worth Analysis of Equal-life Alternatives
- Annual Worth Analysis of Non-equal-life Alternatives
- ◆ Evaluation of Independent Projects
- ◆ Annual Worth of Permanent Projects
- ◆ Using Spreadsheets for Annual Worth Computation
- ◆ Summary
- ◆ Discussion Questions and Problems

Motivation for Using Annual Worth Analysis

A Benefit-Cost Analysis Tool for Assessing Guardrail Needs for Two-Lane Rural Road was developed by the Virginia Department of Transportation in 2015. The analysis states, “Although it is relatively straightforward to estimate the direct costs of guardrail installation, it is considerably more difficult to quantify the safety benefits associated with the treatment.”

Times Standard Columns,
May 15, 2016

In the previous chapter, we discussed the present worth analysis approach to evaluating projects.

This approach is very effective for single projects, mutually exclusive projects with the same lifespan, and independent projects. However, as mentioned in Chapter 5, when it comes to comparing two projects that have unequal lives, present worth analysis becomes complicated. For such a scenario, we must assume the projects in question are repeatable. Consequently, we need to find the least common multiple (LCM) of their lives and repeat the projects to be able to evaluate their overall present worth throughout the length of LCM.

For instance, consider a company that needs to acquire a new aircraft for its operations. Usually based on the range, performance, commonality with other fleet and some other factors, operators can narrow down their options to a given make and model. Let’s assume the company decided to get a Boeing 737–700C to carry its personnel and also equipment between operation sites. The Boeing 737–700 is a short to medium range, narrow-body jet airliner. The 700C variation is a convertible version which can switch between carrying passengers and cargo. The seats can be removed to convert the main cabin to cargo capacity and the aircraft has a large door on the left side for cargo loading. This option is particularly important for some companies that need to satisfy their logistics needs as well as providing transportation service to personnel. The US Navy and US Air Force have used the variations of Boeing 737–700 for transportation of generals and for cargo delivery. The next critical question would be whether they should purchase an aircraft or opt for leasing one. The current price for a new 737–700C is approximately \$80 million with a life expectancy of 25 years. On the other hand, the monthly leasing rate for a 2001-made 737–700C can be estimated at around \$250,000 per month for a 10-year lease. Now, to use present worth analysis, since the lives of these options are not equal, we need to consider a 50-year time horizon that covers buying the aircraft two consecutive times or leasing it five times. This approach not only requires more extensive calculations, but also makes it harder to interpret. This complexity can be avoided by using annual worth analysis.

Annual worth analysis provides a yearly worth of doing a project. This approach does not require LCM for projects with different life spans and is easier to interpret for companies that do annual budgeting.

Evaluation of Single Projects

Although present worth analysis is probably the most popular technique to evaluate single, equal lives mutually exclusive, and independent projects, annual worth is commonly used to evaluate mutually exclusive alternatives with different lives, because annual worth analysis does not require least common multiple (LCM) to select among competing alternatives with unequal lives. In performing the annual worth analysis, we rely on the fundamental equations that we discussed in Chapter 2. Essentially in this approach, we first convert all cash flows including first cost, non-recurring cost, and salvage value to an equivalent annuity worth for the life span of the project. Then we add them all together with annual cash flows such as annual operating cost and annual revenue, so long as they are a constant amount per year, to calculate the annual worth of the project. If annual cost or annual revenue are not constant per year, then we have to convert those to equivalent annuity worth for the purpose of calculating annual worth of the project. Table 6.1 presents the criteria for accepting or rejecting a project using AW values.

The following examples clarify the process of computing annual worth for a single project with different cash flow formats.

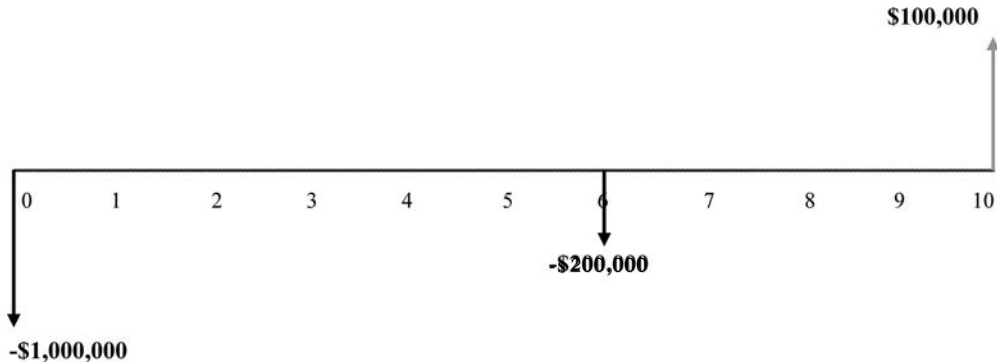
Example 6.1

Johnson Regional Airport (JRA) is a small airport offering domestic commercial services all over the United States. Due to significant increase in the number of flights and high passenger traffic, the JRA is considering a proposal to build a new rental car complex. The initial investment is estimated to be \$1,000,000 with a salvage value of \$100,000 at the end of the 10-year life of the project. In addition, there is a single non-recurring cost of \$200,000 incurred in year 6 for repairs. Compute the annual worth of this investment assuming JRA uses a MARR of 8 percent. The cash flow diagram of this project is presented in Figure 6.1.

To compute AW value of the JRA investment in a rental car building, we first convert all cash flows associated with the project to an equivalent annuity worth and

Table 6.1 Accept and Reject Criteria for Annual Worth Analysis

Annual Worth	Selection Criteria
$AW > 0$	Accept the project, since the AW of revenue exceeds the AW of costs and therefore, it is economically justified.
$AW = 0$	Accept the project, since the AW of revenue is equal to the AW of costs and therefore, it is economically justified.
$AW < 0$	Reject the project, because the AW of revenue is less than the AW of costs and therefore, it is <u>not</u> economically justified.

Figure 6.1 Cash Flow Diagram

then we add them together. The value of compound interest factors is taken from Table A14, Appendix A for 8 percent compound interest.

Annuity of the first cost = $-\$1,000,000 (A/P, 8\%, 10) = -\$1,000,000 (0.14903) = -\$149,030$

Annuity of the salvage value = $\$100,000 (A/F, 8\%, 10) = \$100,000 (0.06903) = \$6,903$

Annuity of the repair cost = $-\$200,000 (P/F, 8\%, 6) (A/P, 8\%, 10)$
 $= -\$200,000 (0.6302) (0.14903) = -\$18,783.74$

$AW = -149,030 + 6,903 - 18,783.74$

$AW = -160,910.74$

The AW value means that it would cost JRA about \$160,910.74 per year for 10 years for the investment in the rental car complex. This AW value also means that JRA should collect revenue of approximately \$160,910.74 per year from the rental car complex to break even on this investment. In other words, this investment is economically justified if JRA collects at least a revenue of \$160,910.74 per year.

Example 6.2

Majestic Aircraft Company (MAC) is a manufacturer of single engine, high performance turboprop aircraft. MAC is considering purchasing composite wing fixtures for the assembly of its signature aircraft. The cost of the assembly system is \$3 million with life expectancy of 10 years, annual operating cost of \$200,000 with zero salvage value. MAC anticipates a stream of \$1,000,000 in revenue per year for 10 years from this project. Using annual worth analysis and assuming MAC uses a MARR of 10 percent, is this investment economically justified? The cash flow diagram of this project is presented in Figure 6.2.

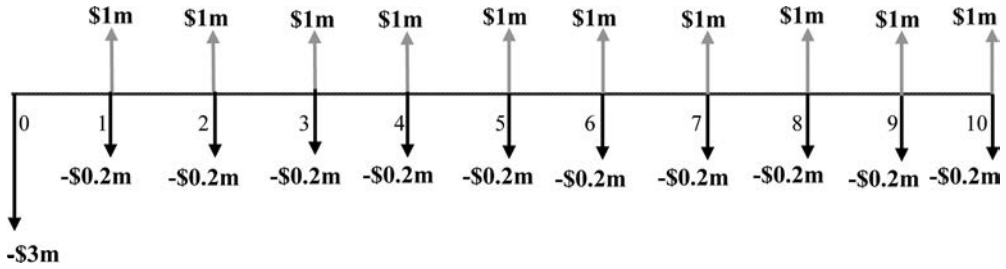
First we calculate net annual cash flows (NACF) for year 1 through 10.

$$NACF = \$1,000,000 - \$200,000 = \$800,000$$

Since NACF are constant per year, to calculate AW of the project, we simply compute the annuity of the first cost and add it to the NACF.

$$AW = -\$3,000,000 (A/P, 10\%, 10) + \$800,000$$

$$AW = -\$3,000,000 \times (0.16275) + \$800,000$$

Figure 6.2 Cash Flow Diagram

$$AW = -\$488,250 + \$800,000$$

$$AW = \$311,750$$

In this example, because AW value is greater than zero, then the project is economically justified.

Example 6.3

In Example 6.2, assume there is no annual operating cost, but there are rebuild costs of $\$800,000$ in year 3 and $\$1,200,000$ in year 7. Assume everything else remains the same; re-calculate the AW of the project. The cash flow diagram is presented in Figure 6.3.

Because revenues are constant per year, to calculate AW of the project, we compute the equivalent annuity values of the first cost and rebuild costs and simply add them all to the annual revenue.

$$AW = \text{Equivalent annuity values of all costs} + \text{Annual revenue}$$

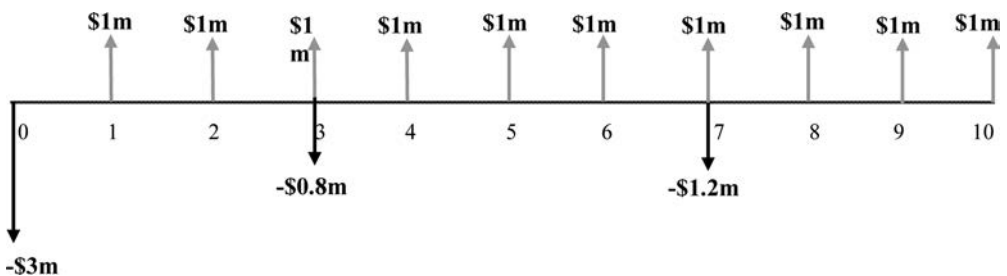
To calculate the equivalent annual values of all costs, we shift all costs to present time by calculating the present worth of all costs (PW_{Cost}) and then compute the equivalent annuity of that amount over a 10-year period.

$$PW_{Cost} = -\$3,000,000 - \$800,000 (P/F, 10\%, 3) - \$1,200,000 (P/F, 10\%, 7)$$

$$PW_{Cost} = -\$3,000,000 - \$800,000 \times (0.7513) - \$1,200,000 \times (0.5132)$$

$$PW_{Cost} = -\$3,000,000 - \$601,040 - \$615,840$$

$$PW_{Cost} = -\$4,216,880$$

Figure 6.3 Cash Flow Diagram

Equivalent Annuity Values of all cost = $-\$4,216,880 (A/P, 10\%, 10)$
 Equivalent Annuity Values of all cost = $-\$4,216,880 \times (0.16275)$
 Equivalent Annuity Values of all cost = $-\$686,297$

To compute the annual worth of the project, we simply add the AW of all costs to the annual revenue.

$$AW = -\$686,297 + \$1,000,000$$

$$AW = +\$313,703$$

Because AW value of the project is greater than zero, this project is economically justified. As you notice, depending on the format of the cash flows, there are many ways to calculate the AW of a project. In fact, when it comes to time value of money, there is more than one way to find the solution to any problem.

Example 6.4

In Example 6.2, assume there is no annual operating cost, but there are rebuild costs of \$800,000 in year 3 and \$1,200,000 in year 7. In addition, there is no uniform annual revenue, but revenue of \$4,000,000 and \$5,000,000 occurs in years 4 and 8 respectively. Assume everything else remains the same; re-calculate the AW of the project. The cash flow diagram is presented in Figure 6.4.

To calculate the AW value of the project, we first shift all cash flows to the present time by calculating the PW of all cash flows as shown below.

$$PW = -\$3,000,000 - \$800,000 (P/F, 10\%, 3) - \$1,200,000 (P/F, 10\%, 7)$$

$$+ \$4,000,000 (P/F, 10\%, 4) + \$5,000,000 (P/F, 10\%, 8)$$

$$PW = -\$3,000,000 - \$800,000 \times (0.7513) - \$1,200,000 \times (0.5132)$$

$$+ \$4,000,000 \times (0.6830) + \$5,000,000 \times (0.4665)$$

$$PW = -\$3,000,000 - \$601,040 - \$615,840 + \$2,732,000 + \$2,332,500$$

$$PW = +\$847,620$$

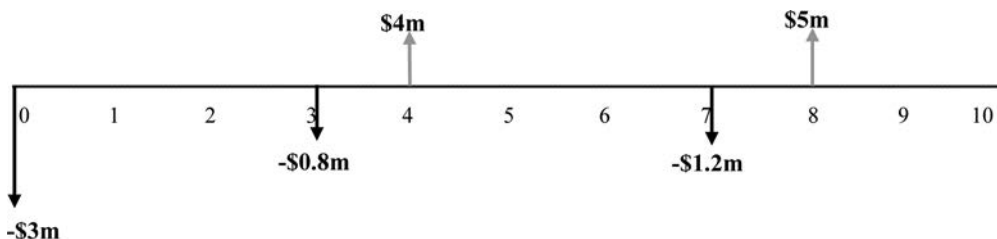
Now, we compute the AW of the present amount over the life of the project.

$$AW = \$847,620 (A/P, 10\%, 10)$$

$$AW = \$847,620 \times (0.16275)$$

$$AW = \$137,950$$

Figure 6.4 Cash Flow Diagram



Because $AW > 0$, this project is economically justified.

Annual worth of a project can also be calculated by adding the following two separate components as shown by the equation below.

- a. Capital recovery (CR) of the first cost and all other non-recurring costs
- b. Annuity of the net annual cash flows ($A \text{ of } NACF$)

$$CR + A \text{ of } NACF = AW$$



Capital Recovery

Capital recovery of investment in equipment is the uniform annual revenue needed to recover the cost of the equipment (first cost), any non-recurring costs, and the salvage value at a given interest rate over the life of the project. In other words, capital recovery is the annuity of the first cost, any non-recurring costs and the salvage value over the economic life of the equipment.

This concept is particularly of interest in aircraft financing for airlines. Purchasing a new aircraft imposes various acquisition and maintenance costs on airlines. Airlines make a down payment for ordering a new aircraft and often arrange some payment plan with a financial company to pay for the aircraft. Through the life cycle of the aircraft, aside from recurring maintenance and operating costs, there are multiple C and D checks that impose notable cost. Eventually, airlines are expected to sell their airplanes to recover a salvage value based on the market price of the aircraft at the time of retirement. Accordingly, in view of these expenditures, airlines are interested in estimating the annual cost to recover the capital invested in acquiring the new aircraft.

This concept is closely related to break-even analysis, which we will discuss in detail in Chapter 9.

The following formula is used to calculate the capital recovery:

$$CR = [-P - C_{NR} (P/F, i, n_t)] \times [(A/P, i, n)] + S (A/F, i, n)$$

Where: CR = Capital recovery
 P = First cost (initial investment)
 C_{NR} = Non-recurring cost occurs in year n_t
 S = Salvage value
 i = MARR
 n = Time period

Example 6.5

Considering Example 6.1 for Johnson Regional Airport (JRA), compute the CR for this equipment. The following is the summary of the information in Example 6.1.

$$\begin{aligned}P &= \$1,000,000 \\C_{NR} &= \$200,000 \text{ in year six } (n_t = 6) \\S &= \$100,000 \\n &= 10 \text{ years} \\MARR &= 8\%\end{aligned}$$

We utilize the Capital Recovery equation to compute the CR for JRA as follows:

$$\begin{aligned}CR &= [-\$1,000,000 - \$200,000 (P/F, 8\%, 6)] [(A/P, 8\%, 10)] + \$100,000 (A/F, 8\%, 10) \\CR &= [-\$1,000,000 - \$200,000 \times (0.6302)] \times [0.1490] + \$100,000 \times (0.0690) \\CR &= [-\$1,000,000 - \$126,040] \times [0.1490] + \$6,900 \\CR &= -\$167,780 + \$6,900 \\CR &= -\$160,880\end{aligned}$$

In other words, it costs the company \$160,880 to recover the capital for this project. The answers in Example 6.1 and Example 6.5 are marginally different which is due to rounding.

Annuity of Net Annual Cash Flows

The NACF can be either the annual operating cost or the annual revenue, or a combination of both. The distribution of NACF can be an annuity series, a gradient series, or have no particular format. If the NACF is an annuity series, then the A of NACF will be the same as the NACF per year. Otherwise, we have to compute the annuity of the NACF.

Example 6.6

Consider Example 6.2 for Majestic Aircraft Company (MAC), where the NACF is an annuity series. Compute the CR, annuity of NACF, and the AW of this project.

First, we compute the CR using a simplified version of the equation, where there is no non-recurring cost and no salvage value.

$$\begin{aligned}CR &= -P (A/P, i, n) \\CR &= -\$3,000,000 (A/P, 10\%, 10) \\CR &= -\$3,000,000 \times (0.16275) \\CR &= -\$488,250\end{aligned}$$

Second, because NACF is constant per year, the A of NACF is the same as NACF per year.

$$A \text{ of NACF} = NACF = \$1,000,000 - \$200,000 = \$800,000$$

Third, we utilize the equation to compute the AW.

$$\begin{aligned}
 AW &= CR + A \text{ of } NACF \\
 AW &= -\$488,250 + \$800,000 \\
 AW &= \$311,750
 \end{aligned}$$

Because $AW > 0$, the project is economically justified.

Evaluation of Mutually Exclusive Projects

In Chapter 5, we discussed the meaning of mutually exclusive projects. These are the project options among which we can only choose one. Thus, selecting one implies foregoing the others.

As a case in point, Boeing faced such a decision in the late 1990s. In competition with Airbus 3XX (later A380), Boeing considered two options: 747X or Sonic Cruiser. The first option, 747X, was the extension of the 747-400 series with more capacity and better fuel efficiency. The alternative was a Sonic Cruiser with distinctive delta wing canard that promised a 15–20 percent faster speed than the conventional airliner.

Considering the required capital and market demand it only made sense to pursue one of these options. The market interest in 747X was moderate to low. Boeing eventually chose the Sonic Cruiser that later on became the Boeing 787, Dreamliner.

Congestion has been a major problem at many major airports around the world. Congestion imposes costs on both the passengers and the airline industry. A traditional solution is to expand the current infrastructure in the most afflicted areas such as runways, gates, terminals and parking areas. And the expanding of the current facilities may entail a multi-year capital expansion effort. The current landing fees solely depend on aircraft weight. Another way to reduce the congestion is through peak-load pricing of landing charges on airlines charging a higher landing fee in accordance with the higher demand during the peak time. The pricing strategy is an attempt to shift demand from peak time to off-peak to accommodate runway slots.

To evaluate mutually exclusive alternatives, we simply compute the AW of each alternative and select the one alternative with the highest numerical AW value. In the following subsections, we evaluate mutually exclusive projects with equal and non-equal lives.

Aviation Snippets

New Orleans airport's new \$807 million terminal

Construction of an \$807 million terminal at Louis Armstrong International Airport, a project hailed as the most important construction for the city. . . . The \$598 million agreed-upon price covers a 760,500-square-foot terminal with 30 gates, which is bigger than the original 650,000-square-foot design. Airport officials said the size was increased to accommodate the higher number of passengers flying on bigger jets. The costs also include a 2,000-car parking garage, a central utility plant, and a ground transportation staging area. It's expected to create 13,000 temporary construction jobs.

The Times-Picayune,
December 2015

Annual Worth Analysis of Equal-life Alternatives

Annual worth analysis of equal-life alternatives is one of the most used analysis techniques for engineer economists. The Annual Equivalent Worth criterion provides a basis for measuring investment worth by determining equal payments on annual basis. Annual worth of equal-life alternatives is a measure of cash flows in terms of equivalent equal payments made on an annual basis for different projects. As shown in Example 6.7, we simply compute the AW of each project and select the one alternative that numerically has the highest AW.

Example 6.7

The Kirby Chambliss Red Bull Air Race Team wishes to evaluate the two alternative machines presented in Table 6.2 for engine testing. Use the AW method with MARR of 12 percent per year to select the best alternative.

To select the best alternative, we compute annual worth of machine A and machine B and select the one alternative with higher AW value.

$$AW = CR + A \text{ of } AOC$$

$$AW_A = [-\$250,000 (A/P, 12\%, 5) + \$20,000 (A/F, 12\%, 5)] - \$80,000$$

$$AW_A = [-\$250,000 \times (0.27741) + \$20,000 \times (0.15741)] - \$80,000$$

$$AW_A = [-\$69,353 + \$3,148] - \$80,000$$

$$AW_A = -\$66,205 - \$80,000$$

$$AW_A = -\$146,205$$

$$AW_B = [-\$350,000 (A/P, 12\%, 5) + \$25,000 (A/F, 12\%, 5)] - \$50,000$$

$$AW_B = [-\$350,000 \times (0.27741) + \$25,000 \times (0.15741)] - \$50,000$$

$$AW_B = [-\$97,094 + \$3,935] - \$50,000$$

$$AW_B = -\$93,159 - \$50,000$$

$$AW_B = -\$143,159$$

Because $AW_B > AW_A$, the machine B is selected.

Annual Worth Analysis of Non-Equal-life Alternatives

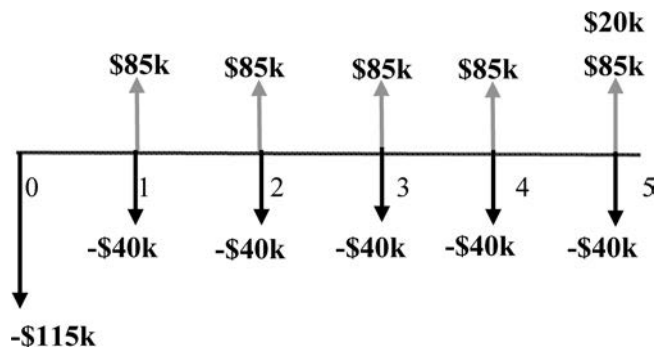
As mentioned earlier, the advantage of annual worth analysis over present worth analysis is that the equal-life service requirement is met without using the least common multiples (LCM) of alternatives with different lives because the annual worth calculated over one life cycle is exactly the same for any multiple life cycles. Example 6.8 demonstrates the repeatability of the AW value over multiple life cycles.

Table 6.2 Mutually Exclusive Alternatives

	Machine A	Machine B
First cost	−\$250,000	−\$350,000
Annual operating cost	−\$80,000	−\$50,000
Life, years	5	5
Salvage value	\$20,000	\$25,000

Table 6.3 Cash Flows for Cessna 150

First cost	\$115,000
Annual operating cost	\$40,000
Annual revenue	\$85,000
Salvage value	\$20,000
Life, years	5

Figure 6.5 Cash Flows for One Life Cycle**Example 6.8**

Johnson is purchasing a Cessna 150 for \$115,000 to conduct primary flight instruction. The cash flows associated with this project are presented in Table 6.3. Evaluate this project using AW analysis over one and two life cycles. Johnson uses a MARR of 10 percent.

- a. AW value computation over one life cycle. Figure 6.5 shows the cash flows for one life cycle.

$$AW = -\$115,000 (A/P, 10\%, 5) + \$20,000 (A/F, 10\%, 5) - \$40,000 + \$85,000$$

Plug in the value of interest factors from Table A16 in Appendix A in the above relationship

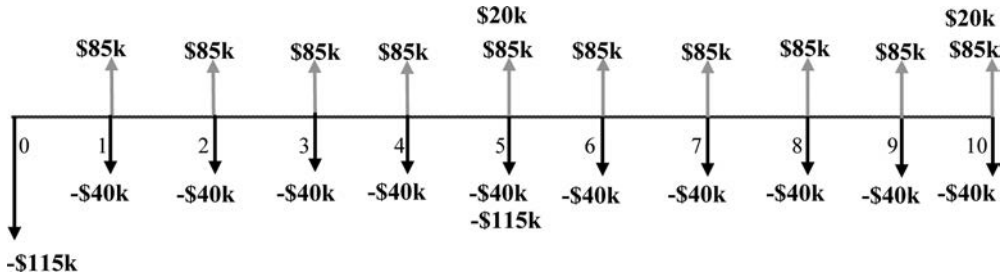
$$AW = -\$115,000 \times (0.2638) + \$20,000 \times (0.1638) + \$45,000$$

$$AW = -\$30,337 + \$3,276 + \$45,000$$

$$AW = \$17,939$$

Because AW value is greater than zero, it is a good, economically justified project.

- b. AW value computation over two life cycles. Figure 6.6 shows cash flows for two life cycles.

Figure 6.6 Cash Flows for Two Life Cycles

$$AW = [-\$115,000 - \$115,000 (P/F, 10\%, 5)] \times [(A/P, 10\%, 10)] + [\$20,000 (F/P, 10\%, 5) + \$20,000] \times [(A/F, 10\%, 10)] - \$40,000 + \$85,000$$

Plug in the value of interest factors from Table A16 in Appendix A in the above relationship.

$$\begin{aligned} AW &= [-\$115,000 - \$115,000 \times (0.6209)] \times [0.16275] + [\$20,000 \times (1.6105) + \\ &\quad \$20,000] \times [0.06275] + \$45,000 \\ AW &= -\$30,337 + \$3,276 + \$45,000 \\ AW &= \$17,939 \end{aligned}$$

As you notice, the AW value is the same for one life cycle and for two life cycles. The AW is also the same over multiple life cycles.

Evaluation of Independent Projects

Independent projects are those projects for which, when we accept or reject one of them, it does not have any impact on accepting or rejecting of the rest of the projects. For example, the decision by an airline to replace an old aircraft would be considered independent of a decision to replace the computer reservation system. A mutually exclusive project is the one where acceptance of one project will have an effect on the acceptance of another project.

It is important to be able to determine which projects are to be approved under these constraints. The annual worth approach to evaluate independent projects is to compute the AW of each project and accept all projects that have an AW of greater than or equal to zero, so long as there is no budget constraint. If there is a budget limitation, then we use the PW approach as explained in Chapter 5.

Example 6.9

Majestic Oil Company is producing jet fuel from crude oil in its refinery in Texas. It is considering producing synthetic rubber, rubber cement, car battery cases, and linoleum from byproducts of the crude oil. The cash flows for the above byproducts are presented in Table 6.4. Using the AW approach and assuming a MARR of 7 percent, which of the following alternatives should be selected by Majestic, if any?

Table 6.4 Majestic Byproducts Independent Projects

	Synthetic Rubber (SR)	Rubber Cement (RC)	Car Battery Cases (CB)	Linoleum (L)
First cost	\$100,000	\$150,000	\$200,000	\$85,000
Annual operating cost	\$30,000	\$40,000	\$75,000	\$50,000
Annual revenue	\$65,000	\$80,000	\$100,000	\$65,000
Salvage value	\$10,000	\$12,000	\$15,000	\$8,000
Life, years	8	8	8	8

AW of synthetic rubber (SR)

$$AW_{SR} = -\$100,000 (A/P, 7\%, 8) + \$10,000 (A/F, 7\%, 8) - \$30,000 + \$65,000$$

$$AW_{SR} = -\$100,000 \times (0.16747) + \$10,000 \times (0.09747) - \$30,000 + \$65,000$$

$$AW_{SR} = -\$16,747 + \$975 - \$30,000 + \$65,000$$

$$AW_{SR} = \$19,228 > 0$$

AW of rubber cement (RC)

$$AW_{RC} = -\$150,000 (A/P, 7\%, 8) + \$12,000 (A/F, 7\%, 8) - \$40,000 + \$85,000$$

$$AW_{RC} = -\$150,000 \times (0.16747) + \$12,000 \times (0.09747) - \$40,000 + \$85,000$$

$$AW_{RC} = -\$25,121 + \$1,170 - \$40,000 + \$85,000$$

$$AW_{RC} = \$21,049 > 0$$

AW of car battery cases (CB)

$$AW_{CB} = -\$200,000 (A/P, 7\%, 8) + \$15,000 (A/F, 7\%, 8) - \$75,000 + \$100,000$$

$$AW_{CB} = -\$200,000 \times (0.16747) + \$15,000 \times (0.09747) - \$75,000 + \$100,000$$

$$AW_{CB} = -\$33,494 + \$1,462 - \$75,000 + \$100,000$$

$$AW_{CB} = -\$7,032 < 0$$

AW of linoleum (L)

$$AW_L = -\$85,000 (A/P, 7\%, 8) + \$8,000 (A/F, 7\%, 8) - \$50,000 + \$65,000$$

$$AW_L = -\$85,000 \times (0.16747) + \$8,000 \times (0.09747) - \$50,000 + \$65,000$$

$$AW_L = -\$14,235 + \$780 - \$50,000 + \$65,000$$

$$AW_L = \$1545 > 0$$

Majestic should accept synthetic rubber, rubber cement, and linoleum projects since they have positive AW values. The car battery cases project should be rejected due to its negative AW value.

Annual Worth of Permanent Projects

As explained in Chapter 3, sometimes the life of projects is long enough that we can consider them as permanent projects. Let us consider Next Generation Air Transportation System (NextGen). NextGen proposes to transform America's air traffic control system from a radar-based system with radio communication to a satellite-based system with GPS technology. The GPS technology will be used to shorten routes, save time and fuel, reduce traffic delays, increase capacity, and permit controllers to monitor and manage aircraft with greater safety margins. Radio communications will be increasingly replaced by data exchange and an automation system that reduces the amount of information the air crew must process at one time. As a result of these changes, planes will be able to fly closer together, take more direct routes and avoid delays caused by airport "stacking" as planes wait for an open runway.¹ NextGen requires massive federal investment for R&D, feasibility study, development, implementation, and maintenance.

To compute the AW of an infinite life project, we use the capitalized cost (CC) concept and start with the equation as follows:

$$CC = \frac{AW}{i}$$

$$AW = CC \times (i)$$

For an infinite life project, the first cost becomes the capitalized cost and AW of the first cost is calculated as:

$$AW = PV \times (i)$$

For recurring cash flows in a permanent life project, we compute the AW of recurring cash flows over the life cycle of their occurrences. All other non-recurring cash flows are shifted to present time value and then, using the equation above, their annual worth is computed.

Example 6.10

Johnson International Airport (JIA) is considering building a new five-story parking garage with a first cost of \$100 million. This project is considered to be permanent with an estimated annual operating cost of \$500,000 per year and a resurfacing cost of \$2,000,000 every five years. The annual revenue from this parking garage is estimated to be \$8,000,000 per year for the first five years and \$12,000,000 per year for the next five years, and \$15,000,000 per year thereafter. Johnson International Airport uses a MARR of 10 percent. Use AW analysis to determine if this project is economically justified.

$$AW_P = -\$100,000,000 \times (0.10) = -\$10,000,000$$

$$AW_{AOC} = -\$500,000$$

$$AW_{Resurfacing} = -\$2,000,000 (A/F, 10\%, 5)$$

$$AW_{Resurfacing} = -\$2,000,000 \times (0.1638)$$

$$AW_{Resurfacing} = -\$327,600$$

$$\begin{aligned}
 AW_{\text{Revenue}} &= \$8,000,000 + \$4,000,000 (P/F, 10\%, 5) + \$3,000,000 (P/F, 10\%, 10) \\
 AW_{\text{Revenue}} &= \$8,000,000 + \$4,000,000 \times (0.6209) + \$3,000,000 \times (0.3855) \\
 AW_{\text{Revenue}} &= \$8,000,000 + \$2,483,600 + \$1,156,500 \\
 AW_{\text{Revenue}} &= \$11,640,100
 \end{aligned}$$

To calculate the total annual worth of the project, we add the AW_P , AW_{AOC} , $AW_{\text{Resurfacing}}$, and AW_{Revenue} together as follows:

$$\begin{aligned}
 AW_{\text{Total}} &= -\$10,000,000 - \$500,000 - \$327,600 + \$11,640,100 \\
 AW_{\text{Total}} &= \$812,500 > 0
 \end{aligned}$$

Because AW value of the parking garage project is positive, it is, therefore, economically justified.

Using Spreadsheets for Annual Worth Computation

This section presents how to run the financial equation (AW) using Microsoft Excel functions. The AW can be calculated using the Excel financial function for payment and is denoted as PMT as follows:

$$=PMT(i\%,n,PV,FV)$$

To calculate the AW of a project, we plug the following function into a cell unit of an Excel spreadsheet as shown in the equation below:

$$=PMT(i\%,n,PV,FV) + NACF$$

Where i is the MARR, n is the time period, P is the first cost, F is the salvage value, and NACF is annual cash inflows less annual cash outflows.

Example 6.11

An investment in equipment costing \$100,000 results in net annual cash flow (NACF) of \$50,000 per year for five years and a salvage value of \$20,000 at the end of five years when the equipment will be disposed of. Assuming a MARR of 10 percent, compute the AW of this investment using the Excel financial equation.

Figure 6.7 shows the Excel spreadsheet solution for the project in Example 6.11. The cash flow in cell B3 is the first cost, the cash flow in cell B4 is the salvage value, and the cash flow in cell B7 is the NACF. The function to calculate the AW is presented in cell C9, and the solution will appear instantly in that cell as soon as you hit the return key. As you notice, the NACF represents a uniform series, first cost has a positive sign, and salvage value has a negative sign. In order to get a correct answer, you need to pay close attention to the cash flow signage.

Example 6.12

EZJET Airlines purchased a tow truck for \$300,000. The operating cost will be \$5,000 per year. The airline expects to sell the tow truck for \$50,000 five years from now. What is the equivalent annual worth of the truck assuming the interest rate is 10 percent?

$$AW = -\$300,000 \times (A/P, 10\%, 5) + \$50,000 \times (A/F, 10\%, 5) - \$5,000$$

$$AW = -\$300,000 \times (0.2638) + \$50,000 \times (0.1638) - \$5,000 = -\$75,950$$

To calculate the Annual Worth for this example using Excel, enter the cost of equipment (the first cost) in cell B3, the salvage value in cell B4 with a negative sign, and the operating cost per year in cell B7 as the NACF. The function to calculate the AW is presented in cell C9, and the solution will appear instantly in that cell as soon as you hit the return key. The results are presented in Figure 6.8.

Figure 6.7 Annual Worth Computation Using Excel Spreadsheets

	A	B	C	D
1				
2				
3	P	\$100,000		
4	S	-\$20,000		
5	n	5		
6	MARR	10%		
7	NACF	50,000		
8				
9	AW	=PMT(B6,B5,B3,B4)+B7		
10		PMT(rate, nper, pv, [fv], [type])		

	A	B	C	D
1				
2				
3	P	\$100,000		
4	S	-\$20,000		
5	n	5		
6	MARR	10%		
7	NACF	50,000		
8				
9	AW	\$26,896.20		
10				

Figure 6.8 Annual Worth Computation in Excel Spreadsheets

	A	B	C
1			
2			
3	P	\$300,000	
4	S	-\$50,000	
5	n	5	
6	MARR	10%	
7	NACF	-5,000	
8			
9	AW	=PMT(B6,B5,B3,B4)+B7	
10		PMT(rate, nper, pv, [fv], [type])	

	A	B	C
1			
2			
3	P	\$300,000	
4	S	-\$50,000	
5	n	5	
6	MARR	10%	
7	NACF	-5,000	
8			
9	AW	(\$75,949.37)	
10			

Summary

Annual worth analysis is a simple technique used by professional engineers to evaluate single, multiple, competing alternatives, and independent projects. To evaluate a single project, the annual worth analysis consists of calculating the annual equivalent amount (annuity amount) of all cash flows, both inflows and outflows, associated with a project and simply adding them together. If AW of a project is equal to or greater than zero, the project is economically justified and accepting it increases the

value of the firm. If AW is negative, it should be rejected as it negatively affects the value of the firm.

Annual worth analysis is generally preferred over a present worth analysis due to its simplicity in calculating the worth and not requiring equal service lives for comparison of mutually exclusive alternatives.

Annual worth of a project can also be calculated by adding the capital recovery (CR) to the annuity of net annual cash flows (A of NACF). Capital recovery of an investment in equipment is the uniform annual revenue needed to recover the cost of the equipment, and any non-recurring cash flows over the life of the project. Capital recovery can be computed by simply adding the equivalent annuity of the first cost, any non-recurring costs and the salvage value over the economic life of the equipment. The NACF can be of annual operating cost, annual revenue, or the difference between the annual revenue and annual operating cost. The distribution of NACF can be an annuity series, a gradient series, or have no particular format. If the format of NACF is an annuity series, then the A of NACF will be the same as the NACF per year. Otherwise, we have to compute the annuity of the NACF to calculate the annual worth of a project.

To evaluate mutually exclusive alternatives using AW analysis, we simply compute the AW of each alternative over its original life and select the one alternative with the highest numerical AW value.

The last section of this chapter discusses the use of Excel spreadsheets to calculate the annual worth of the project.

Discussion Question and Problems

- 6.1 Royal Aviation Corporation is undertaking a project that requires purchase of equipment with the first cost of \$5,000,000, one time rebuild cost of \$400,000 at the end of year 5, and a salvage value of \$250,000 at the termination of the project in 10 years. Royal Aviation uses a MARR of 9 percent. Compute the annual worth of cash flows associated with this project.
- 6.2 Royal Aviation Corporation (RAC) is considering purchasing a piece of equipment to manufacture a part of the wing fixture for an aircraft. The cost of the system is \$5 million with life expectancy of 10 years and with no salvage value. The annual operating cost of the system is \$400,000 and RAC anticipates a stream of \$1,500,000 in revenue at the end of each year for 10 years. Using AW analysis and assuming a MARR of 10 percent, is this investment economically justified?
- 6.3 Majestic Heavy Industries (MHI) purchased a diagnostic system for \$500,000. The life of the system is eight years with a salvage value of \$100,000. The annual operating cost is \$50,000 in year 1 increasing by \$20,000 per year starting in year 2 through the end of year 8. Calculate the CR, A of NACF, and the AW for this diagnostic system assuming MHI uses a MARR of 9 percent.
- 6.4 The Cirrus SR22 is equipped with the Cirrus Airframe Parachute System (CAPS), a whole-plane parachute recovery system capable of lowering the entire aircraft to the ground in an emergency situation (Zimmerman, 2015). Assume the cost of the Parachute System installation is \$4 million with life

expectancy of 10 years, salvage value of \$200,000, and no annual operating cost; but there are rebuild costs of \$500,000 at the end of year 4 and \$600,000 at the end of year 8. If the added revenue stream of \$1,200,000 at the end of each year for 10 years is anticipated, given a MARR of 8 percent, is this investment economically justified?

- 6.5** Royal Aviation Services (RAS) is a premier aviation service provider of ground handling, passenger services, cargo aircraft handling, loading and offloading, etc. Royal Aviation is considering purchasing equipment with a cost of \$400,000, and with a salvage value of \$50,000 at the end of its eight-year life. The annual operating cost is \$20,000. There is no annual revenue, but revenues of \$300,000 and \$400,000 are expected in years 3 and 6 respectively. Royal Aviation uses a MARR of 10 percent. Is the purchase of this equipment economically justified?
- 6.6** Inflight Internet Services Company is a provider of in-flight broadband internet service and other connectivity services for commercial and business aircraft. It is considering a new system of satellite solution Wi-Fi technology. The cost of the system is \$4 million, with a rebuild cost of \$500,000 in year 5 and a salvage value of \$200,000 at the end of the 10-year life of the system. Assuming a MARR of 12 percent, what is the capital recovery of this system?
- 6.7** Majestic Aircraft Company (MAC) manufactures a single-seat, single-jet engine, multi-role light fighter jet. It is supersonic and highly maneuverable, and is the smallest and lightest in its class of contemporary combat aircraft. MAC undertook a project with a first cost of \$50 million for construction of aluminum-lithium alloys, carbon-fiber composites, and titanium alloy steels for airframe and fuselage to make the aircraft lighter and stronger. The annual operating cost was \$3 million; the salvage at the end of 25 years of operation was \$4 million. Assuming a MARR of 10 percent, compute the CR and AW of the costs.
- 6.8** Johnson Aerospace Company (JAC) purchased a diagnostic system for \$2,000,000. The life of the system is eight years with a salvage value equal to 10 percent of the first cost. The annual operating cost is \$100,000 in year 1 increasing by \$50,000 per year starting in year 2 through the end of year 8. Calculate the CR, A of NACF, and the AW for this diagnostic system assuming JAC uses a MARR of 8 percent.
- 6.9** Johnson Aerospace Company (JAC) wishes to evaluate the following two alternative machines for engine testing of its signature aircraft. Use the AW method with MARR of 11 percent per year to select the most cost-efficient alternative.

	Machine A	Machine B
First cost	−\$500,000	−\$800,000
Annual operating cost	−\$120,000	−\$40,000
Life, years	10	10
Salvage value	\$50,000	\$100,000

- 6.10** Amazing Flight Academy (AFA) is purchasing 10 Cessna 150 for \$1,100,000 to conduct primary flight training. The cash flows associated with this project are presented below. Evaluate this project using AW analysis over a 10-year period. AFA uses a MARR of 10 percent.

First cost	\$1,100,000
Annual operating cost	\$100,000 in the first year increasing by \$50,000 per year
Annual revenue	\$300,000 in year one increasing by \$75,000 per year
Salvage value	\$80,000
Life, years	10

- 6.11** Royal Mobile Company is producing jet fuel from crude oil in its refinery in Texas. It is considering three mutually exclusive alternatives to improve the efficiency of its refinery process. The cash flows for the three alternatives are shown in the following table. Using the AW approach and assuming a MARR of 7 percent, determine which alternative should be selected by Royal Mobile.

	Alternative 1	Alternative 2	Alternative 3
First cost	\$300,000	\$500,000	\$800,000
Annual operating cost	\$85,000	\$65,000	\$45,000
Salvage value	\$50,000	\$70,000	\$100,000
Life, years	4	6	8

- 6.12** Dutch Mobile Company is producing jet fuel from crude oil in its refinery in Mexico. It is considering three mutually exclusive alternatives to improve the efficiency of its refinery process. The cash flows for the three alternatives are shown below. Using AW approach and assuming a MARR of 8 percent, determine which alternative should be selected by Dutch Mobile.

	Alternative 1	Alternative 2	Alternative 3
First cost	\$300,000	\$500,000	\$800,000
Annual operating cost	\$50,000 in year 1 increasing by \$20,000 per year	\$65,000	\$30,000 in year 1 growing at 5% per year
Salvage value	\$50,000	\$70,000	\$100,000
Life, years	4	6	10

- 6.13** Synthetic Product Manufacturing (SPM) is considering four capital investment proposals with no budget limitation. The projects are independent from each other and have the following cash flows (see table). SPM is using a MARR of 6 percent. Based on AW analysis, which of the following projects should be undertaken, if any?

	Project A	Project B	Project C	Project D
First cost	\$100,000	\$150,000	\$200,000	\$250,000
Annual operating cost	\$30,000	\$40,000	\$75,000	\$80,000
Annual revenue	\$65,000	\$80,000	\$100,000	\$120,000
Salvage value	\$10,000	\$12,000	\$15,000	\$20,000
Life, years	10	10	10	10

- 6.14** Legacy Regional Airport is considering building a new five-story parking garage with a first cost of \$180 million. This project is considered to be permanent with an estimated annual operating cost of \$2 million per year for the first 10 years and \$4 million per year thereafter, and a resurfacing cost of \$5 million every four years. The annual revenue from this parking garage is estimated to be \$15 million per year for the first four years, \$20 million per year for the next six years, and \$30 million per year thereafter. Legacy Regional Airport uses a MARR of 10 percent. Is this project economically justified?
- 6.15** Legacy Regional Airport (LRA) is considering the following two mutually exclusive projects:
- To build a concession area to offer food, beverage, gift merchandise, etc. The first cost of this project is \$25,000,000 and it is a permanent project. The annual operating cost is estimated to be \$400,000 per year and annual revenue is projected to be at least \$3,000,000 per year to infinity.
 - To build an in-line system of conveyer belts to process up to 700 bags per hour. The life of this project is 10 years with a first cost of \$5,000,000 and zero salvage value. The annual operating cost is \$200,000 in year 1 with an increase of 5 percent per year starting the second year over the life of the project. The revenue (cost savings) of the conveyer belt system is \$1,200,000 in year 1 with an increase of \$200,000 per year starting the second year over the life of the project. LRA uses a MARR of 8 percent. Using annual worth technique, which of these two alternatives should be selected?
- 6.16** An aviation enthusiast has donated \$1,000,000 to endow a university professorial chair in one of the leading aeronautical universities. If the money is invested at 8 percent, how much can be withdrawn each year to pay for this position?

Note

- 1 Federal Aviation Authority, NextGen Fact Sheet, 2007.

References

- FAA (2007). Performance and Accountability Highlights: The Next Generation of Flight. Federal Aviation Administration. Retrieved on March 15, 2016. http://www.faa.gov/about/plans_reports/media/2007_highlights.pdf.
- Zimmerman, J. (2015). Fatal Cirrus Crashes Are Way Down—Thank the Parachute. *Air Facts Journal*. Retrieved on March 15, 2016. <http://airfactsjournal.com/2015/02/fatal-cirrus-crashes-way-thank-parachute>.

7

Analysis of Rate of Return

This is a concern as the industry will not be able to attract capital requirements for aircraft purchases and other non-aircraft capital expenditure or provide adequate returns to shareholders. The capital-intensive nature of the industry and the fact that about 75% of the world's airlines are majority owned by the private sector require that returns on invested capital are high enough to attract investments.

T. Cederholm, December, 2014



Rate of return analysis is yet another popular technique used by professional engineers to evaluate single projects and multiple projects that are either competing alternatives or independent from each other. This technique is used often by investors to evaluate the rate of return of their investments in a single asset or security, portfolio of securities, or mutual funds. So long as the rate of return on an investment is equal to or greater than the minimum acceptable rate of return, the investment is considered to be economically justified. Rate of return analysis, similar to present worth analysis, requires equivalent service lives for comparison of mutually exclusive alternatives. In addition, mutually exclusive projects are evaluated two at a time on the basis of the incremental cash flows evaluation to select the optimal project.

The focus of this chapter is to enable the readers to use rate of return analysis to evaluate single projects, select the optimal project among mutually exclusive alternatives on the basis of incremental cash flows and equivalent service lives, and select economically justified independent projects.

The following sections briefly address the calculation of rate of return for single projects, the criteria for accepting or rejecting projects, and the comparison and selection of mutually exclusive alternatives and independent projects. In addition, the chapter presents the possibility of multiple rates of return existing and describes the use of modified rate of return analysis.

- ♦ Rate of Return Calculation
 - Interpretation of Rate of Return Values
 - Trial and Error Method
- ♦ Rate of Return of Bonds
- ♦ Evaluating Mutually Exclusive Projects
 - Incremental Cash Flows
 - Cost-based Alternatives
 - Revenue-based Alternatives
 - Equal-life Service Requirement
- ♦ Evaluating Independent Projects
- ♦ Complications with Rate of Return
 - Multiple Rates of Return Scenario
 - ROR Reinvestment Rate Assumption
 - Modified Rate of Return Analysis
- ♦ Using Spreadsheets for Rate of Return Computation
- ♦ Summary
- ♦ Discussion Questions and Problems

Rate of Return Calculation

Rate of return (ROR) analysis is used often by professional engineers and investors to evaluate the rate of return of their investments in a single asset or security, a portfolio of assets or securities, or mutual funds. Newnan (Newnan et al. 2012) defines rate of return as the interest rate at which the present worth and the annual worth are equal to zero. But Blank (Blank and Tarquin 2014) defines rate of return as “the rate paid on the unpaid balance of

borrowed money, or the rate earned on the uncovered balance of an investment, so that the final payment or receipt brings the balance to exactly zero with interest considered.”

In this book, the ROR, which is also called internal rate of return (IRR), is defined as the interest rate (i^*) that sets present worth of all cash flows, both inflows and outflows, to zero. We will use this definition as presented in the equation below to calculate the rate of return.

$$PW = \sum_{t=0}^n \left[\frac{CF^t}{(1+i^*)^t} \right] = 0$$

Where: CF_t = cash flow in period t
 i^* = rate of return
 n = number of periods

Aviation Snippet

Airlines have been a serial killer of capital. We have done a good job for consumers of selling our product at less than cost. In our view the industry is a giant charity.

John Owen, JetBlue

The advantages of ROR analysis over PW and AW analyses, are as follows:

- ROR measures the project attractiveness in terms that are commonly and easily understood by both engineers and business leaders.
- You need to accurately estimate the minimum acceptable rate of return (MARR) for PW and AW calculation, while for ROR computation, MARR is not needed.

Example 7.1

DirectJet Airline is a major carrier in the US and has an extensive network of operation. DirectJet has borrowed \$1,228.92 million from a local bank to expand its fleet of B787s. DirectJet will pay back this loan with payments of \$200 million per year for 10 years. What is the rate of return of this loan to the local bank?

Figure 7.1 shows the cash flow diagram. To find ROR, we set the PW of all cash flows to zero.

$$\begin{aligned} PW &= 0 \\ PW &= -\$1,228.92 + \$200 (P/A, i^*, 10) = 0 \\ \$200 (P/A, i^*, 10) &= \$1,228.92 \end{aligned}$$

We divide both sides of the above relationship by \$200,000,000 to find the value of the interest factor.

$$\begin{aligned} \frac{\$200(P/A, i^*, 10)}{\$200} &= \frac{\$1,228.92}{\$200} \\ (P/A, i^*, 10) &= 6.1446 \end{aligned}$$

Knowing the value of the interest factor is 6.1446, we go to Appendix A in the textbook for interest factor values and in each table we look for the value in column under P/A for a 10-year period. We find the interest factor value of 6.1446 in Appendix A, Table A16 for 10 percent interest rate and we conclude that the rate of return is 10 percent.

$$ROR = i^* = 10\%$$

If we could not find the exact interest factor value in the tables, then we would interpolate between the two closest values.

Figure 7.1 Cash Flow Diagram

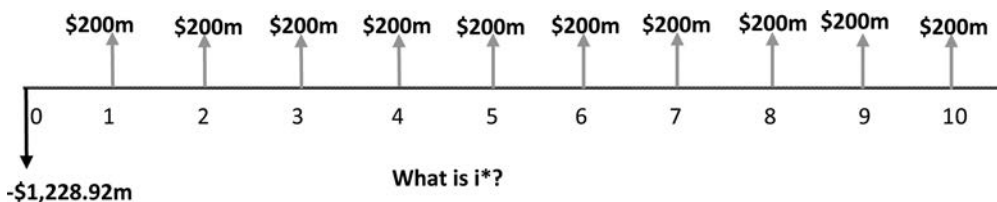


Table 7.1 Accept and Reject Criteria for Rate of Return

ROR Value	Selection Criteria
$ROR > MARR$	Accept the project, because it is economically justified. In other words it produces a return that is greater than MARR.
$ROR = MARR$	Indifference about the project. It is still economically justified. It produces a return that is equal to MARR.
$ROR < MARR$	Reject the project, because it is <u>is not</u> economically justified. In other words it produces a return that is <u>less than</u> MARR.

Interpretation of Rate of Return Values

The range for numerical value of ROR is from -100 percent to infinity. The extreme down side return is when $ROR = -100$ percent. This means that the investment is totally lost. The upside return is open and theoretically unlimited.

The criteria for selection or rejection of a project are summarized in Table 7.1.

Trial and Error Method

In Example 7.1, there was only one interest factor, $(P/A, i^*, 10)$, and that allowed us to compute the value of the interest factor and reference it to the interest factor values in Appendix A to find the corresponding interest rate. In Example 7.2, there are two interest factors, $(P/A, i^*, 10)$ and $(P/F, i^*, 10)$, therefore, we have to use the trial and error method to find the correct rate of return.

Example 7.2

In Example 7.1, assume that DirectJet Airline pays \$500 million at the end of 10 years in addition to annual payments of \$200 million. What is the rate of return on this loan to the local bank? If the local bank uses a 10 percent MARR, is this investment economically justified?

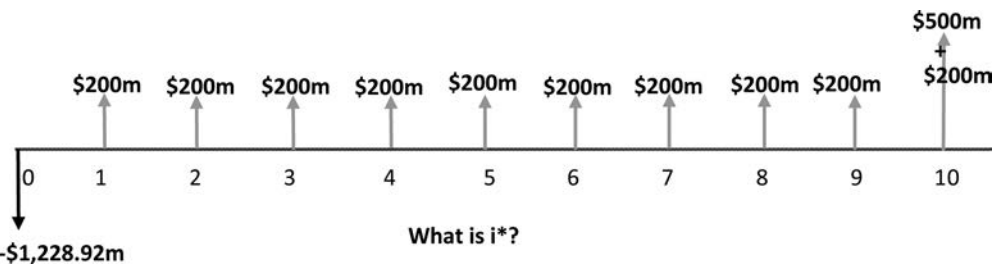
Figure 7.2 shows the cash flow diagram for Example 7.2.

Similar to Example 7.1, we set the PW of all cash flows to zero to find the ROR.

$$PW = 0$$

$$PW = -\$1,228.92 + \$200 (P/A, i^*, 10) + \$500 (P/F, i^*, 10) = 0$$

Figure 7.2 Cash Flow Diagram



The trial and error method consists of applying different interest rates to calculate the PW of the cash flows; the one interest rate that sets the PW of all cash flows to zero is the ROR. If there is no such interest rate that sets the PW to zero, then we select the two interest rates that provide the PW values closest to zero and then we will interpolate.

Normally we start with an initial interest rate of 10 percent. We plug this value for i^* into the PW function. If the PW value is a positive number, then we increase the interest rate. If PW value is a negative number, then we decrease the interest rate. For a typical investment, as we increase the interest rate, the PW value is expected to decrease and as we decrease the interest rate, the PW value is expected to increase. In Example 7.2, because the PW value at 10 percent interest rate is a positive number, \$192.75 million, we increase the interest rate to 12 percent for the second trial.

This trial also results to a positive PW value of \$62.12 million. For the third trial of 13 percent, we get another positive value for PW, but it is gradually getting closer to zero. With the fourth trial at 14 percent, the calculated PW value is a negative number. We know that i^* lies between the 13 percent and 14 percent. This actually provides enough information to make a decision, because the MARR is 10 percent, but we are asked to compute the ROR on this loan. The trial and error process is shown below.

In the first trial, we start with initial interest rate of 10 percent for i^* and plug that into the PW equation.

Try $i^* = 10\%$

$$PW = -\$1,228.92 + \$200 (P/A, 10\%, 10) + \$500 (P/F, 10\%, 10)$$

$$PW = -\$1,228.92 + \$200 (6.1446) + \$500 (0.3855)$$

$$PW = -\$1,228.92 + \$1,228.92 + \$192.75$$

$$PW = +\$192.75$$

Because PW value is a positive number, we increase the interest rate to 12 percent for the second trial.

Try $i^* = 12\%$

$$PW = -\$1,228.92 + \$200 (P/A, 12\%, 10) + \$500 (P/F, 12\%, 10)$$

$$PW = -\$1,228.92 + \$200 (5.6502) + \$500 (0.3220)$$

$$PW = -\$1,228.92 + \$1,130.04 + \$161.00$$

$$PW = +\$62.12$$

Because PW value is a still positive number, we increase the interest rate to 13 percent for the third trial.

Try $i^* = 13\%$

$$PW = -\$1,228.92 + \$200 (P/A, 13\%, 10) + \$500 (P/F, 13\%, 10)$$

$$PW = -\$1,228.92 + \$200 (5.4262) + \$500 (0.2946)$$

$$PW = -\$1,228.92 + \$1,085.24 + \$147.30$$

$$PW = +\$3.62$$

Because PW value is a still positive number, we increase the interest rate to 14 percent for the fourth trial.

Try $i^* = 14\%$

$$PW = -\$1,228.92 + \$200 (P/A, 14\%, 10) + \$500 (P/F, 14\%, 10)$$

$$PW = -\$1,228.92 + \$200 (5.2161) + \$500 (0.2697)$$

$$PW = -\$1,228.92 + \$1,043.22 + \$134.85$$

$$PW = -\$50.85$$

Because the PW value changes sign from a positive value at 13 percent interest rate to a negative value at 14 percent interest rate, it provides accurate information that i^* value lies between 13 percent and 14 percent. Now, we will interpolate to determine the value for i^* .

Usually the relationship between interest rates and their corresponding present worth values is not a linear function as shown in Figure 7.3.

However, for interest rates that are not too far apart, with one or possibly two percentages differential, we can assume that the relationship is linear and we can interpolate to determine the approximate value for i^* as presented in Figure 7.4.

Figure 7.3 Present Worth as a Function of the Interest Rate

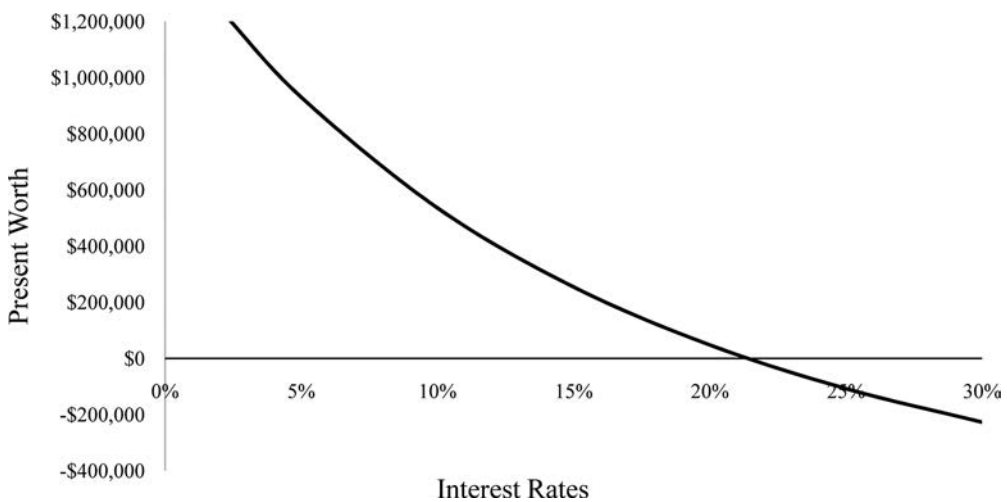


Figure 7.4 Endpoints for Interpolating the ROR

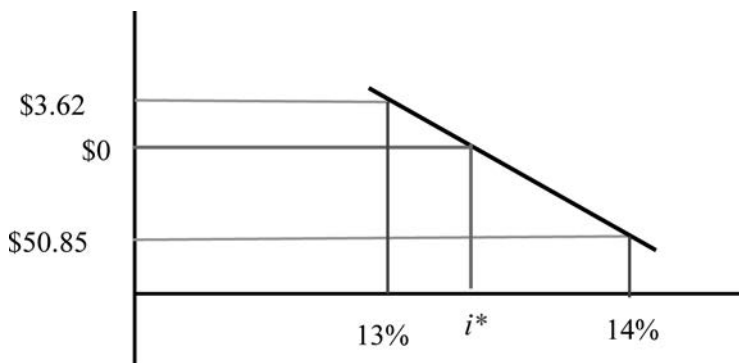


Table 7.2 Information for Interpolation

i^*	PW
13%	\$3.62
13% + X%	0
14%	−\$50.85

To interpolate between 13 percent and 14 percent, we set up information as shown in Table 7.2.

Now we form two equal fractions. For the fraction on the left side, the numerator is the difference between 13 percent and 13 percent + X percent, which is X percent, and the denominator is the difference between 13 percent and 14 percent, which is 1 percent. Similarly, on the right side, the numerator is the difference between the \$3.62 million and the 0 value which is \$3.62 million and denominator is the difference between \$3.62 million and −\$50.85 million which is \$54.47 million.

$$\frac{X\%}{1\%} = \frac{3.62}{54.47}$$

Now we cross multiply and solve for X percent.

$$X\% = \frac{3.62}{54.47}$$

$$X\% = 7\%$$

Therefore, $ROR = i^* = 13.07\%$

Since ROR exceeds the MARR of 10 percent, therefore, this investment by the local bank is economically justified.

Because the relationship between different ranges or boundaries is not linear, to get the accurate ROR through interpolation, we have to interpolate between the two closest ranges. In the above example interpolating between 10 percent and 14 percent would not have given the accurate answer. Rule of thumb is to interpolate between one to two percentage boundaries.

Rate of Return of Bonds

Many airlines including financially well-established airlines issue bonds to finance the purchase of the new aircraft. At the 2015 Dubai Airshow, Emirates Airline's chairman announced a bond issue of \$1 billion to finance 30 new aircraft purchases with additional cash infusion to help finance the acquisition of aircraft. The airline has around 270 additional aircraft

Aviation Snippets

Bombardier has a series of bonds that yield 11.55% and mature March 15, 2025. The bonds trade at a 987.79-point spread versus T-bonds maturing Feb. 14, 2026.

H. Osborne, April 2016

on order including 46 Boeing 777-300ERs, 115 Boeing 777-9Xs, 35 Boeing 777-8Xs, and 74 Airbus A380s.¹

Individuals and companies tend to invest in bonds because bonds are considered to be much safer than stocks. With bonds you receive the annual interest and the face value of bond at maturity. There is some degree of uncertainty associated with bonds as well. The value of bonds may change as interest rates change. Since most bonds are sold at discount, below their par value, one would wonder about the rate of return earned on bonds. Example 7.3 shows how to calculate the rate of return on bonds. The ROR on bonds can be compared to the required rate of return or MARR to determine whether or not the investment in the bond is economically justified.

Example 7.3

DirectJet Airline has a new bond issue with face value of \$1,000, coupon rate of 6 percent, with 10 years to maturity. If you pay \$850 for this bond, what would be your rate of return on this bond if interest is paid semi-annually?

We use the equations below to calculate the periodic interest payment, set the PW value equal to zero, and use trial and error method to solve for the interest rate.

$$I = \frac{(\$1,000) \times (0.06)}{2} = \$30$$

$$PW = -\$850 + \$30 (P/A, \frac{i^*}{2}, 10 \times 2) + \$1,000 (P/F, \frac{i^*}{2}, 10 \times 2) = 0$$

$$PW = -\$850 + \$30 (P/A, \frac{i^*}{2}, 20) + \$1,000 (P/F, \frac{i^*}{2}, 20) = 0$$

Try $i^* = 10\%$

$$PW = -\$850 + \$30 (P/A, \frac{10\%}{2}, 20) + \$1,000 (P/F, \frac{10\%}{2}, 20)$$

$$PW = -\$850 + \$30 (P/A, 5\%, 20) + \$1,000 (P/F, 5\%, 20)$$

$$PW = -\$850 + \$30 \times (12.4622) + \$1,000 \times (0.3769)$$

$$PW = -\$850 + \$373.87 + \$376.9$$

$$PW = -\$99.23$$

Try $i^* = 8\%$

$$PW = -\$850 + \$30 (P/A, 8\%/2, 20) + \$1,000 (P/F, 8\%/2, 20)$$

$$PW = -\$850 + \$30 (P/A, 4\%, 20) + \$1,000 (P/F, 4\%, 20)$$

$$PW = -\$850 + \$30 \times (13.5903) + \$1,000 \times (0.4564)$$

$$PW = -\$850 + \$407.71 + \$456.40$$

$$PW = \$14.11$$

We interpolate between 8 percent and 10 percent.

i^*	PW
8%	\$14.11
8% + X%	0
10%	-\$99.23

$$\frac{X}{2} = \frac{14.11}{113.34}$$

$$X\% = \frac{\$14.11 \times 2}{\$113.34} = \frac{\$28.22}{\$113.34} = 0.25\%$$

Therefore, ROR on DirectJet bond is 8.25 percent.

In bond valuation, to find the rate of return, we may use approximate rate instead of the trial and error method. Approximate rate is a rate that is very close to the ROR. Once we compute the approximate rate, we can interpolate between the two whole percentage boundaries of the approximate rate. The equation below computes the approximate rate.

$$\text{Approximate Rate} = \frac{I + \frac{V - P}{n}}{2}$$

Where: I = Dollar amount of periodic interest
 V = Bond face value
 P = Bond market price or value
 n = Number of years to maturity

The approximate rate for Example 7.3 is calculated below.

$$\text{Approximate Rate} = \frac{\$30 + \frac{(\$1000 - \$850)}{20}}{\frac{(\$1000 + \$850)}{2}}$$

Semi-annual Approximate Rate = 4.05%

$$\frac{i^*}{2} = 4.05\%$$

Annual Approximate rate $i^* = 8.10\%$

Now we can interpolate between 8 percent and 10 percent as we did above to compute the rate of return on this bond which becomes 8.25 percent.

Evaluating Mutually Exclusive Projects

As we described earlier, mutually exclusive projects are those projects that cannot be executed simultaneously, and thus selection of one project results in the rejection of the rest of the projects even though they might be profitable. Using ROR analysis to select the best project among the competing alternatives, we need to arrange the alternatives in order of their first cost from the lowest to the highest, and evaluate two projects at a time using their incremental cash flows. The survivor of these two alternatives is compared to the next alternative in line. This process continues until the

best alternative is selected which is the surviving alternative. With revenue-based projects, first, the project with the lowest first cost is compared with do-nothing. The survivor of these two alternatives is compared to the next alternative in line. If none of the projects survive, then do-nothing will be selected.

Incremental Cash Flows

Professional engineers often use ROR to select among mutually exclusive alternatives. A complication often arises because there can be selection conflicts between the PW and the ROR analysis of evaluated alternatives. In other words, we might find ourselves between two projects that both have positive PW values and RORs that are greater than the MARR. But one project may have a higher PW value whereas the other has a higher ROR. Which criterion should we use, the higher PW value or the greater ROR? This conflict occurs when size of the projects or cash flow patterns of alternatives are significantly different.

Example 7.4

Consider the two mutually exclusive alternatives shown in Table 7.3. Alternative A is a larger size project whereas alternative B is a smaller size project. Assuming a MARR of 10 percent, which alternative should be selected?

Notice that both projects have positive PW values and RORs that are greater than 10 percent which is the MARR. But PW criterion is in favor of the project B whereas ROR criterion favors the project A.

Example 7.5

Now, consider the two mutually exclusive projects C and D shown in Table 7.4. Alternative C has a decreasing cash inflow pattern whereas Alternative D has an increasing cash inflow pattern. Assuming a MARR of 10 percent, which alternative should be selected?

As was the case in Example 7.4, in Example 7.5 both projects C and D have positive PW values and RORs that are greater than MARR. But the PW criterion is in favor of project D, whereas the ROR criterion favors project C.

Because the selection based on the individual project's ROR may not result in selection of the best project, as shown in Example 7.4 and Example 7.5, we are going to calculate the incremental cash flows between the larger and the smaller first cost

Table 7.3 Mutually Exclusive Projects of Different Sizes

	Project A	Project B
First cost	−\$1,000,000	−\$10,000,000
AOC	−\$50,000	−\$500,000
Annual revenue	\$300,000	\$2,500,000
Life	10	10
PW	\$536,142	\$2,289,134
ROR	21.41%	15.10%

Table 7.4 Mutually Exclusive Projects with Cash Flows Pattern Differential

Year	Project C	Project D
0	−\$9,000,000	−\$10,000,000
1	\$5,000,000	\$1,000,000
2	\$4,000,000	\$2,000,000
3	3,000,000	\$3,000,000
4	2,000,000	\$4,000,000
5	1,000,000	\$5,000,000
6	500,000	\$6,000,000
PW	\$3,374,369	\$4,039,432
ROR	27.50%	19.71%

Table 7.5 Accept and Reject Criteria for the Δ Project

Δi^* Value	Selection Criteria
$\Delta i^* > \text{MARR}$	Accept the Δ project, because it is economically justified. Select the alternative with the larger first cost
$\Delta i^* = \text{MARR}$	Indifferent about the Δ project. Indifferent about the alternatives. You may select the smaller or larger first cost alternative
$\Delta i^* < \text{MARR}$	Reject the Δ project, because it is <u>not</u> economically justified. Select the alternative with the smaller first cost

alternatives, calling it the Δ project, and evaluate the Δ project or the extra project. If the Δ project is economically justified, we select the project with the larger first cost. Otherwise, we select the project with the smaller first cost.

The criteria for selecting the Δ project using ROR analysis are presented in Table 7.5.

Table 7.6 shows the incremental cash flows for both alternatives A and B and the Δ project in Example 7.4.

$$\Delta \text{ Project} = \text{Project B} - \text{Project A}$$

In other words,

$$\text{Project B} = \text{Project A} + \Delta \text{ Project}$$

The Δ project has a positive PW value and has a rate of return (Δi^*) that is greater than MARR, therefore, it is a profitable project or it is economically justified. Because Δ project is a profitable project, we are going to select project B. By selecting project B, we are theoretically selecting project A and an additional profitable project (Δ Project).

Table 7.6 Cash Flows for Projects A and B and the Δ Project

	Project A	Project B	Δ Project
First cost	−\$1,000,000	−\$10,000,000	−\$9,000,000
AOC	−\$50,000	−\$500,000	−\$450,000
Annual revenue	\$300,000	\$2,500,000	\$2,200,000
Life	10	10	10
PW	\$536,142	\$2,289,134	\$1,752,992
ROR	21.41%	15.10%	$\Delta i^* = 14.36\%$

Table 7.7 Cash Flows for Projects C and D and the Δ Project

Year	Project C	Project D	Δ Project
0	−\$9,000,000	−\$10,000,000	−\$1,000,000
1	\$5,000,000	\$1,000,000	−\$4,000,000
2	\$4,000,000	\$2,000,000	−\$2,000,000
3	3,000,000	\$3,000,000	0
4	2,000,000	\$4,000,000	\$2,000,000
5	1,000,000	\$5,000,000	\$4,000,000
6	500,000	\$6,000,000	\$5,500,000
PW	\$3,374,369	\$4,039,432	\$1,665,063
ROR	27.50%	19.71%	$\Delta i^* = 12.70\%$

On the other hand if we select project A we are actually giving up a profitable project that would have otherwise increased the value of the firm.

Table 7.7 shows the cash flows for projects C and D and incremental cash flows for the Δ project in Example 7.5. In this example, because the Δ project has a positive PW value and has a rate of return (Δi^*) that is greater than MARR, we are going to select project D with the larger first cost.

Examples 7.4 and 7.5 illustrate why we should use incremental cash flows to evaluate mutually exclusive alternatives rather than comparing alternatives simply based on their individual rate of returns.

Cost-based Alternatives

Cost-based competing alternatives are projects that provide the same service and produce the same amount of revenue in the same time period. The focus of ROR analysis is to determine the most cost-efficient project. In cost-based alternatives,

usually the revenue is not provided, and because one project is needed, do-nothing is not an option. The selection criteria are based on the evaluation of the Δ project.

Example 7.6

Mega Cargo airline is considering express delivery to Hanoi in Vietnam and therefore is looking for leasing storage facilities in Hanoi. It is evaluating the lease options for the two locations in Table 7.8. Using ROR analysis and assuming a MARR of 12 percent, which location should be selected by Mega?

Table 7.9 shows the incremental cash flows for the Δ project.

$$PW\Delta = -\$1,500,000 + \$350,000 (P/A, \Delta i^*, 8) + \$500,000 (P/F, \Delta i^*, 8) = 0$$

$$\text{Try } \Delta i^* = 12\%$$

$$PW\Delta = -\$1,500,000 + \$350,000 (P/A, 12\%, 8) + \$500,000 (P/F, 12\%, 8) = 0$$

$$PW\Delta = -\$1,500,000 + \$350,000 \times (4.9676) + \$500,000 \times (0.4039)$$

$$PW\Delta = -\$1,500,000 + \$1,738,660 + \$201,950$$

$$PW\Delta = \$440,610$$

Since at Δi^* equal to MARR, the PW value of the Δ project is positive, it indicates that $\Delta i^* > \text{MARR}$, and the Δ project is a profitable project and economically justified and therefore, the project with larger first cost, location Y, should be selected.

If there are more than two alternatives, we first place them in order of their first cost from the lowest to the highest. Then we compare the first two alternatives. The survivor of the first two alternatives is compared with the third alternative. This process continues until one project has survived and that is the selected alternative.

Table 7.8 Mega Storage Facility Options in Hanoi, Vietnam

	Location X	Location Y
First cost	−\$2,000,000	−\$4,000,000
Annual operating cost (AOC)	−\$500,000	−\$150,000
Deposit return	\$500,000	\$1,000,000
Life, year	8	8

Table 7.9 Mega Storage Facility Options in Hanoi, Vietnam

	Location X	Location Y	Δ Project
First cost	−\$2,000,000	−\$3,500,000	−\$1,500,000
Annual operating cost (AOC)	−\$500,000	−\$150,000	+\$350,000
Deposit return	\$500,000	\$1,000,000	+\$500,000
Life, year	8	8	8

Revenue-based Alternatives

When a group of mutually exclusive projects that are being evaluated produce different amounts of revenue in the same time period, they are called mutually exclusive revenue-based alternatives. The ROR analysis is performed to determine the most profitable alternative. In the revenue-based alternative, do-nothing becomes an option as potentially none of the projects may be economically justified. The selection criteria, similar to cost-based alternatives, are based on the evaluation of the Δ project.

Example 7.7

DirectJet Airline is a major carrier in the US and has an extensive network of operation. It is considering purchasing a number of aircraft to expand its operations to either Central America or Latin America. The estimated initial investment, annual operating cost, annual revenue, and potential salvage value for both alternatives are presented in Table 7.10. Which alternative should DirectJet Airline select given a MARR of 8 percent, using ROR analysis and a study period of 10 years?

Because these projects are revenue-based alternatives, Do-nothing (DN) becomes an option. This means that the smaller first cost project is compared with DN first and the survivor of those two is compared to the second project. Table 7.11 shows incremental cash flows between Do-nothing and Central America Expansion.

$$PW_{\Delta} = -\$300 + (\$100 - \$50) (P/A, \Delta i^*, 10) + \$30 (P/F, \Delta i^*, 10) = 0$$

Table 7.10 Airline Expansion Plan

	Central America Expansion (CAE)	Latin America Expansion (LAE)
Initial investment (first cost)	−\$300 million	−\$700 million
Annual operating cost (AOC)	−\$50 million	−\$30 million
Annual revenue	\$100 million	\$150 million
Salvage value	\$30 million	\$50 million
Life, years	10	10

Table 7.11 Airline Incremental Cash Flows for $\Delta_{\text{CAE-DN}}$ Project

	Do-nothing	Central America Expansion (CAE)	$\Delta_{\text{CAE-DN}}$ Project
Initial investment (first cost)	\$0.0	−\$300 million	−\$300 million
Annual operating cost (AOC)	\$0.0	−\$50 million	−\$50 million
Annual revenue	\$0.0	\$100 million	\$100 million
Salvage value	\$0.0	\$30 million	\$30 million
Life, years	10	10	10

Table 7.12 Airline Incremental Cash Flows for $\Delta_{\text{LAE-CAE}}$ Project

	Central America Expansion (CAE)	Latin America Expansion (LAE)	$\Delta_{\text{LAE-CAE}}$ Project
Initial investment (first cost)	−\$300 million	−\$700 million	−\$400 million
Annual operating cost (AOC)	−\$50 million	−\$30 million	+\$20 million
Annual revenue	\$100 million	\$150 million	+\$50 million
Salvage value	\$30 million	\$50 million	+\$20 million
Life, years	10	10	10

Using trial and error method, we start with the MARR.

Try $\Delta i^* = 8\%$

$$PW_{\Delta} = -\$300 + (\$100 - \$50) (P/A, 8\%, 10) + \$30 (P/F, 8\%, 10)$$

$$PW_{\Delta} = -\$300 + \$50 \times (6.7101) + \$30 \times (0.4632)$$

$$PW_{\Delta} = -\$300 + \$335.51 + \$13.90$$

$$PW_{\Delta} = \$49.401$$

Since PW of the Δ project is positive, it indicates that $\Delta i^* > \text{MARR}$ and that the Δ project is an economically justified project. Therefore, we eliminate Do-nothing and the Central America Expansion project survives. Now we compare Central America Expansion with Latin America Expansion.

Table 7.12 shows incremental cash flows between Central America Expansion and Latin America Expansion.

$$PW_{\Delta} = -\$400 + (\$50 + \$20) (P/A, \Delta i^*, 10) + \$20 (P/F, \Delta i^*, 10) = 0$$

Using the trial and error method, we start with the MARR.

Try $\Delta i^* = 8\%$

$$PW_{\Delta} = -\$400 + \$70 (P/A, 8\%, 10) + \$20 (P/F, 8\%, 10)$$

$$PW_{\Delta} = -\$400 + \$70 \times (6.7101) + \$20 \times (0.4632)$$

$$PW_{\Delta} = -\$400 + \$469.71 + \$9.26$$

$$PW_{\Delta} = \$78.97$$

Because PW of the Δ project is positive, it indicates that $\Delta i^* > \text{MARR}$ and that the Δ project is an economically justified project. Therefore, we eliminate Central America Expansion and the Latin America Expansion project survives and should be selected by DirectJet Airline.

Equal-life Service Requirement

The rate of return analysis of mutually exclusive projects not only requires the evaluation of incremental cash flows but, similar to PW analysis, requires equal life service comparison. Example 7.8 shows the comparison process of non-equal-life mutually exclusive alternatives.

Example 7.8

Legacy Regional Airport (LRA) serves traffic within a relatively lightly populated area in Southern France. The airport is considering two mutually exclusive projects for purchasing a weather monitoring system with five-year life or a jet fuel service station with approximately 10-year life. The cash flows for both alternatives are presented in Table 7.13. Legacy uses a MARR of 10 percent. Which alternative should Legacy Regional Airport select based on the ROR analysis?

Because these projects are revenue-based alternatives, Do-nothing (DN) becomes an option. So the smaller first cost project is first compared with DN and the survivor of those two is compared to the second project. Table 7.14 shows incremental cash flows between Do-nothing and the Weather Monitoring System (WMS).

$$PW_{\Delta} = -\$200,000 + \$80,000 (P/A, \Delta i^*, 5) + \$20,000 (P/F, \Delta i^*, 5) = 0$$

Using the trial and error method, we start with the MARR.

Try $\Delta i^* = 8\%$

$$PW_{\Delta} = -\$200,000 + \$80,000 (P/A, 10\%, 5) + \$20,000 (P/F, 10\%, 5)$$

$$PW_{\Delta} = -\$200,000 + \$80,000 \times (3.7908) + \$20,000 \times (0.6209)$$

$$PW_{\Delta} = -\$200,000 + \$303,264 + \$12,418$$

$$PW_{\Delta} = \$115,682$$

Since PW of the Δ project is positive, it indicates that $\Delta i^* > \text{MARR}$ and Δ project is an economically justified project. Therefore, we eliminate the Do-nothing option and the

Table 7.13 Legacy Regional Airport Projects

	Weather Monitoring System (WMS)	Jet Fuel Service Station (JFSS)
Initial investment (first cost)	−\$200,000	−\$500,000
Net annual cash flow (NACF)	\$80,000	\$150,000
Salvage value	\$20,000	\$50,000
Life, years	5	10

Table 7.14 Legacy Regional Airport Incremental Cash Flows for $\Delta_{\text{WMS-DN}}$ Project

	Do-nothing	Weather Monitoring System (WMS)	$\Delta_{\text{WMS-DN}}$ Project
Initial investment (first cost)	\$0.0	−\$200,000	−\$200,000
Net annual cash flow (NACF)	\$0.0	\$80,000	\$80,000
Salvage value	\$0.0	\$20,000	\$20,000
Life, years	5	5	5

Table 7.15 Legacy Regional Airport Incremental Cash Flows for $\Delta_{\text{JFS-WMS}}$ Project

Year	Weather Monitoring System (WMS)		Jet Fuel Service Station (JFS)	$\Delta_{\text{JFS-WMS}}$ Project
	First Cycle	Second Cycle		
0	−\$200,000		−\$800,000	−600,000
1	\$80,000		\$170,000	\$90,000
2	\$80,000		\$170,000	\$90,000
3	\$80,000		\$170,000	\$90,000
4	\$80,000		\$170,000	\$90,000
5	\$80,000 + \$20,000	−\$200,000	\$170,000	\$270,000
6		\$80,000	\$170,000	\$90,000
7		\$80,000	\$170,000	\$90,000
8		\$80,000	\$170,000	\$90,000
9		\$80,000	\$170,000	\$90,000
10		\$80,000 + \$20,000	\$170,000 + \$50,000	\$120,000

Weather Monitoring System (WMS) project survives. Now we compare the Weather Monitoring System (WMS) with the Jet Fuel Service Station (JFS) alternative.

Table 7.15 shows the cash flows for both projects over a life of 10 years. We compare two life cycles of the weather monitoring system with one life cycle of the jet fuel service station. This table also shows the incremental cash flows between the two alternatives (Δ project).

To evaluate the ROR for Δ project (Δi^*), we set the PW of all cash flows for the Δ project to zero as follows:

$$PW_{\Delta} = -\$600,000 + \$90,000 (P/A, \Delta i^*, 10) + \$180,000 (P/F, \Delta i^*, 5) + \$30,000 (P/V, \Delta i^*, 10)$$

$$\text{Try } \Delta i^* = \text{MARR} = 10\%$$

$$PW_{\Delta} = -\$600,000 + \$90,000 (P/A, 10\%, 10) + \$180,000 (P/F, 10\%, 5) + \$30,000 (P/V, 10\%, 10)$$

$$PW_{\Delta} = -\$600,000 + \$90,000 \times (6.1446) + \$180,000 \times (0.6209) + \$30,000 \times (0.3855)$$

$$PW_{\Delta} = -\$600,000 + \$553,014 + \$111,762 + \$11,565$$

$$PW_{\Delta} = \$76,341$$

Because PW_{Δ} at MARR of 10 percent is positive, it indicates that the rate of return of the Δ project (Δi^*) is greater than MARR, and therefore, the Δ project is economically justified. Hence, the Weather Monitoring System should be eliminated and the Jet Fuel Service Station should be selected.

Evaluating Independent Projects

Projects are considered to be independent when the decision to accept or to reject one project does not have any impact on the rest of the projects. Independent projects evaluation does not require common lives. The ROR approach to evaluate independent projects is to compute the ROR of each project, eliminate those projects that have a ROR that is less than MARR and accept the rest of them because they contribute to the growth or value of the firm as long as there is no budget limitation. If there is a budget constraint, then we use the PW approach as explained in Chapter 5. Examples of independent projects are:

- ♦ Investing in a new runway
- ♦ Construction of a new terminal
- ♦ Building a new car park
- ♦ Investment in weather monitoring equipment
- ♦ Building a new concession area, etc.

The issue of whether one project is better than another is not crucial and the only condition for selecting a project is whether the respective B/C ratio is greater or equal to 1.0.

Example 7.9

Legacy Regional Airport is considering four independent projects with cash flows presented in Table 7.16. The airport uses a MARR of 9 percent and has no budget constraint. Using a ROR analysis, which of the projects in Table 7.16 should be selected to maximize the value of the airport?

The ROR for each project is computed by setting the PW of all cash flows to zero and solving for i^* .

New parking garage

$$\begin{aligned}
 PW &= 0 \\
 -\$200 - 10(P/A, 9\%, 10) + \$50(P/A, 9\%, 10) &= 0 \\
 \$200 &= (\$50 - \$10)(P/A, 9\%, 10) \\
 \$200 &= \$40(P/A, 9\%, 10) \\
 (P/A, 9\%, 10) &= 5
 \end{aligned}$$

Table 7.16 Legacy Regional Airport Independent Projects

	New Parking Garage	New Runway	Expanding C Pier	Expanding F Pier
First cost	−\$200 million	−\$300 million	−\$500 million	−\$800 million
Estimated annual operating cost (AOC)	−\$10 million	−\$20 million	−\$25 million	−\$30 million
Estimated annual revenue	\$50 million	\$45 million	\$85 million	\$100 million
Life, years	10	20	25	25

In Appendix A, compound interest factors, we look for the value of 5.0 in column P/A for a 10-year period with different interest rates until we find an interest rate at which the interest factor is equal to 5. If we cannot find the exact interest factor value in the tables, then we will interpolate between the two closest interest factor values as shown below.

i^*	$(P/A, i^*, 10)$
15%	5.0188
15% + X%	5.0
16%	4.8332

$$\frac{X}{1} = \frac{0.0188}{0.1856}$$

$$X\% = \frac{0.0188}{0.1856} = 0.10\%$$

Therefore, $ROR = i^* = 15.10\% > MARR$

The new parking garage is economically justified.

New runway

$$\begin{aligned} PW &= -\$300 - 20 (P/A, 9\%, 20) + \$45 (P/A, 9\%, 20) = 0 \\ \$300 &= (\$45 - \$20) (P/A, 9\%, 20) \\ \$300 &= \$25 (P/A, 9\%, 20) \\ (P/A, 9\%, 10) &= 12 \end{aligned}$$

i^*	$(P/A, i^*, 20)$
5%	12.4622
5% + X%	12.0
6%	11.4699

$$\frac{X}{1} = \frac{0.4622}{0.9923}$$

$$X = \frac{0.4622}{0.9923}$$

$$X = 0.47\%$$

Therefore, $ROR = i^* = 5.47\% < MARR$

The new runway is not economically justified.

Expanding C pier

$$\begin{aligned} PW &= -\$500 - 25 (P/A, 9\%, 25) + \$85 (P/A, 9\%, 25) = 0 \\ \$500 &= (\$85 - \$25) (P/A, 9\%, 25) \\ \$500 &= \$60 (P/A, 9\%, 25) \\ (P/A, 9\%, 25) &= 8.3333 \end{aligned}$$

$$i^* \quad (P/A, i^*, 25)$$

$$11\% \quad 8.4217$$

$$11\% + X\% \quad 8.3333$$

$$12\% \quad 7.8431$$

$$\frac{X}{1} = \frac{0.0884}{0.5786}$$

$$X = \frac{0.0884}{0.5786}$$

$$X = 0.15\%$$

Therefore, $ROR = i^* = 11.15\% > MARR$

The expanding of C pier is economically justified.

Expanding F pier

$$PW = -\$800 - 30 (P/A, 9\%, 25) + \$100 (P/A, 9\%, 25) = 0$$

$$\$800 = (\$100 - \$30) (P/A, 9\%, 25)$$

$$\$800 = \$70 (P/A, 9\%, 25)$$

$$(P/A, 9\%, 25) = 11.4286$$

$$i^* \quad (P/A, i^*, 10)$$

$$7\% \quad 11.6536$$

$$7\% + X\% \quad 11.4286$$

$$8\% \quad 10.6748$$

$$\frac{X}{1} = \frac{0.2250}{0.9788}$$

$$X = \frac{0.2250}{0.9788} = 0.23\%$$

Therefore, $ROR = i^* = 7.23\% < MARR$

The F pier is not economically justified.

Out of the four projects only the new parking garage and C pier expansion are economically justified and Legacy Regional Airport should only undertake those two projects.

Complications with Rate of Return

Most investments start with an initial investment followed by a stream of positive cash flows over the life of the project. For projects with this pattern of cash flows, where the initial investment or first cost is negative and the rest of the cash flows are positive, there is only one unique ROR for that project. If the ROR is greater than the firm's MARR, then both PW and AW are positive and all three techniques

mutually indicate that the project is economically justified and the firm should undertake the project. The complications with the rate of return analysis are associated with multiple rates of return and the reinvestment rate assumption as discussed below.

Multiple Rates of Return Scenario

For unconventional projects where there is more than one infusion of capital (initial investment) over the original life of the project or there are multiple sign changes in the cash flow stream, there is a possibility of more than one rate of return. In that case none of the rates of return are valid and we should use other techniques such as PW and AW, where only one PW and one AW exist for each project. Alternatively, we could use the modified rate of return analysis as explained later in this chapter.

Example 7.10

Consider a project with the unconventional cash flow stream presented in Table 7.17. Compute the project's rate of return.

Table 7.18 presents different interest rates and their corresponding present worth value. Note that PW becomes zero at interest rates of 2.387 percent and 16.75 percent. By definition, those two interest rates are the two rates of return for this project.

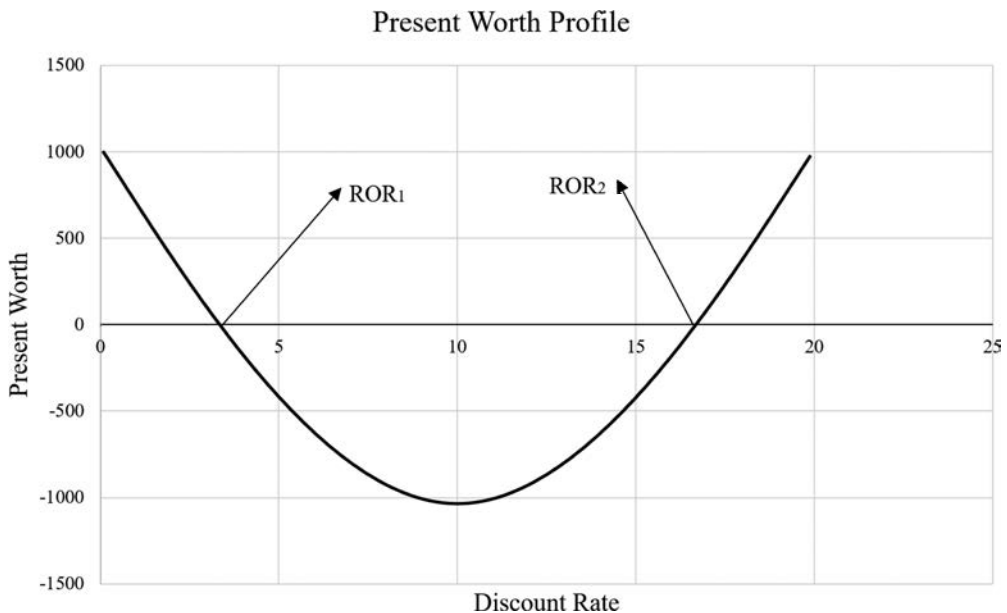
The two rates of return for Example 7.10 are also depicted in Figure 7.5.

Table 7.17 Cash Flow Stream of an Unconventional Project

Year	Cash Flows
0	251,000
1	−550,000
2	300,000

Table 7.18 Comparing Interest Rates and Their Corresponding PW

Interest Rate	Present Worth
0.00%	\$1,000
2.38%	\$0
5%	−\$701
10%	−\$1,066
15%	−\$418
16.75%	\$0
20%	\$1,000

Figure 7.5 Multiple Rates of Return

ROR Reinvestment Rate Assumption

PW analysis implicitly assumes that the generated cash flows from a project can be reinvested at the rate of MARR; while the ROR analysis implies that the firm has the opportunity to reinvest the generated cash flows from a project at the ROR. Although there may be many projects available to yield MARR, the firm may not find another project that provides a rate of return equal to ROR. Therefore, the reinvestment rate assumption of the PW analysis seems more reasonable than reinvestment rate assumption of the ROR technique.

Modified Rate of Return Analysis

Modified rate of return (MROR) was developed to allow decision makers the intuitive appeal of the ROR, but let them eliminate the problem of multiple RORs with the ability to directly specify the appropriate reinvestment rate. The idea behind MROR is: 1) to shift all negative cash flows after first cost to time zero at the rate of MARR and add them to the first cost; and 2) to shift all positive cash flows to the end of the project at the rate of MARR and add them to the salvage value, calling this terminal cash flow. The MROR is an interest rate (mi^*) that sets present worth of all cash flows, first cost and terminal cash flow, to zero. The criteria for selection or rejection of a project are summarized in Table 7.19.

Example 7.11

In Example 7.10, calculate the modified rate of return assuming a MARR is 10 percent.

$$\text{First cost} = -\$550,000 (P/F, 10\%, 1)$$

$$\text{First cost} = -\$550,000 \times (0.9091)$$

$$\text{First cost} = -\$500,005$$

Table 7.19 Accept and Reject Criteria for Modified Rate of Return

MROR (mi^*) Value	Selection Criteria
MROR (mi^*) > MARR	Accept the project, because it is economically justified. In other words it produces a return that is greater than MARR
MROR (mi^*) = MARR	Indifference about the project. It is still economically justified. It produces a return that is equal to MARR
MROR (mi^*) < MARR	Reject the project, because it is <u>not</u> economically justified. In other words it produces a return that is <u>less than</u> MARR

$$\text{Terminal cash flow} = \$251,000 (F/P, 10\%, 2) + \$300,000$$

$$\text{Terminal cash flow} = \$251,000 \times (1.2100) + \$300,000$$

$$\text{Terminal cash flow} = \$303,710 + \$300,000$$

$$\text{Terminal cash flow} = \$603,710$$

$$PW = 0$$

$$-500,005 + \$603,710 (F/P, mi^*, 2) = 0$$

$$\$500,005 = \$603,710 (F/P, mi^*, 2)$$

$$(F/P, mi^*, 2) = 0.8282$$

We now go to Appendix A for compound interest factors and in each table we look for the value of 0.8282 in the column under F/P for a two-year period. If we cannot find the exact interest factor value in the tables, then we will interpolate between the two closest values.

$$i^* \quad (F/P, mi^*, 2)$$

$$9\% \quad 0.8417$$

$$9\% + X\% \quad 0.8282$$

$$10\% \quad 0.8264$$

$$\frac{X}{1} = \frac{0.0135}{0.0153}$$

$$X = \frac{0.0135}{0.0153} = 0.88\%$$

Therefore, $MROR = mi^* = 9.88\% < MARR$

This project is not economically justified.

Using Spreadsheets for Rate of Return Computation

This section presents how to run the financial equation to calculate ROR using Microsoft Excel functions. The ROR equation in Excel is called Internal Rate of Return

and is denoted as IRR. To calculate IRR, we plug in the following function into an Excel spreadsheet cell unit:

$$= \text{IRR}(\text{Cell [0]:Cell [n]})$$

Where (Cell [0]:Cell [n]) is the range of cash flows from period [0] to period [n]. Cell [0] is the first cost or initial investment (Year 0), Cell [n] is the NACF in year n (the last year).

Example 7.12

The cash flows for an investment are presented in Table 7.20 for five years. Compute the ROR of this investment using the Excel financial equation for ROR analysis.

Figure 7.6 shows the Excel spreadsheet solution for the project in Example 7.12. The cash flow in year zero is the first cost and NACFs are in years 1 through 5. The function presented in cell C9 should be entered in cell B9 and the solution will appear in that cell as soon as you hit the return key. As you note, the NACFs do not represent a uniform series and as soon as you enter them in an Excel spreadsheet and plug in the Excel function for IRR, the result will appear instantly. Microsoft Excel makes it very easy relative to the trial and error method.

Table 7.20 Net Annual Cash Flows

Year	NACF
0	−\$200,000
1	\$40,000
2	\$60,000
3	\$80,000
4	\$60,000
5	\$40,000

Figure 7.6 Rate of Return Computation in Excel Spreadsheets

	A	B		A	B
1			1		
2	Year	NACF	2	Year	NACF
3	0	(\$200,000)	3	0	(\$200,000)
4	1	\$40,000	4	1	\$40,000
5	2	\$60,000	5	2	\$60,000
6	3	\$80,000	6	3	\$80,000
7	4	\$60,000	7	4	\$60,000
8	5	\$40,000	8	5	\$40,000
9	ROR	=IRR(B3:B8)	9	ROR	12.26%

Summary

Rate of return (ROR) analysis is used often by professional engineers and individual investors to evaluate the return on their investments and select the best alternative among the competing alternatives. Rate of return is defined as the interest rate (i^*) that sets the present worth of all cash flows, both inflows and outflows, to zero. Although present worth and annual worth are easier to calculate, the rate of return is the most frequently used technique and is easily understood by professional engineers, leaders, and individual investors. If the ROR is greater than MARR, the project is to be accepted; if it is less than MARR, the project is to be rejected. If ROR is equal to MARR, we are indifferent about the project.

When rate of return analysis is used to select the best project among mutually exclusive alternatives, the selection based on an individual project's ROR may not result in selection of the best project, as is shown earlier in this chapter. Therefore, the incremental cash flows between the larger and the smaller alternatives based on their first cost (known as Δ project), need to be calculated, and the Δ project or the extra project is to be evaluated. If the Δ project is economically justified, the project with the larger first cost is selected. Otherwise, we select the project with smaller first cost. If ROR is greater than or equal to MARR, then the Δ project is economically justified and the project with higher first cost is to be selected. If ROR is less than MARR, then the Δ project is not economically justified and the project with lower first cost is to be selected.

The comparison of mutually exclusive alternatives is not only based on evaluating incremental cash flows or the Δ project but also requires equivalent service lives. In comparing cost-based competing alternatives, do-nothing is not an option, whereas for revenue-based alternatives do-nothing becomes an option.

If there are more than two alternatives, we first place them in order of their first cost from the lowest to the highest. Then, we compare the first two alternatives. The survivor of the first two alternatives is compared with the third alternative. This process continues until one project has survived and that is the selected alternative.

The ROR approach to evaluating independent projects is to compute the ROR of each project, eliminate those projects that have a ROR that is less than MARR and accept the rest of them because they contribute to the growth or value of the firm as long as there is no budget limitation. If there is a budget constraint, then we use the PW approach as explained in Chapter 5.

There are complications with rate of return analysis due to the possibility of multiple rates of return and the underlying reinvestment rate assumption. A modified rate of return (MROR) has been presented to eliminate the ROR complications. The selection criteria are the same for both ROR and MROR.

The ROR equation in Excel spreadsheets is called Internal Rate of Return and denoted as IRR. To calculate ROR using Excel, we simply plug the project's cash flows into the IRR function.

Discussion Questions and Problems

- 7.1 Define rate of return.
- 7.2 What are the rate of return criteria for accepting or rejecting a project?

- 7.3** What are the differences between ROR and PW analysis in evaluating mutually exclusive alternatives?
- 7.4** What are the complications of rate of return analysis and how can they be resolved?
- 7.5** Heritage Airways is planning to expand its fleet by purchasing five B737s at a price of \$465 million. A local bank is providing a loan to Heritage in the amount of \$465 million for the purchase price of the five B737s, with an agreement that Heritage pay the bank \$80 million per year for the next 10 years. What is the rate of return to the local bank on this investment (loan)?
- 7.6** In Problem 7.5, assume that Heritage pays \$100 million at the end of 10 years in addition to the annual payments of \$80 million. What would be the rate of return on this loan to the local bank? If the bank's MARR is 14 percent, is this investment economically justified?
- 7.7** Avian Air Cargo is considering an investment of \$10 million to purchase facilities with life expectancy of 10 years and an annual operating cost of \$400,000 in year 1 with an increase of \$50,000 per year starting the second year through the life of the project. Avian Air Cargo anticipates a stream of \$2,500,000 in revenue per year from this project for 10 years. Compute the rate of return for this project and explain whether or not this project is economically justified. Assume MARR is 12 percent.
- 7.8** Legacy Airport has a new bond issue with face value of \$1,000, coupon rate of 8 percent, and 10 years to maturity. As an investor, if you pay \$850 to purchase this bond, what would be your rate of return on it, assuming interest is paid semi-annually?
- 7.9** DirectJet has a bond issue with face value of \$1,000, coupon rate of 8 percent, and 10 years to maturity. The interest is paid annually. You are considering purchasing this bond at \$1,050. Assuming your minimum acceptable rate of return is 6 percent compounded semi-annually, what is your ROR on this bond and is this investment economically justified?
- 7.10** DirectJet is considering leasing one of the following hangars for 10 years. Using ROR analysis and assuming a MARR of 12 percent, which hangar should be leased by the airline?

	Hangar A	Hangar B
First cost	−\$2,000,000	−\$3,000,000
Annual operating cost (AOC)	−\$500,000	−\$200,000
Deposit return	\$400,000	\$800,000
Life, years	10	10

- 7.11** Heritage Airways is considering a new route to Ho Chi Minh City, Vietnam or Sydney, Australia. The first cost, annual operating cost, expected annual revenue, and potential salvage value for both alternatives are presented below. Assume a MARR of 10 percent and a study period of 10 years. Which alternative route should Heritage Airways select?

	Ho Chi Minh City Route Vietnam	Sydney Route Australia
Initial investment (first cost)	−\$300 million	−\$500 million
Annual operating cost (AOC)	−\$50 million	−\$80 million
Annual revenue	\$100 million	\$180 million
Salvage value	\$30 million	\$50 million
Life, years	10	10

- 7.12** Legacy Airport is considering two mutually exclusive projects of purchasing a weather monitoring system with five-year life or a jet fuel service station with an approximately 10-year life. The cash flows for both alternatives are presented below. Legacy uses a MARR of 8 percent. Which alternative should Legacy select based on ROR analysis?

	Weather Monitoring System (WMS)	Jet Fuel Service Station (JFS)
Initial investment (first cost)	−\$300,000	−\$600,000
Annual operating cost	−\$40,000	−\$20,000
Annual revenue	\$120,000	\$180,000
Salvage value	\$20,000	\$50,000
Life, years	5	10

- 7.13** Mid-East Airfreight is considering two mutually exclusive projects for operational improvement. The cash flows for both alternatives are presented below. Mid-East Airfreight uses a MARR of 10 percent. Which alternative should Mid-East Airfreight select based on ROR analysis?

	Project C	Project D
First cost	−1,000,000	−6,000,000
Net annual cash flow	\$500,000	\$1,500,000
Salvage value	\$100,000	\$400,000
Life, years	3	9

- 7.14** Legacy Regional Airport is considering three independent projects with cash flows presented in the following table. The airport uses a MARR of 12 percent and has no budget constraint. Using ROR analysis, which of the independent projects, if any, should be selected to maximize the value of the airport?

	New Parking Garage	Expanding Existing Terminal	Building New Terminal
First cost	−\$120 million	−\$540 million	−\$780 million
Estimated annual operating cost (AOC)	−\$5 million	−\$25 million	−\$30 million
Estimated annual revenue	\$25 million	\$85 million	\$150 million
Life, years	20	25	25

7.15 Legacy International Airport is considering an investment of \$5 million in constructing facilities to provide jet fuel to the airlines, with a life expectancy of 10 years and annual operating cost of \$300,000 in year 1 with an increase of \$50,000 per year starting the second year through the life of the project. Legacy anticipates a stream of \$2.5 million revenue in year 1 with an annual increase of 8 percent per year through the end of 10 years. Compute the modified rate of return (MROR) for this project and explain whether or not this project is economically justified. Legacy is using a MARR of 10 percent.

Notes

- 1 Dania Saadi, *The National Business*, March 2016.

References

- Blank, L. and Tarquin, A. (2014). *Basics of Engineering Economy*, 2nd Edition. New York: McGraw-Hill.
- Newnan, D.G., Eschenbach, T.G., and Lavelle, J.P. (2012). *Engineering Economic Analysis*, 11th Edition. New York: Oxford University Press.

8

Public Sector Projects and Benefit to Cost Analysis

We are a mixed economy. We will remain a mixed economy. The public and private sector will continue to play a very important role. The private sector in our country has very ample scope and I am confident that India's entrepreneurs have the capacity, and the will to rise to the occasion.

Manmohan Singh



The transition from government ownership to the free market system has gone especially well for the air transport industry. From the start, the government's objective has been to create strong partnerships between the public and private sectors (PPP). Throughout the world, governments are turning over to private sectors everything from airports and airlines to public utilities (Goodman and Loveman, 1991). From time to time, outdated air navigation systems,¹ crumbling bridges, or shabby airport terminals remind us of the need for increased investment in the maintenance of transport infrastructure. In the United States, most airports request funding from the Federal Aviation Administration (FAA) for airport improvement projects or expanding the airport operations by, for instance, adding a new terminal or expanding the existing terminal to accommodate the enplanement growth. One of the decisive requirements of the FAA if they are to consider these requests is the presentation of the benefit/cost analysis of those projects. The techniques used in the previous chapters are commonly applied to projects in the private sector, for both profit and non-profit companies and institutions. Public sector projects are often evaluated based on the benefit/cost analysis in which the public benefits are compared to the costs of projects. As long as the public benefits exceed the costs to the public, or ratio of public benefits to public costs is equal to or greater than one, the public project is economically justified. The benefit/cost analysis, similar to the rate of return analysis, is based on evaluation of the

incremental cash flows when mutually exclusive projects are being evaluated. Present or annual worth values are utilized to calculate the benefit/cost ratios. If present worth values are applied, then equivalent service lives for mutually exclusive alternatives are required.

The focus of this chapter is to enable readers and practitioners to use the benefit/cost analysis technique to evaluate single public projects, compare mutually exclusive public alternatives, and select among independent public projects.

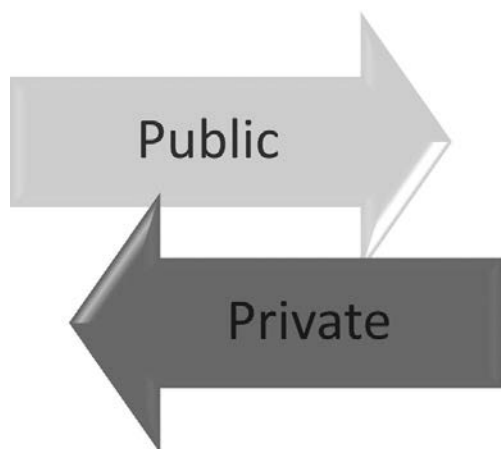
The following sections briefly address the calculation of benefit/cost ratios for a single public project using the present worth or annual worth values, the criteria to accept or to reject projects, and the comparison and selection of mutually exclusive alternatives and independent public projects. In addition, they describe the use of modified benefit/cost ratio.

- ♦ Public Sector versus Private Sector Projects
- ♦ Benefit/Cost Analysis of Single Projects
- ♦ Evaluation of Cost-based Mutually Exclusive Alternatives
- ♦ Evaluation of Benefit-based Mutually Exclusive Alternatives
- ♦ Evaluation of Independent Projects
- ♦ Modified Benefit and Cost Analysis
- ♦ Using Spreadsheets for Benefit/Cost Ratio Computation
- ♦ Summary
- ♦ Discussion Questions and Problems

Public Sector versus Private Sector Projects

In previous chapters, we have evaluated projects in the private sector and the criterion we used was the economic justification of the projects. In other words, as long as projects had positive present worth, we accepted them. Public projects are owned by the public sector, comprised of the federal, state, county, city, and other government agencies. The investments made by the public sector are usually developed to benefit the public in some way.

The majority of public sector investments are in infrastructure, for example, airports, highways, roads, bridges, dams, hospitals, police and fire departments, courts and prisons, public housing, and investment in clean energy projects. The Federal Aviation Administration (FAA) has provided clear and thorough guidelines to airport authorities for project-level benefit/cost analysis.² This guidance facilitates the computation of benefit/cost ratios for competing airport projects that can be used by FAA in its consideration for funding under the Airport Improvement Program



(AIP). All airport improvement project proposals requesting AIP discretionary funding must be shown to have total discounted benefits that exceed total discounted costs.

Other public sector investments are designed to support the leisure activities of the public such as parks and recreation, lakes, museums, sport arenas, convention centers, and libraries. The government also invests in projects to defend the country such as military bases, tanks, fighter jets, helicopters, submarines, and navy ships. Although these projects are very important, we cannot evaluate them based on maximizing worth. Generally, the public sector or government exists to serve the public and the investment decisions they make should reflect that goal. The goal of public sector projects is to provide services to the public at no profit. Public sector projects are often evaluated based on the benefit and cost comparison method in which the public benefits are compared to the costs of projects. As long as the public benefits exceed the costs to the public, or the ratio of public benefits to public costs is equal to or greater than one, the public project is economically justified.

Benefit/cost analysis has also recently been used by governments for consideration of regulatory proposals that would be costly to implement but that would have potentially large benefits to society. Examples of these regulatory actions are the Clean Air Act and the Clean Water Act.³

The characteristics of public projects are as follows:

- ◆ **Size of public projects.** Most public projects, such as airports, highways, and public transportation, require significant initial investment, possibly distributed over several years. In recent years, we have seen accelerated partnerships between public entities and private companies, as funding for large public projects becomes more challenging.
- ◆ **Life of public projects.** Most public projects are considered permanent, such as airports, bridges, dams, and highways.
- ◆ **The investments made by the public sector are usually developed to benefit the public in some way.** While the goal of the private sector is to maximize the company's worth, the goal of the public project is to provide services to the public at minimized cost and no profit.

A report published in 2013 found that worldwide, \$57-trillion in new infrastructure investment would be required in the period up to 2030, simply to keep up with projected GDP growth. This estimate suggests a requirement of \$3.2 trillion investment a year.

McKinsey Global Institute, 2013

Some investments made by the public sector may positively impact one group of people while negatively impacting other groups. The negative consequences of a public project are called disbenefits. Disbenefit is defined as expected economic disadvantages or undesirable and negative consequences of public projects to the public. These economic disadvantages are usually estimated in monetary units.

Let's look at the benefits and disbenefits of construction of an international airport as a public project.

Table 8.1 Economic Impacts (Benefits) of Commercial Airports in the US in 2013

Impact Measure	On-Airport	CIP	Visitor	US Total
Employment	3,490,640	176,160	5,929,540	9,596,340
Payroll	\$177,019 m	\$5,114 m	\$175,727 m	\$357,860 m
Economic output	\$617,630 m	\$31,600 m	\$485,949 m	\$1,135,179 m

Source: ACI-NA 2014

The major benefits of an international airport are as follows:

- ♦ Vast economic impact. Commercial airports are often considered the economic engines that drive the local, regional, and national economies. Airports are significant generators of economic activities including creating jobs, payroll, and total output for the economy. Table 8.1 presents total impacts of the 485 commercial airports, broken down into on-airport activities, capital improvement projects (CIP), and visitor categories. Total impacts show the combined effects of the direct and multiplier impact. These economic impacts are a considerable contribution to the national economy. In total, this analysis found that the commercial airports in the US create an annual payroll of \$358 billion, produce a \$1.135 trillion annual output, and support 9.6 million jobs.
- ♦ Access to the global air transportation network. Airports help to accommodate the travel needs of business and leisure visitors in the country and beyond. With an airport, a community has access to virtually every major town in the country and, broadly speaking, to all of the airports in the world. The community includes businesses, industry, and the general public.
- ♦ Airport development is considered a catalyst for business growth. The construction of a new airport increases the value added by manufacturers, retail trades, and the wholesale trade, and also increases the volume of the service business.
- ♦ Airports are considered industry magnets. Airports are commonly found in the center of commercial and industrial development. Some industries locate at or near airports because they have a direct need for access to air transport facilities. New industries not only provide more employment and improve the community's economy, but also serve to increase tax revenues for local governments.
- ♦ Retaining existing industry. Air transportation has become increasingly more important to local industry representatives by helping them retain and improve their competitive positions in local, national, and international markets.
- ♦ Commercial airports drive the new urban form. Airports are no longer just aviation infrastructures. They are becoming multimodal, multifunctional enterprises generating significant commercial development within and well beyond their boundaries. Major airports are expanding to include shopping

- mall, entertainment amenities, theme restaurants, electronic shops, internet cafés, and other business development to form an airport city.
- ♦ Airports play an integral role for cargo logistics and supply chain business. Global supply chains are becoming firms' new competitive edge. Speed, reliability, and agility are becoming as important as price to compete in many high-value sectors.

The major disbenefits of an international airport are as follows:

- ♦ Noise. Noise from aircraft and from traffic going to and from airports is probably the most obvious environmental impact of the airport because it is easily perceived and annoying, especially when this occurs frequently. Aircraft noise is generated by both the engine and the airframe and is most evident during landing and take-off and under frequently used flight paths.
- ♦ Air pollution. Airports generate air pollution through combustion of jet fuel, inefficiency of engine on approach, fuel dumping by aircraft releases, vehicles traveling to and from the airport, construction of airport-related projects that can lead to dust, and emissions from asphalt laying. These activities contribute to emission of carbon monoxide, nitrogen oxide, nitrogen dioxide, and ozone.
- ♦ Climate changes. The emission of carbon monoxide, nitrogen oxide, nitrogen dioxide, and ozone contributes to the concentration of greenhouse gases, and leads to higher levels of global warming or climate change.
- ♦ Biodiversity impact (impacts on plants and animals). Habitat loss occurs when previously "green" areas are built on, destroying the habitats of the plants and animals that lived there.
- ♦ Traffic. Traffic can cause congestion, severance, and environmental problems such as noise and air pollution.
- ♦ Community impact. House values surrounding airports may fall due to noise, air pollution, landscape, and other impacts. The number of children attending schools in the area could fall, as could the number of people participating in community activities.

In contrast to airport benefits, which can be converted into monetary value relatively easily, the disbenefits may not be easily estimated and converted to monetary units.

There are other examples where disbenefit is clearly estimable. For example, a city official issues a \$50 million, 10-year bond with coupon rate of 5 percent paid annually to build a new high school to accommodate the growing number of new middle school graduates and high school-age children of families who are moving to this city. The city increases property taxes per year in equal amounts for the next 10 years so that it can both pay off the interest every year for 10 years and pay off the bond face value of \$50 million to bond holders in 10 years. In this example, the benefits are received by high school children and their families, teachers, maintenance crew, etc. But the disbenefits are levied on property owners, of whom some may not have any kids in high school.

While the benefits are not as clearly estimable, the disbenefits can be clearly computed. If the city is able to earn 5 percent on its investment, the annual disbenefit

comes to \$6,475,000, which is the AW of the \$50 million today at 5 percent interest rate as calculated below.

$$\begin{aligned} AW &= PW (A/P, 5\%, 10) \\ AW &= \$50,000,000 (0.1295) \\ AW &= \$6,475,000 \end{aligned}$$

Benefit/Cost Analysis of Single Projects

Public sector projects are often evaluated based on the benefit/cost ratio in which the public benefits are compared to the costs of the project to determine whether the benefits of a proposed project justify its costs.

Effective Practices for Preparing Airport Improvement Program Benefits-Cost Analysis defines benefit/cost analysis as follows:

Benefit-Cost Analysis is a tool to aid agencies in project selection and prioritization. At its core, it is a computational tool to determine whether an investment will generate benefits that will exceed its costs. This analysis is accomplished by expressing all current and future benefits and costs on an equivalent basis, which is their 'present value' (PV). A project is economically efficient if it has a positive 'Net Present Value' (calculated as the PV of benefits minus the PV of costs), and hence also a 'Benefit-Cost Ratio' exceeding one.⁴

There are a number of publications by the FAA and Transportation Research Board (TRB) that define and describe best practices and techniques to assess project benefits by airports. Since some types of benefits are difficult to convert into monetary values, the FAA application process for airport capacity funding allows other techniques in addition to benefit/cost analysis.

There are two types of benefit/cost ratio: the Conventional Benefit/Cost ratio which will be discussed in this section and the Modified Benefit/Cost ratio which will be discussed later in this chapter. When we use benefit/cost ratio, we are referring to the conventional benefit/cost ratio. PW, AW, or FW values are utilized to calculate the benefit/cost ratios. In other words, benefits, disbenefits, and all costs must be converted to a common equivalent worth (present worth, annual worth, or future worth) at some interest rate (MARR), and plugged into the equation below to calculate the B/C ratio.

$$\begin{aligned} \text{Conventional B/C Ratio} &= \frac{\text{Benefits} - \text{Disbenefits}}{\text{First cost} + \text{AOC} - \text{Salvage value (Savings)}} \\ \text{Conventional B/C Ratio} &= \frac{B - D}{P + \text{AOC} - S} \end{aligned}$$

Where:

B = Benefits

D = Disbenefits

P = First cost, or initial investment

AOC = Annual operating cost

S = Salvage value or savings

Table 8.2 Accept and Reject Criteria for Benefit/Cost Ratio

B/C Ratio	Selection Criteria
$B/C > 1$	Accept the project, because it is economically justified. In other words, it produces benefits that are greater than costs.
$B/C = 1$	Indifference about the project. It is still economically justified. It produces benefits that are equal to costs.
$B/C < 1$	Reject the project, because it is <u>not</u> economically justified. In other words, it produces benefits that are <u>less than</u> costs.

If present worth values are applied, then equivalent service lives for mutually exclusive alternatives are required. The benefit/cost analysis, similar to the rate of return analysis, is based on evaluation of the incremental cash flows when mutually exclusive projects are being evaluated. *In this chapter, and only this chapter, costs will have a positive sign.*

The criteria to select or to reject a project are summarized in Table 8.2.

Example 8.1

Legacy Regional Airport (LRA) is owned and operated by the Legacy Airport Authority (LAA). The airport is conveniently located in the southwest of the United States and offers commercial services all over the United States and to a few foreign destinations. Due to a significant increase in the number of flights and high passenger traffic, the LAA is considering a proposal to build a new rental car complex. The initial investment is estimated to be \$200 million with an annual operating cost of \$5 million. This is a 20-year project with annual benefit (revenue from car rental companies) of \$35 million,

Seattle Monorail—Benefit/Cost Analysis in Practice

The Elevated Transportation Company developed plans for a monorail project in and near downtown Seattle. The new “Green Line” would extend 14 miles and serve Seattle Center, downtown, and the stadium district. The project is intended to enhance transit options in a congested area. The predicted benefits include faster, more frequent, and more reliable transit service; savings in automobile operating and parking costs for drivers who switch to the monorail; and reduction in accidents. Three groups stand to benefit from the monorail project: transit riders, former drivers who switch to the monorail, and continuing auto users who may experience a slight decrease in traffic delays because bus and car trips have moved off the road.

Forecasted Benefits from Monorail Project, 2020

Benefit Type	Benefit Values Millions
Value of travel time savings	\$77.1
Parking savings	\$28.7
Reduced auto operating/ ownership costs	\$11.2
Reliability	\$7.7
Road capacity for drivers	\$4.6
Reduction in bus-related accidents	\$3.7
Reduction in auto-related accidents	\$2.6
2020 Benefits	\$135.6

Brian Slack, *Cost/Benefit Analysis In Practice, The Geography of Transportation Systems* (2002).

disbenefit to the public of \$2 million per year, and a salvage value of \$20 million. Using conventional benefit/cost ratio with MARR of 8 percent, is this project economically justified?

In order to calculate the B/C ratio, we need to convert all costs, benefits, and disbenefits to a common equivalent worth. Because most of the information is provided in annual worth, we convert the rest to annual worth values.

AW of Benefits = \$35 million

AW of Disbenefits = \$2 million

AW of AOC = \$5 million

AW of Initial Investment (P) = \$200 (*A/P*, 8%, 20) = \$200 (0.10185) = \$20.37 million

AW of Salvage = \$20 (*A/F*, 8%, 20) = \$20 (0.02185) = \$0.44 million

$$B/C = \frac{\$35 - \$2}{\$20.37 + \$5 - \$0.44}$$

$$B/C = \frac{\$33}{\$24.93}$$

$$B/C = 1.32 > 1$$

Because the B/C ratio of the car rental complex project is greater than one, it is economically justified.

Evaluation of Cost-based Mutually Exclusive Alternatives

Cost-based competing alternatives have no direct benefits. The implied benefits come from cost savings to the public when we compare the alternatives. In other words, the public saves costs for using one alternative relative to the other one and those costs savings are considered an implied benefit.

When we are comparing mutually exclusive alternatives, the problem of size differential reemerges. Projects with a larger initial investment may have a greater PW value, but have a lower B/C ratio than projects with a smaller initial investment. Similar to ROR analysis in Chapter 7, to overcome this problem we are evaluating mutually exclusive alternatives based on the incremental cash flows or creating a Δ project and then evaluating it. If there are more than two alternatives, then the survivor of the first two alternatives is compared to the next alternative in line. This process continues until one project survives and that project is the selected alternative. If alternatives have different lives and we use PW values to calculate the B/C ratio, then equal service lives for comparison are required. Although most public projects are considered permanent and therefore generally have equal lives.

To select among the cost-based mutually exclusive alternatives using conventional B/C ratio, we use the following procedures:

- ♦ Arrange alternatives in order of their total cost expressed in equivalent worth.
- ♦ Compare two alternatives at a time.

- ♦ Create the Δ project based on incremental Cash flows.
- ♦ Calculate B/C ratio of the Δ project ($\Delta_{B/C}$)
- ♦ Based on the Δ project selection criteria in Table 8.3, select the best alternative.
- ♦ Finally, for cost-based mutually exclusive alternatives, Do-nothing is not an option since there is no direct benefit is associated with the alternatives.

The Δ project selection criteria are presented in Table 8.3.

Example 8.2

Legacy Regional Airport (LRA) is considering solar and power line alternatives to light the airport. The costs associated with each alternative are shown in Table 8.4. Use conventional B/C analysis with AW values to determine which alternative should be selected. LRA uses a MARR of 10 percent.

Since these alternatives are cost-based projects, do-nothing is not an option. The benefits come from the AOC saving. Table 8.5 presents the incremental cash flows for the Δ project.

$$AW \text{ of } P = \$10 (A/P, 10\%, 10) = \$10 (0.16275) = \$1.6275$$

$$AW \text{ of Benefits} = \$3$$

$$\text{Modified B/C} = \$5 (A/F, 10\%, 10)$$

$$\text{Modified B/C} = \$5 (0.06275) = \$0.3136$$

Table 8.3 Accept and Reject Criteria for the Δ Project

$\Delta B/C$ Ratio	Selection Criteria
$\Delta B/C > 1$	Since Δ project is economically justified, select the alternative with the larger total cost.
$\Delta B/C = 1$	Indifferent about the Δ project. Indifferent about the alternatives. You may select any of the two alternatives.
$\Delta B/C < 1$	Since the Δ project is not economically justified, select the alternative with the smaller total cost.

Table 8.4 Cash Flows for Both Alternatives

	Power Line (PL)	Solar Cell (SC)
Initial investment (first cost)	\$10 million	\$20 million
Annual operating cost (AOC) to public	\$5 million	\$2 million
Salvage value	\$0	\$5 million
Estimated life, years	10	10

Table 8.5 Cash Flows for Both Alternatives and Incremental Cash Flows for the Δ Project

	Power Line (PL)	Solar Cell (SC)	Δ_{SC-PL}
Initial investment, P	\$10 million	\$20 million	\$10 million
Annual operating cost (AOC)	\$5 million	\$2 million	\$3 million (Benefits)
Salvage value	\$0	\$5 million	\$5 million
Estimated life, years	10	10	10

$$\Delta B/C = \frac{\Delta B - \Delta D}{\Delta TC - \Delta S}$$

$$\Delta B/C = \frac{\$3 - \$0}{\$1.6275 - \$0.3136}$$

$$\Delta B/C = \frac{\$3}{\$1.314}$$

$$\Delta B/C = 2.28 > 1$$

Because the $\Delta B/C$ ratio is greater than one, the Δ project is economically justified and therefore select the Solar Power alternative with the largest total cost.

Evaluation of Benefit-based Mutually Exclusive Alternatives

The benefit-based projects are projects with direct benefits. That means the project will provide some benefits to the public. To evaluate benefit-based mutually exclusive alternatives using a B/C ratio, we follow the same procedure discussed for evaluation of cost-based mutually exclusive alternatives except that Do-nothing is now an option. First, the project with the least total cost is compared with Do-nothing. The survivor of these two alternatives is compared to the next alternative in line. This process continues until the best alternative is selected which is the surviving alternative.

Example 8.3

Legacy Regional Airport (LRA) is owned and operated by the Legacy Airport Authority (LAA). The airport is conveniently located in the southwest of the United States offering commercial services all over the United States and to a few foreign destinations. Due to significant increase in the number of flights and high passenger traffic, the LAA is considering three mutually exclusive alternatives shown in Table 8.6 (in millions) to accommodate the increasing traffic growth. Using conventional B/C ratio with MARR of 8 percent, which project should LRA select?

Because projects have different lives, it is best to use AW values and therefore we first convert all cash flows to AW values and present them in Table 8.7.

Table 8.6 Legacy Regional Airport Mutually Exclusive Projects

	New Economy Parking Garage (NEPG)	Concourse A Expansion (CAE)	Concourse B Expansion (CBE)
First cost	\$200	\$500	\$300
Estimated annual operating cost (AOC)	\$10	\$20	\$15
Benefits (B), estimated annual revenue	\$50	\$100	\$85
Disbenefit (D), incurs at the time zero	\$20	\$10	\$10
Salvage value (S)	\$20	\$50	\$30
Life, years	15	25	20

Table 8.7 Legacy Regional Airport Mutually Exclusive Projects in AW Values (in millions)

	New Economy Parking Garage (NEPG)	Concourse A Expansion (CAE)	Concourse B Expansion (CBE)
AW of first cost	\$23.37	\$46.84	\$30.56
AW of AOC	\$10	\$20	\$15
AW of benefits	\$50	\$100	\$85
AW of disbenefits	\$2.34	\$0.94	\$1.02
AW of salvage value	\$0.74	\$0.68	\$0.66
Life, years	10	10	10

AW of TC for NEPG = \$200 (*A/P*, 8%, 15) = \$200 (0.11683) = \$23.37

AW of TC for CAE = \$500 (*A/P*, 8%, 25) = \$500 (0.09368) = \$46.84

AW of TC for CBE = \$300 (*A/P*, 8%, 20) = \$300 (0.10185) = \$30.56

AW of D for NEPG = \$20 (*A/P*, 8%, 15) = \$20 (0.11683) = \$2.34

AW of D for CAE = \$10 (*A/P*, 8%, 25) = \$10 (0.09368) = \$0.94

AW of D for CBE = \$10 (*A/P*, 8%, 20) = \$10 (0.10185) = \$1.02

AW of S for NEPG = \$20 (*A/F*, 8%, 15) = \$20 (0.03683) = \$0.74

AW of S for CAE = \$50 (*A/F*, 8%, 25) = \$50 (0.01368) = \$0.68

AW of S for CBE = \$30 (*A/F*, 8%, 20) = \$30 (0.02185) = \$0.66

Because all cash flows are converted to AW values and AW values are used to calculate conventional B/C ratios, then the lives of the projects become irrelevant and therefore are eliminated.

Because these projects are benefit-based alternatives, Do-nothing becomes an option. First we calculate total costs of all projects and rearrange them in order of

increasing total costs in AW values and we add Do-nothing as the first alternative. Then we compare the first two alternatives. The survivor of the first two alternatives is compared to the next alternative in line and the survivor of those two will be compared to the last alternative. The project which survived in the last comparison is the selected alternative. We now rearrange projects in Table 8.8 and add Do-nothing as the first alternative.

$$\Delta \text{ Project} = \text{NEPG} - \text{DN}$$

Table 8.9 presents the cash flows for Do-nothing and for the New Economy Parking Garage, and the incremental cash flows for the Δ Project.

$$\Delta B/C = \frac{\$50 - \$2.34}{\$33.37 - \$0.74}$$

$$\Delta B/C = \frac{\$47.66}{\$32.63}$$

$$\Delta B/C = 1.46 > 1$$

Table 8.8 Legacy Regional Airport Projects in Order of Their Total Costs (in millions)

	Do-nothing (DN)	New Economy Parking Garage (NEPG)	Concourse B Expansion (CBE)	Concourse A Expansion (CAE)
AW of TC	\$0	\$33.37	\$45.56	\$66.84
AW of benefits	\$0	\$50	\$85	\$100
AW of disbenefits	\$0	\$2.34	\$1.02	\$.94
AW of salvage value	\$0	\$0.74	\$0.66	\$0.68
Life, years	10	10	10	10

Table 8.9 Comparison of Do-nothing with New Economy Parking Garage (in millions)

	Do-nothing (DN)	New Economy Parking Garage (NEPG)	Δ Project NEPG – DN
AW of TC	\$0	\$33.37	\$33.37
AW of benefits	\$0	\$50	\$50
AW of disbenefits	\$0	\$2.34	\$2.34
AW of salvage value	\$0	\$0.74	\$0.74
Life, years	10	10	10

Because $\Delta B/C > 1$, the Δ project is economically justified. Therefore the Do-nothing alternative is eliminated, and the New Economy Parking Garage project survives. Now, we compare the New Economy Parking Garage with the Concourse B Expansion.

$$\Delta \text{ Project} = \text{CBE} - \text{NEPG}$$

Table 8.10 presents the cash flows for the New Economy Parking Garage and the Concourse B Expansion, and incremental cash flows for the Δ Project.

$$\Delta B/C = \frac{\$35 - (-\$1.32)}{\$12.19 - (-\$0.08)}$$

$$\Delta B/C = \frac{\$36.32}{\$12.27}$$

$$\Delta B/C = 2.96 > 1$$

Because $\Delta B/C > 1$, the Δ project is economically justified. The New Economy Parking Garage is eliminated, and the Concourse B Expansion survives. Now, we compare the Concourse B Expansion project with the Concourse A Expansion.

$$\Delta \text{ Project} = \text{CAE} - \text{CBE}$$

Table 8.11 presents the cash flows for the Concourse B Expansion, the Concourse A Expansion, and the Δ Project.

$$\Delta B/C = \frac{\$15 - (-\$0.08)}{\$21.28 - \$0.02}$$

$$\Delta B/C = \frac{\$15.08}{\$21.26}$$

$$\Delta B/C = 0.71 < 1$$

Table 8.10 Comparison of New Economy Parking Garage and Concourse B Expansion (in millions)

	New Economy Parking Garage (NEPG)	Concourse B Expansion (CBE)	Δ Project CBE – NEPG
AW of TC	\$33.37	\$45.56	\$12.19
AW of benefits	\$50	\$85	\$35
AW of disbenefits	\$2.34	\$1.02	–\$1.32
AW of salvage value	\$0.74	\$0.66	–\$0.08
Life, years	10	10	10

Table 8.11 Comparison of Concourse B Expansion and Concourse A Expansion (in millions)

	Concourse B Expansion (CBE)	Concourse A Expansion (CAE)	Δ Project CAE – CBE
AW of TC	\$45.56	\$66.84	\$21.28
AW of benefits	\$85	\$100	\$15
AW of disbenefits	\$1.02	\$0.94	–\$0.08
AW of salvage value	\$0.66	\$0.68	\$0.02
Life, years	10	10	10

Because $\Delta B/C < 1$, the Δ project is not economically justified. Therefore, the Concourse A Expansion is eliminated, and the Concourse B Expansion survives. LAA should select the Concourse B Expansion for Legacy Regional Airport.

Evaluation of Independent Projects

The B/C approach to evaluate independent projects is to compute the B/C ratio of each project, eliminate those projects that have a B/C ratio that is less than one and accept the rest of them because they contribute to the growth of the public institution as long as there is no budget limitation. If there is a budget constraint, then we use PW approach as explained in Chapter 5. Examples of independent projects for an airport include: construction of a new terminal, concourse expansion, building a new runway, investment in weather monitoring equipment, building a new concession area, constructing a food court, building a new parking garage, airport flight information screen, building a rental car complex, hangar expansion, etc.

Example 8.4

Legacy Airport Authority (LAA) is entertaining four independent proposals presented in Table 8.12 for Legacy Regional Airport. LAA uses a MARR of 10 percent and has no budget constraint. Using B/C ratio and PW values, which projects in Table 8.12 are economically justified and should be selected to maximize the services to the public?

First, calculate the conventional B/C ratios of all alternatives using PW values at 10 percent MARR.

B/C ratio of New Parking Garage:

$$PW(P) = \$100$$

$$PW(AOC) = \$5 (P/A, 10\%, 20) = \$5 (8.5136) = \$42.57$$

$$PW(B) = \$30 (P/A, 10\%, 20) = \$30 (8.5136) = \$255.41$$

$$PW(D) = \$5$$

$$PW(S) = \$10 (P/F, 10\%, 20) = \$10 (0.1486) = \$1.49$$

Table 8.12 Legacy Regional Airport Independent Projects (in millions)

	New Parking Garage	Flight Information Screen	Gate Expansion	Hangar Expansion
First cost (P)	\$100	\$2	\$40	\$8
Annual operating cost (AOC)	\$5	\$0.2	\$2	\$0.4
Annual benefits (B)	\$30	\$0.8	\$4	\$3
Disbenefits (D)	\$5	\$0	\$1	\$1
Salvage value (S)	\$10	\$0.2	\$2	\$0.5
Life, years	20	6	25	10

$$B/C = \frac{\$255.41 - \$5}{\$100 + \$42.57 - \$1.49}$$

$$B/C = \frac{\$250.41}{\$141.08}$$

$$B/C = 1.77 > 1$$

B/C ratio of Flight Information Screen:

$$PW(P) = \$2$$

$$PW(AOC) = \$0.2 (P/A, 10\%, 6) = \$0.2 (4.3553) = \$0.87$$

$$PW(B) = \$0.8 (P/A, 10\%, 6) = \$0.8 (4.3553) = \$3.48$$

$$PW(D) = \$0$$

$$PW(S) = \$0.2 (P/F, 10\%, 6) = \$0.2 (0.5645) = \$0.11$$

$$B/C = \frac{\$3.48 - \$0}{\$2 + \$0.87 - \$0.11}$$

$$B/C = \frac{\$3.48}{\$2.76}$$

$$B/C = 1.26 > 1$$

B/C ratio of Gate Expansion:

$$PW(P) = \$40$$

$$PW(AOC) = \$2 (P/A, 10\%, 25)$$

$$PW(AOC) = \$2 (9.0770) = \$18.15$$

$$PW(B) = \$4 (P/A, 10\%, 25) = \$4 (9.0770) = \$36.31$$

$$PW(D) = \$1$$

$$PW(S) = \$2 (P/F, 10\%, 25) = \$2 (0.0923) = \$0.18$$

$$B/C = \frac{\$36.31 - \$1}{\$40 + \$18.15 - \$0.18}$$

$$B/C = \frac{\$35.31}{\$57.97}$$

$$B/C = 0.61 < 1$$

B/C ratio of Hangar Expansion:

$$PW(P) = \$8$$

$$PW(AOC) = \$0.4 (P/A, 10\%, 10) = \$0.4 (6.1446) = \$2.46$$

$$PW(B) = \$3 (P/A, 10\%, 10) = \$3 (6.1446) = \$18.43$$

$$PW(D) = \$1$$

$$PW(S) = \$0.5 (P/F, 10\%, 10) = \$0.5 (0.3855) = \$0.19$$

$$B/C = \frac{\$18.43 - \$1}{\$8 + \$2.46 - \$0.19}$$

$$B/C = \frac{\$17.43}{\$10.27}$$

$$B/C = 1.70 > 1$$

Out of four proposals under review by Legacy Airport Authority, the New Parking Garage, Flight Information Screen, and Hangar Expansion for Legacy Regional Airport have B/C ratios that are greater than one and therefore they are economically justified and the Port Authority should undertake those projects. The Gate Expansion has a B/C ratio of less than one, so it is not economically justified and therefore should be eliminated from consideration.

Modified Benefit and Cost Analysis

Modified benefit and cost analysis is another technique to evaluate public projects. Modified B/C ratio is the profitability index of a public project and indirectly measures the return of a public project to the public. Modified B/C ratio is calculated by a ratio where the denominator consists of only the initial investment and the salvage value associated with the initial investment (first cost) and all other cash flows are placed in the numerator. In other words, modified B/C ratio compares the net benefits to initial investment as shown in the equation below.

$$\text{Modified B/C} = \frac{\text{Net Benefits}}{\text{Initial Investment}}$$

Where:

Net benefits = benefits – disbenefits – operating cost + savings

Initial investment = first cost + non-recurring cost – salvage value

Plugging net benefit details into the above equation results in the following equation which is the expanded formula of the above equation.

$$\text{Modified B/C} = \frac{B - D - AOC + \text{Savings}}{P - \text{Salvage Value}}$$

This equation is also used to determine the rate of return of public investments. It is often used to rank the projects in order of highest to lowest modified benefit/cost ratio. Public institutions usually fund the project with highest modified benefit/cost ratio first and then fund the next highest, and next until the entire budget is utilized.

Although modified B/C ratio yields a different value than conventional B/C ratio, it does not change the decision to accept or reject a project. In other words, both methods of conventional and modified B/C ratios consistently provide the same answer to accept or reject a project. The accept or reject criteria for the modified B/C ratio are the same as for the conventional B/C ratio.

Example 8.5

US Airports Authority (USAA) owns and operates three airports in the southeast of the United States. The airports are conveniently located in a triangle format. Gator International Airport (GIA) is the largest airport, with enplanement of 60 million per year, and offers commercial services to major cities within the central and eastern parts of the United States and major cities abroad. Gator Regional Airport (GRA), with enplanement of 30 million, is located approximately 100 miles southwest of GIA, and offers domestic commercial services to the southwest, west, and northwest of the United States. Gator Reliever Airport (GRA) is located approximately 30 miles from GIA and offers commercial services to all cities in the southeast, east, and northeast part of the United States.

USAA has received a proposal for the construction of a new food court for Gator International Airport. The information about this proposal is summarized in Table 8.13. USAA uses a MARR of 12 percent. Use both conventional and modified B/C ratios with PW values to determine if the food court project is economically justified.

Conventional B/C ratio:

$$PW \text{ of First cost} = \$6,000,000 + \$1,500,000 (P/F, 12\%, 5)$$

$$PW \text{ of First cost} = \$6,000,000 + \$1,500,000 (0.5674)$$

$$PW \text{ of First cost} = \$6,851,100$$

Table 8.13 Food Court Project for Gator International Airport

First cost	\$6,000,000 now and \$1,500,000 upgrade cost five years from now
AOC	\$500,000 in the first year with increase of \$100,000 per year starting the second year
Annual benefits (revenue)	\$3,000,000
Disbenefits at the time zero	\$1,000,000
Salvage value	\$1,200,000
Life, years	10

$$PW \text{ of } AOC = \$500,000 (P/A, 12\%, 10) + \$100,000 (P/G, 12\%, 10)$$

$$PW \text{ of } AOC = \$500,000 (5.6502) + \$100,000 (20.2541)$$

$$PW \text{ of } AOC = \$2,852,100 + \$2,025,410$$

$$PW \text{ of } AOC = \$4,877,510$$

$$PW \text{ of } Benefits = \$3,000,000 (P/A, 12\%, 10)$$

$$PW \text{ of } Benefits = \$3,000,000 (5.6502)$$

$$PW \text{ of } Benefits = \$16,950,600$$

$$PW \text{ of } Disbenefits = \$1,000,000$$

$$PW \text{ Salvage value} = \$1,200,000 (P/F, 12\%, 10)$$

$$PW \text{ Salvage value} = \$1,200,000 (0.3220)$$

$$PW \text{ Salvage value} = \$386,400$$

$$\text{Conventional } B/C = \frac{\$16,950,000 - \$1,000,000}{\$6,851,100 + \$4,877,510 - \$386,400}$$

$$\text{Conventional } B/C = \frac{\$15,950,000}{\$11,342,210}$$

$$\text{Conventional } B/C = 1.41 > 1$$

Modified B/C Ratio:

$$\text{Modified } B/C = \frac{\$16,950,000 - \$1,000,000 - \$4,877,510}{\$6,851,100 - \$386,400}$$

$$\text{Modified } B/C = \frac{\$11,072,490}{\$6,464,700}$$

$$\text{Modified } B/C = 1.71 > 1$$

Both conventional B/C ratio and modified B/C ratio are greater than one indicating that the food court project is economically justified and should be undertaken by GIA.

Using Spreadsheets for Benefit/Cost Ratio Computation

This section presents how to run an equation in Excel spreadsheets to calculate conventional or modified B/C ratios using PW values. Unlike the previous methods, Excel does not have a financial function to compute B/C ratio. We can, however, insert the equation shown below into an Excel spreadsheet to compute conventional B/C ratio.

$$B/C = \frac{PW \text{ of } B - PW \text{ of } D}{PW \text{ of } P + PW \text{ of } AOC - PW \text{ of } S}$$

Figure 8.1 Benefit/Cost Computation in Excel Spreadsheets

	A	B	C
1	A	B	C
2			
3	First cost	\$2,000,000	
4	Salvage value	\$400,000	
5	Life/ years	10	
6	MARR	10%	
7			
8	Year	AOC	Benefits - Disbenefits
9	1	100,000	500,000
10	2	100,000	500,000
11	3	100,000	500,000
12	4	150,000	800,000
13	5	150,000	800,000
14	6	150,000	800,000
15	7	150,000	800,000
16	8	100,000	500,000
17	9	100,000	500,000
18	10	100,000	500,000
19			
20	Conv. B/C Ratio	1.31	=NPV(10%,C9:C18)/(B3+NPV(10%,B9:B18)- PV(10%,10,,B4))
21			
22	Mod. B/C Ratio	1.42	=(NPV(10%,C9:C18)-NPV(10%,B9:B18))/(B3- PV(10%,10,,B4))
23			

We can also insert this equation into an Excel spreadsheet to compute modified B/C ratio:

$$B/C = \frac{PW \text{ of } B - PW \text{ of } D - PW \text{ of } AOC + PW \text{ of } Savings}{PW \text{ of } P - PW \text{ of } S}$$

Example 8.6

US Airport Authority (USAA) has received a proposal for a Hangar Expansion at Gator Regional Airport. The cash flows associated with the Hangar Expansion are as follows:

First cost is \$2,000,000. AOCs are \$100,000 per year for the first three years, \$150,000 per year for years 4–7, and \$100,000 per year for years 8–10. The benefits less disbenefits are \$500,000 per year for first three years, \$800,000 per year for years 4–7, and \$500,000 per year for years 8–10. Salvage value is \$400,000, life is 10 years, and MARR is 10 percent. Using Excel spreadsheets, compute both conventional and modified B/C ratios and determine whether or not the project is economically justified.

Figure 8.1 shows the Excel spreadsheet solutions for both conventional and modified B/C ratios.

The conventional B/C ratio equation shown in cell C20 has been actually entered in B20 and the solution appears in cell B20. Similarly, the modified B/C ratio equation shown in cell C22 has been actually entered in B22 and the solution appears in cell B21. Both conventional B/C ratio and modified B/C ratio are greater than one and therefore the Hangar Expansion is economically justified.

Summary

Public projects are owned by the public sector comprised of the federal, state, county, city, and other government agencies. The investments made by the public sector are

usually developed to benefit the public in some way. Generally, the public sector or the government exists to serve the public and the investment decisions it makes should reflect that goal. The goal of public sector projects is to provide services to the public at no profit. Public sector projects are often evaluated based on the benefit and cost comparison method in which the public benefits are compared to the costs of projects. As long as the public benefits exceed the costs to the public, or ratio of public benefits to public costs is equal to or greater than one, the public project is economically justified.

Benefits, disbenefits, and all costs must be estimated accurately and be converted to a common equivalent worth (PW, AW, or FW) at some interest rate (minimum acceptable rate of return), and be plugged into the conventional or modified benefit/cost ratio to evaluate a single project. If B/C ratio is greater or equal to one, the project is to be selected since the benefits exceed the costs of the project. If B/C ratio is less than one, then the project is to be rejected since the costs of the project exceed its benefits.

The comparison of mutually exclusive alternatives is based on the incremental cash flows of the larger total cost project less cash flows of the project with the smaller total cost. All incremental cash flows must be converted to a common equivalent worth and equal service lives enforced if PW values are utilized.

Cost-based mutually exclusive alternatives do not have direct benefits. The implied benefits come from a cost savings to the public when we compare the alternatives. In other words, the public saves costs for using one alternative relative to the other one and those costs savings are considered as implied benefits. When cost-based mutually exclusive alternatives are being evaluated, do-nothing is not an option since there is no direct benefit associated with the alternatives. If there are more than two alternatives, then the survivor of the first two alternatives is compared to the next alternative in line. This process continues until one project survives and that project is the selected alternative.

To evaluate benefit-based mutually exclusive alternatives using B/C ratio, we follow the same procedure discussed for evaluation of cost-based mutually exclusive alternatives except that Do-nothing is now an option. First, the project with the least total cost is compared with Do-nothing. The survivor of these two alternatives is compared to the next alternative in line. This process continues until the best alternative is selected, which is the surviving alternative.

If $\Delta B/C$ ratio is greater or equal to one, the project with higher total cost is to be selected. If $\Delta B/C$ ratio is less than one, the project with lower total cost is to be selected.

Modified B/C ratio is calculated like a profitability index, where the denominator is only the initial investment and all other cash flows are placed in the numerator. In other words, modified B/C ratio compares the net benefits to initial investment.

Although modified B/C ratio yields a different value than conventional B/C ratio, it does not change the decision to accept or reject a project.

Discussion Questions and Problems

- 8.1 What are the characteristics of private sector and public sector projects?
- 8.2 What are the benefits of an airport to its community?

- 8.3** What are the disbenefits of an airport to its community?
- 8.4** Explain the process of evaluating cost-based mutually exclusive alternatives.
- 8.5** Explain the process of evaluating benefit-based mutually exclusive alternatives.
- 8.6** Legacy Airport Authority (LAA) is considering construction of a reliever airport in Faridville County, which is about 30 miles away from Legacy International Airport (LIA), to reduce current congestion at LIA. The initial investment is estimated to be \$1,200 million with an annual operating cost of \$50 million in year 1 increasing by \$10 million per year. This is a 20-year project with annual benefit (airport revenues) of \$200 million in year 1 increasing by \$20 million per year and a disbenefit to the public of \$5 million per year. LAA is planning to sell the airport to Faridville County at a price of \$500 million at the end of 20 years which is considered the salvage value of the project. Using conventional benefit/cost ratio with PW values and a MARR of 8 percent, determine whether or not this reliever airport project is economically justified?
- 8.7** US Airport Authority (USAA) is considering the construction of a new terminal at Gator Regional Airport. This project is considered to be permanent and has an initial cost of \$800 million with additional investment of \$100 million every 10 years. No salvage value is expected. The revenues from the new terminal (benefits) are expected to be \$150 million per year and disbenefits are estimated to be \$4 million per year. Annual operating costs are \$25 million per year. USAA uses a MARR of 12 percent. Using a conventional B/C ratio with AW values, determine whether or not this project is economically justified.
- 8.8** Legacy Airport Authority (LAA) has received a proposal for construction of a new conveyer belt to increase the efficiency of transferring luggage to passengers. The information about this proposal is summarized in the table below. LAA uses a MARR of 10 percent. Using both conventional and modified B/C ratios with PW values, determine if the conveyer belt project is economically justified.

First cost	\$4 million now and \$2 million five years from now
AOC	\$200,000 in the first year with increase of \$50,000 per year starting the second year
Annual benefits (revenue)	\$2.5 million
Disbenefits at the time zero	\$500,000
Salvage value	\$200,000
Life, years	10

- 8.9** Busan Global is considering two alternative bridges to connect the city to an island. The costs associated with both bridges are shown below. The life of the bridges is considered permanent and MARR is 8 percent. Assume that both bridges have equivalent use but Bridge B is shorter than Bridge A with lower users travel cost but higher construction cost. Which alternative should be selected based on a conventional B/C analysis?

(in millions)	Bridge A	Bridge B
Initial investment (first cost)	\$50	\$80
Annual operating cost (AOC)	\$4	\$5
Repair cost every 10 years	\$2	\$2.5
User travel costs per year	\$20	\$10
Estimated life, years	∞	∞

- 8.10** US Airport Authority (USAA) is considering adding a new runway at the Gator Reliever Airport to accommodate the landing of the largest jets. USAA has three options for the runway surface: asphalt, concrete, and a mixture of both. The benefit from all three alternatives is the same. Therefore these projects are cost-based alternatives. Their cash flows are provided in the table below. Using conventional benefit/cost ratio with MARR of 9 percent, which alternative should be selected?

(in millions)	Asphalt (ASP)	Concrete (CON)	Mixture (PEM)
First cost	\$250	\$350	\$400
AW of resurface cost	\$30	\$20	\$10
Disbenefit at construction time	\$5	\$10	\$12
Life, years	20	20	20

- 8.11** Legacy International Airport (LIA) is considering three mutually exclusive alternatives shown in the table below for airport improvement. Using conventional B/C ratio with MARR of 8 percent, which project should LIA select?

(in millions)	Concourse A Expansion (CAE)	Concourse B Expansion (CBE)	New Runway (NRW)
First cost, P	\$300	\$400	\$500
Estimated annual operating cost (AOC)	\$15	\$20	\$25

(in millions)	Concourse A Expansion (CAE)	Concourse B Expansion (CBE)	New Runway (NRW)
Benefits (B), estimated annual revenue	\$85	\$100	\$120
Disbenefit (D), incurs at the time zero	\$15	\$20	\$18
Life, years	∞	∞	∞

8.12 US Airport Authority (USAA) is entertaining three independent proposals presented in the table below for Gator Regional Airport. USAA uses a MARR of 12 percent and has no budget constraint. Using B/C ratio and PW values, which of the following projects are economically justified and should be undertaken by USAA to maximize the services to the public?

(in millions)	New Parking Garage (NPG)	Flight Information Screen (FIS)	Gate Expansion (GE)
First cost (P)	\$120	\$3	\$50
Annual operating cost (AOC)	\$5	\$0.5	\$3
Annual benefits (B)	\$25	\$1.0	\$6
Disbenefits at the time zero (D)	\$5	\$0	\$2
Salvage value (S)	\$15	\$0.4	\$5
Life, years	20	5	25

Notes

- 1 The current air traffic management system is based on radar, voice communications, and ground navigational aids, and will ultimately be unable to deal with predicted air traffic growth.
- 2 1999 FAA Airport Benefit cost analysis guidance (FAA, 1999).
- 3 Part VI: Benefit-Cost Analysis—Centers for Disease Control and Prevention (n.d.).
- 4 ACRP Synthesis 13—Effective Practices for Preparing Airport Improvement Program Benefits-Cost Analysis (Transportation Research Board, 2009).

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9

Analysis of Break-even and Payback Period

But no new A380 order was taken in 2015 – and the current order book stands at 317 units, with the cost break-even point for Airbus now believed to be standing at as high as 420 units (from 270 initially) as a result of delays in production and detrimental currency fluctuations between the euro and the US dollar. The A380 is heir and rival to the Boeing 747 jumbo jet in the so-called Very Large Aircraft (VLA) family, for which the maximum combined market size is believed to be between 700 and 1,700 aircraft.

The Conversation, April, 2016



The aviation industry, including airports, airlines, aircraft manufacturers, and fixed based operators (FBO), is continuously facing a decision of whether or not to buy or lease equipment for its facilities. Break-even analysis can be performed to assist the manager to make that decision easily and accurately.

Break-even analysis is a decision-making technique often used to determine the break-even point quantity (value of a parameter) for a single project that makes total costs equal to total revenue resulting in a zero profit. Therefore, the annual usage rate (annual production or consumption units) must be equal to or greater than the break-even quantity to make the project economically acceptable. Break-even analysis may also be applied to two mutually exclusive alternatives, for instance, buy decision versus make decision. In comparing the alternatives, the intention is to determine the indifference point between the two alternatives. In other words, at break-even point, both alternatives become equally attractive. With larger or smaller parameter values, one or the other alternative becomes more attractive.

Some companies have a required time period as a policy for recovering their investments. Payback period is a simple technique to quickly determine the time period needed to recover the initial investment or the first cost. Conventional payback period assumes that the interest rate is equal to zero. Discounted payback period utilizes the minimum acceptable rate of return.

The focus of this chapter is to empower readers to use the break-even analysis technique to evaluate single projects and select among mutually exclusive alternatives. This chapter also aims to enable readers to quickly utilize the conventional and discounted payback periods to determine the viability of single projects, select the optimal project among mutually exclusive alternatives, and select economically justified independent projects.

The following sections briefly address the calculation of break-even point for single projects, the criteria for accepting or rejecting projects, and the comparison and selection process between two mutually exclusive projects. In addition, this chapter describes calculation of both conventional and discounted payback periods.

- ♦ Introduction to Break-even Analysis
- ♦ Elements of Break-even Analysis
 - Break-even Analysis of a Single Project when Interest Is Equal to Zero
 - Break-even Analysis between Two Alternatives with Interest Rate of Zero
 - Break-even Analysis of a Single Project with Interest Rate Greater than Zero
 - Break-even Analysis of Mutually Exclusive Alternatives with Interest Rate Greater than Zero
- ♦ Airline Break-even Load Factor
- ♦ Introduction to Payback Period Analysis
 - Conventional Payback Period Analysis
 - Payback Period with Uniform Net Annual Cash Flows
 - Payback Period with Non-Uniform Net Annual Cash Flows
 - Discounted Payback Period Analysis
- ♦ Using Spreadsheets for Break-even Analysis and Payback Periods
- ♦ Summary
- ♦ Discussion Questions and Problems

Introduction to Break-even Analysis

The concept of break-even originated from finance, and is commonly used in different fields from economy to business to physics. In essence, break-even analysis is a simple approach to provide an estimation of the minimum quantity of sales (or production) needed to cancel out the cost and investment needed to make them possible. In other words, break-even analysis is trying to find a point where total cost up to that point is equal to the revenue achieved at that point, which yields a zero profit.

Similarly, break-even point can be viewed as the point of indifference between two available options, e.g., selecting between two technologies. Consider Boeing 787; one of the noteworthy changes in the Boeing Dreamliner is the replacement of the traditional aluminum body by new composite material. Composites are capable of providing the required strength but with much less weight. The lighter weight is one of the key factors in the fuel efficiency of the 787 model. According to Boeing, the 787

is about 20 percent more fuel efficient than the Boeing 767 (which it was designed to replace). Consequently, the higher cost of using composite material, in comparison to cheaper aluminum alloys, can be justified by the resulting fuel savings. Now, let us consider this tradeoff between saving on fuel cost and saving on using cheaper material. Since 2014, we have observed a decreasing trend in jet fuel prices. Indeed, as the fuel price decreases there should be a point where the saving of fuel cost becomes equal to the extra cost of adopting composites. Therefore, at that point the traditional aluminum option and the new composite option are economically indifferent. In other words, for a given fuel price (i.e., a parameter) the two options break even.

In general, we can define the break-even point as a quantity of output, value of a parameter, or a decision variable that makes two relationships equal. Here are some other examples of break-even:

At quantity (Q) = 2,000	→	Total revenue = Total costs
At quantity (Q) = 1,000	→	AW of Project A = AW of Project B
At interest rate = 8%	→	PW of Project C = PW of Project D
At interest rate = 10%	→	FW of Project X = FW of Project Y
At MARR = 12%	→	TC of Project M = TC of Project N
At price = \$50 per unit	→	Quantity Demanded = Quantity Supplied

Break-even analysis can be performed considering the time value of money (opportunity cost of money) where the interest rate is greater than zero ($i > 0\%$), or without it where the interest rate is equal to zero ($i = 0\%$). The latter method is simple and used often.

In continuation of our previous discussions on methods used in engineering economics for the evaluation of projects, in this chapter we will elaborate break-even analysis for evaluation and comparison of project(s). We will also discuss the payback period concept in this chapter.

Elements of Break-even Analysis

The break-even point is defined in relation to total cost and total revenue. While cost is particularly discussed in Chapter 12 of this book, this chapter will briefly review the cost concepts necessary for understanding and performing the break-even analysis. The key to understanding break-even analysis is recognizing the difference between two types of cost: fixed costs and variable costs.

In practice, acquiring some resources or assets requires a commitment for a certain period of time when the commitment cannot be nullified or altered. For instance, renting a business space often requires a few months' contract during which a tenant is obligated to pay the property owner for the privilege. This contractual payment for the given length of the contract is fixed. That is, the cost is fixed for the tenant whether he uses the property or not. The type of cost that for the period of the analysis is fixed and is not changed by operator's actions is defined as fixed cost.

In contrast, consider that the tenant is operating a small manufacturing business. Clearly, the rate of consuming raw material depends on its rate of production and consequently the cost of raw material varies based on the production variations. The cost that depends on the actions of the operator and varies with the level of output is defined as a variable cost. Table 9.1 presents a few examples of fixed and variable costs.

Table 9.1 Examples of Fixed and Variable Costs

Fixed Costs	Variable Costs
♦ Building and equipment ♦ Building rental costs ♦ Equipment leases ♦ Depreciation ♦ Insurance ♦ Management salaries ♦ Utilities	♦ Raw materials ♦ Direct labor ♦ Sales commission ♦ Inventory ♦ Packaging supplies ♦ Shipping costs ♦ Marketing and advertising

The fixed costs function by definition is linear. However, as will be discussed in Chapter 12, variable costs can be linear or nonlinear. A linear cost relationship is commonly assumed in this chapter for the presentation of break-even analysis; however, the provided analogy can be extended to accommodate nonlinear variable costs. Figure 9.1 presents the relationships between variable and fixed costs against quantity of output.

Nonlinear variable cost function is more realistic due to the economies of scale. In practice, as production volume increases, the variable cost per unit usually decreases and it creates a nonlinear cost curve as shown in Figure 9.2.

As a nonlinear variable cost function is more realistic than a linear one, so is a nonlinear revenue function. As variable cost decreases per unit, assuming a constant price, the revenue per unit will increase resulting in a nonlinear revenue curve as shown in Figure 9.3. This could mean that there might be two possible solutions to the break-even point, since both total cost and total revenue curves are nonlinear.

After discussing the elements of break-even, we now start by the basic analysis to calculate break-even point for a single project without considering the time value of money.

Figure 9.1 Linear Cost Relations

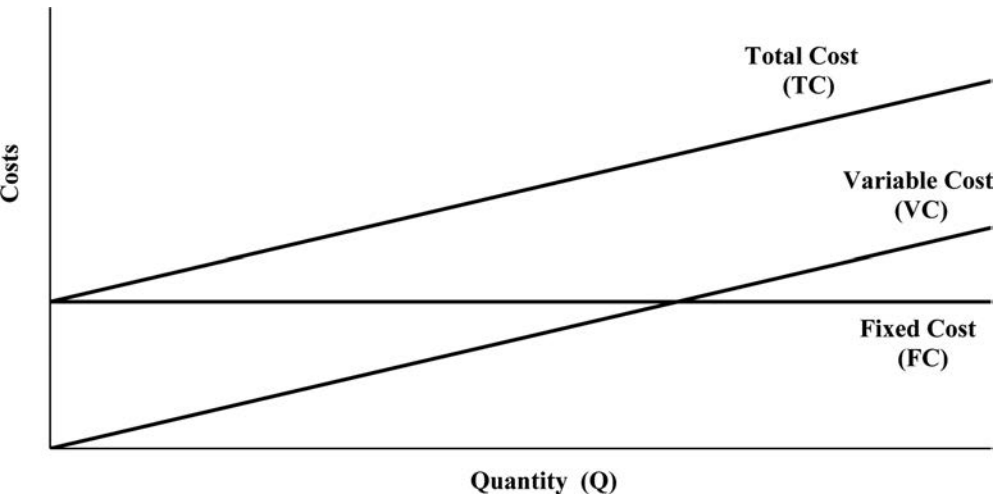
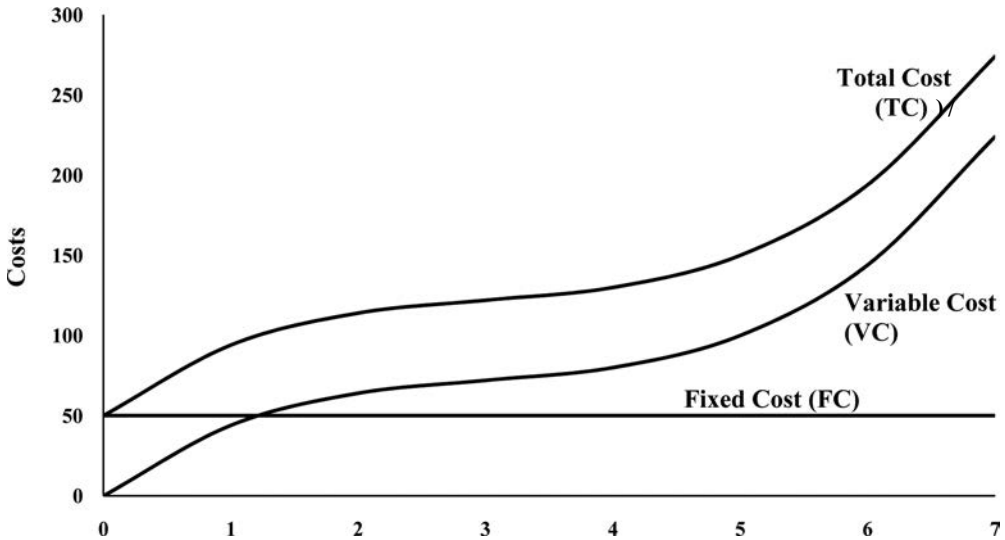
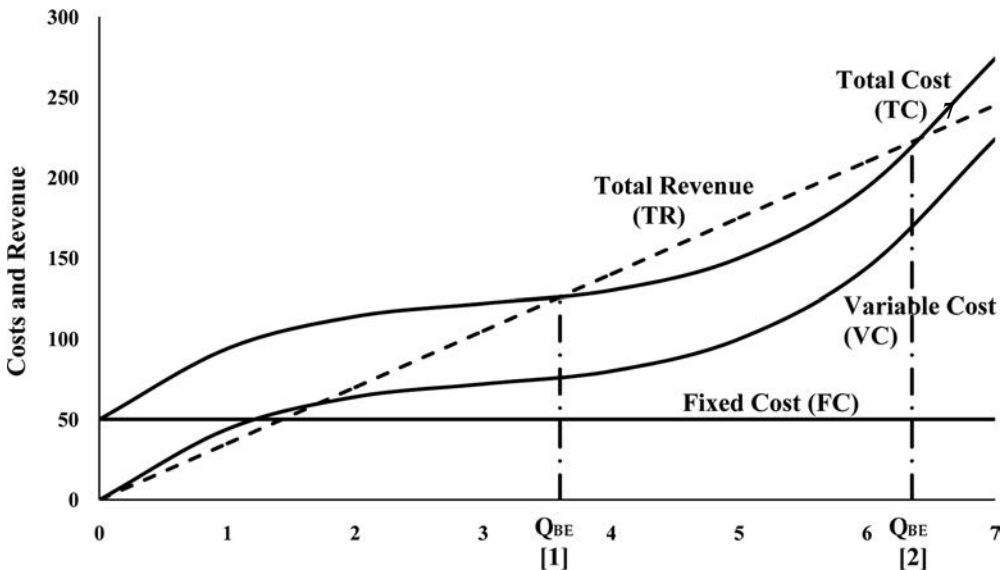


Figure 9.2 Nonlinear Cost Curves**Figure 9.3** Nonlinear Cost Curves and Total Revenue

Break-even Analysis of a Single Project When Interest is Equal to Zero

In this section, we start by performing break-even analysis without considering the time value of money (i.e., interest rate is zero). Break-even point quantity (Q_{BE}) corresponds to the intersection of total revenue and total costs curves. At that point, total revenue is equal to total cost, with profit equal to zero. Figure 9.4 shows the linear relationships between costs and revenue.

$$\text{Total Revenue (TR)} = \text{Quantity of output (Q)} \times \text{Price per unit (P)}$$

$$TR = Q \times P$$

In this section, and only for this section, where the interest rate is equal to zero, P represents the price per unit and not the first cost.

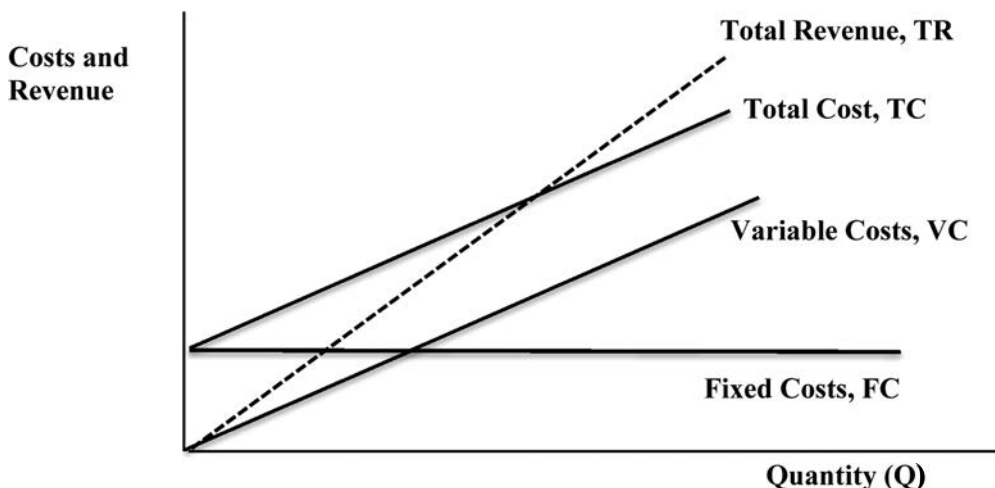
$$\text{Total Costs} = \text{Fixed Costs} + \text{Variable Costs}$$

$$\text{Variable costs} = \text{Quantity of output (Q)} \times \text{Variable cost per unit (V)}$$

$$TC = FC + V \times Q$$

At break-even point quantity (Q_{BE}), $TR = TC$

Figure 9.4 Linear Costs and Revenue Functions



We now solve the equations above for the Q that is the break-even quantity.

$$TR = TC$$

$$QP = FC + Q \times V$$

$$FC = Q \times P - Q \times V$$

$$FC = Q \times (P - V)$$

$$Q_{BE} = \frac{FC}{P - V}$$

As can be seen, break-even analysis is algebraically and graphically easy to understand. As shown in Figure 9.5, at break-even point the profit is zero. At any level of output less than the break-even quantity, the firm incurs a loss; and at any quantity greater than the break-even quantity, the firm makes a profit. The profit is the difference between total revenue and total costs.

$$\text{Profit} = TR - TC$$

$$\text{Profit} = QP - FC - QV$$

$$\text{Profit} = Q \times (P - V) - FC$$

Example 9.1

AirJet International, an aerospace company, has invested \$200 million to build the sharklet wingtips for commercial airplanes. The variable cost per unit is \$200,000 and price per unit is about \$280,000. What is the break-even point, and what is the profit if the company sells 3,000 units per year?

The break-even point is:

$$Q_{BE} = \frac{\$200,000,000}{\$280,000 - \$200,000}$$

$$Q_{BE} = 2,500 \text{ units}$$

And the profit is:

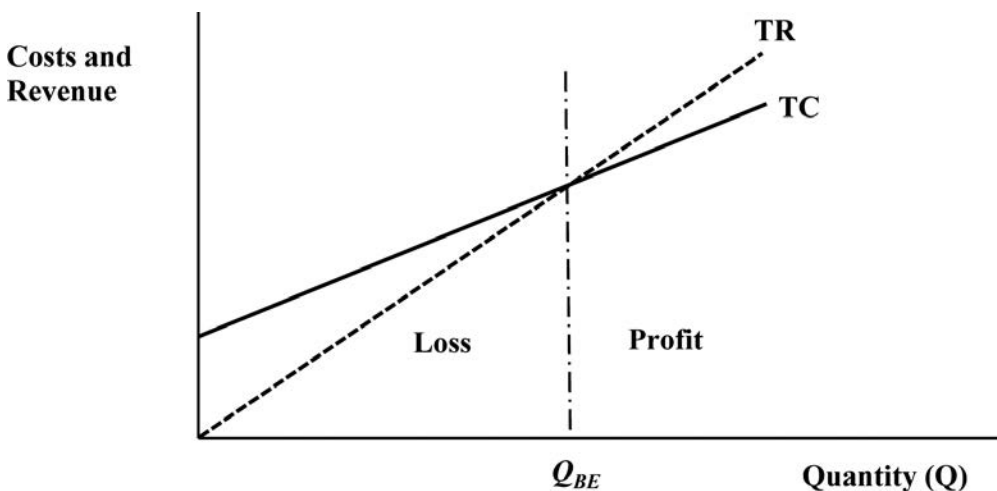
$$\text{Profit} = 3,000 \times (\$280,000 - \$200,000) - \$200,000,000$$

$$\text{Profit} = \$40,000,000$$

Break-even Analysis between Two Alternatives with Interest Rate of Zero

Break-even analysis may also be applied to two mutually exclusive alternatives, such as a purchase decision versus a manufacturing decision. In comparing the alternatives, the intention is to determine the indifference point between them. In other words, at break-even point, both alternatives become equally attractive. With larger or smaller parameter values, one or other alternative becomes more

Figure 9.5 Break-even Point (Q_{BE})



attractive. We set the total costs of one alternative equal to the total costs of the other alternative and solve for the break-even quantity that makes both alternatives equally attractive. Usually one alternative has larger fixed costs and a smaller variable cost per unit relative to the second, which has smaller fixed costs but larger variable cost per unit.

Example 9.2

AvParts, an aircraft manufacturer, has the following two alternatives to manufacture security approved cockpit doors to meet the FAA's new standards for commercial airplanes. The door is designed to resist incursion by one or more persons who try to enter the cockpit using physical force. The FAA requirements include the door, its means of attachment to the surrounding structure, and the attachment structure on the bulkhead itself.

- ♦ Alternative 1: Fixed cost of \$1,000,000 with variable cost per unit of \$8,000.
- ♦ Alternative 2: Fixed cost of \$600,000 with variable cost per unit of \$12,000.

What is the break-even quantity of a cockpit door that allows the manufacturer to be indifferent between these two alternatives?

To find break-even quantity, we set the total cost of alternative 1 equal to the total cost of alternative 2 and solve for the unknown quantity as shown below.

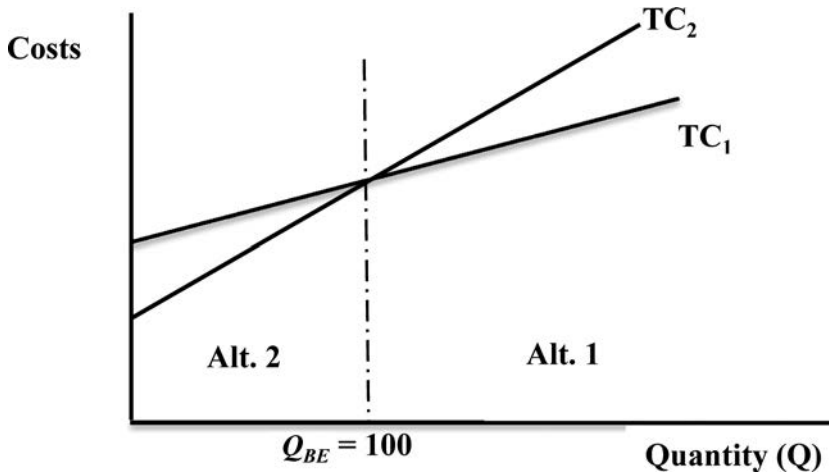
$$\begin{aligned}
 TC_1 &= TC_2 \\
 TC_1 &= \$1,000,000 + \$8,000 Q \\
 TC_2 &= \$600,000 + \$12,000 Q \\
 \\
 \$1,000,000 + \$8,000 Q &= \$600,000 + \$12,000 Q \\
 \$1,000,000 - \$600,000 &= \$12,000 Q - \$8,000 Q \\
 \$400,000 &= \$4,000 Q \\
 Q &= 100
 \end{aligned}$$

Figure 9.6 presents the graphical representation of the alternatives and their relative costs.

As shown in Figure 9.6, the break-even quantity reference point is the intersection of the two linear total cost relations that corresponds to a break-even quantity of 100 units. For an output level of less than 100 units, alternative 2 is preferred with relatively smaller fixed cost and larger variable cost per unit, while for an output level of larger than 100 units, alternative 1 is the selected option with relatively larger FC and smaller variable cost per unit. This argument makes sense because at a break-even point of 100 units, both alternatives have the same total costs. If the manufacturer wants to make more than 100 units, it makes sense to select the alternative with lower variable cost per unit, which is alternative 1.

Break-even Analysis of a Single Project with Interest Rate Greater than Zero

In the previous sections, we ignored the time value of money in evaluating the break-even point. In this section, we consider the scenario that time value of money is

Figure 9.6 Break-even Quantity between Two Alternatives

significant. To calculate the break-even in this setting, we utilized the present worth (PW) and annual worth (AW) methods we discussed in previous chapters. As mentioned, in these approaches we evaluate the overall cash flow during the length of a project, considering positive and negative transactions while accounting for the time value of money.

Now, to compute the break-even point of a single project where interest rate is greater than zero, we set the PW or AW of the project equal to zero and solve for the break-even value of the parameter of interest (e.g., interest rate). In other words, we are interested in finding the value of the parameter of interest that makes the two projects equally attractive.

The parameter of interest in this analysis can be: quantity per year, interest rate, first cost, or any other parameter. For example, the rate of return of a single project (i^*) is the break-even interest rate of that project, as was discussed in Chapter 7. The AW technique is preferred when variable cost is expressed on an annual basis.

Example 9.3

Using Example 9.1, calculate the break-even quantity per year assuming a 20-year project, a MARR of 10 percent, and all other relevant data from Example 9.1.

Because we are asked to compute quantity per year, we set the AW of the project equal to zero.

$$\begin{aligned}
 AW &= 0 \\
 -\$200,000,000 (A/P, 10\%, 20) + (\$280,000 - \$200,000) \times Q &= 0 \\
 \$200,000,000 \times (0.11746) &= \$80,000 Q \\
 \$23,492,000 &= \$80,000 Q \\
 Q_{BE} &= 294 \text{ units per year}
 \end{aligned}$$

The break-even quantity can be now compared with the expected total output per year. If the expected total output is much greater than the break-even quantity, then

there is no room for concern and the aerospace company can comfortably undertake the project. On the other hand, if the expected total output is much less than the break-even quantity, then there is a strong possibility of not breaking even and it should not undertake the project.

Break-even Analysis of Mutually Exclusive Alternatives with Interest Rate Greater than Zero

Break-even analysis can also be utilized as a simple and common technique to compare mutually exclusive alternatives. With interest rate greater than zero, we can set the total cost (TC) of one alternative equal to the TC of the second alternative using PW or AW values to find the value of the parameter that is common to both alternatives. This parameter can be quantity per year, interest rate, first cost, annual operating cost, or any other parameter. The rate of return of the extra project (Δ project), (Δi^*), as calculated in Chapter 7, is the break-even interest rate for two alternatives. At the break-even interest rate, we are indifferent between the two alternatives as both of them provide the same economic value. The AW technique is preferred to PW when the variable costs are expressed on an annual basis or when projects do not have the same lives.

Example 9.4

AvJets designs and manufactures turbine engines for transport category aircraft. Advanced testing systems will ensure that their new signature program entitled “Cool-Start” does indeed work for every engine produced. Let us assume that AvJets has two cost-based testing systems of Turboprop and Turbofan. Table 9.2 presents the costs associated with both alternatives. Assuming a MARR of 8 percent, at what break-even engine quantity per year will AvJets be indifferent about undertaking either of the alternatives?

Table 9.2 Cash Flows for Both Turboprop and Turbofan

	Turboprop (TP)	Turbofan (TF)
Robot and support equipment (first cost)	−\$1,500,000	−\$2,250,000
Annual operating cost (AOC)	−\$700,000	−\$550,000
Rebuild cost in year 3	−\$200,000	\$0
Rebuild cost in year 5	\$0	−\$300,000
Salvage value	\$50,000	\$100,000
Estimated life, years	10	10
Labor wages per hour	\$40	\$30
People	6	8
Number of engines tested per hour	3	6

AW of TC of Turboprop (TP) = AW of TC of Turbofan (TF)

$$\text{Turboprop variable cost per unit } (V_{TP}) = \frac{\$40 \times 6}{3} = \$80$$

$$AW_{TP} = -\$1,500,000 (A/P, 8\%, 10) - \$200,000 (P/F, 8\%, 3) \times (A/P, 8\%, 10)$$

$$+ \$50,000 (A/F, 8\%, 10) - \$700,000 - \$80 Q$$

$$AW_{TP} = -\$1,500,000 \times (0.1490) - \$200,000 \times (0.7938) \times (0.1490)$$

$$+ \$50,000 \times (0.0690) - \$700,000 - \$80 Q$$

$$AW_{TP} = -\$223,500 - \$23,655 + \$3,450 - \$700,000 - \$80 Q$$

$$AW_{TP} = -\$943,705 - \$80 Q$$

$$\text{Turbofan variable cost per unit } (V_{TF}) = \frac{\$30 \times 8}{6} = \$40$$

$$AW_{TF} = -\$2,250,000 (A/P, 8\%, 10) - \$300,000 (P/F, 8\%, 5) \times (A/P, 8\%, 10)$$

$$+ \$100,000 (A/F, 8\%, 10) - \$550,000 - \$40 Q$$

$$AW_{TF} = -\$2,250,000 \times (0.1490) - \$300,000 \times (0.6806) \times (0.1490)$$

$$+ \$100,000 \times (0.0690) - \$550,000 - \$40 Q$$

$$AW_{TF} = -\$335,250 - \$30,423 + \$6,900 - \$550,000 - \$40 Q$$

$$AW_{TF} = -\$908,773 - \$40 Q$$

$$-\$943,705 - \$80 Q = -\$908,773 - \$40 Q$$

$$\$40 Q = \$34,932$$

$$Q = 873$$

Therefore, for an output level of less than 873 engines, the Turboprop system is preferred with relatively smaller fixed cost, while for output level larger than 873 engines the Turbofan is the selected option with relatively smaller variable cost per unit.

Airline Break-even Load Factor

One of the most common, yet simplistic, measurements of an airline's operation is load factor. Load factor simply provides the percentage of an airline's seats that are filled by revenue passengers. For example, when we say that United Airlines (UA) has a load factor of 80 percent on its route from Chicago, Illinois to Phoenix, Arizona, it means on average 80 percent of UA airplane seats were occupied by paying customers. Load factor, however, does not take into consideration any of the revenue or operating

Aviation Snippets

Long-haul budget airline French Blue sets two-year deadline to break even

Airline French Blue plans to give itself two years to break even from low-cost services between Europe and North America, its chairman told Reuters on Thursday.

The latest brand of the Dubreuil group, which also owns Air Caraïbes, aims to start the long-haul services on Sept. 15, initially linking Paris with Punta Cana in the Dominican Republic, followed in June 2017 by services to Reunion Island and Mauritius.

"We will see if it works. We've given ourselves two years," Marc Rochet said. "If it doesn't work, we'll have to adapt."

Reuters, March 2016

costs of the airline. In other words, it does not give any indication whether or not the Dallas–Phoenix route of UA is economically justified. Therefore, load factor alone is not useful in isolation, as the passenger load factor can easily distort an airline's financial image. In order for the load factor to be meaningful, it needs to be compared with the break-even load factor, which is the percentage of the airplane seats that must be filled with paid customers in order for the airline to break even at its current cost structure. The difference between the two load factors indicates the level of profitability of the airline. To calculate load factor and break-even load factor, we need to know the following concepts:

- ♦ Available Seat Mile (ASM) = Number of seats in an airplane times number of miles flown.
- ♦ Revenue Passenger Mile (RPM) = Number of occupied seats by paid passengers times number of miles flown.
- ♦ Total Cost (TC) = Total fixed and variable costs of flights per year.
- ♦ Total Revenue (TR) = Total revenues collected from passengers per year.
- ♦ Cost per Available Seat Mile (CASM) = $\frac{\text{Total Costs}}{\text{ASM}}$
- ♦ Revenue per Revenue Passenger Mile (RRPM) = $\frac{\text{Total Revenue}}{\text{RPM}}$

$$\text{Load Factor (LF)} = \frac{\text{RPM}}{\text{ASM}}$$

$$\text{Break-even LF} = \frac{\text{CASM}}{\text{RRPM}}$$

Example 9.5

DirectJet Airlines has frequent flights from Seoul, Korea to Tokyo, Japan. Now assume the following information, then calculate the load factor, break-even load factor, and determine whether under these assumed scenarios the airline's route is economically justified or not.

ASM = 57,800 million

RPM = 41,616 million

Total revenue = \$6,936 million

Total cost = \$6,358 million

The load factor is to be calculated as follows:

$$\text{Load Factor (LF)} = \frac{41,616}{57,800}$$

$$\text{LF} = 0.72 \text{ or } 72\%$$

To calculate the break-even load factor, we compute the CASM and RRPM and plug them into the equation for break-even load factor

$$CASM = \frac{\text{Total Costs (TC)}}{\text{Available Seat Miles (ASM)}}$$

$$CASM = \frac{\$6,358 \text{ million}}{57,800 \text{ million}} = \$0.110$$

$$RRPM = \frac{\text{Total Revenue (TR)}}{\text{Revenue per Passenger Miles (RPM)}}$$

$$RRPM = \frac{\$6,936 \text{ million}}{\$41,616 \text{ million}} = \$0.167$$

$$\text{Break - even Load Factor} = \frac{\$0.110}{\$0.167} = 0.66 \text{ or } 66\%$$

Because the airline's load factor is greater than break-even load factor, its Seoul–Tokyo route is economically justified.

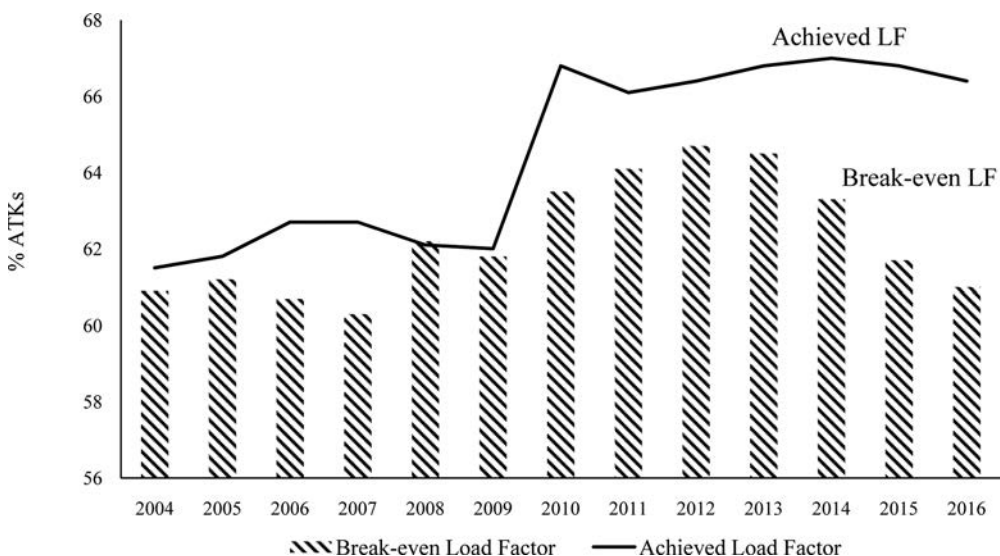
Sometimes for presenting a more comprehensive outlook of airline load factor and break-even threshold, we use available ton kilometer (ATK) that combines the passenger and the cargo capacity of airlines, as they often provide both services. Figure 9.7 presents the load factor and the break-even trend according to IATA.

As can be seen in Figure 9.7, during the financial crisis of 2008–2009 the achieved load factor fell below break-even load factor emphasizing the economic hardship of airlines during these times.

Introduction to Payback Period Analysis

Another analysis that is commonly used in assessing projects is payback period. The objective of this analysis is to estimate the time period needed to recover the initial

Figure 9.7 Average Achieved Load Factor and Break-even Load Factor Trend



investment (i.e., the first cost) considering the cash flow during the life of a project. Clearly, the shorter payback period is more desirable. This method is often used by companies that have a small amount of working capital or by companies in an industry that experiences rapid changes in technology. Another application of payback period analysis is as an initial indicator that a project may be feasible and worthy of a full economic evaluation. We discuss two payback period analyses: conventional and discounted. In the former, we ignore the time value of money while for the latter the interest rate is assumed to be non-zero.

Conventional payback period is very simple to calculate and easy to understand; however, it has two major shortcomings:

- ♦ It ignores cash flows beyond the recovery period. Therefore, it may or may not select the best economically justified alternative.
- ♦ With $i = 0$ assumption, it ignores time value of money, which is the whole basis of the economic evaluation of alternatives.

To eliminate the second shortcoming, we can incorporate the time value of money into payback analysis. This adjustment will result in the discounted payback period analysis, where interest rate is greater than zero.

Conventional Payback Period Analysis

Conventional payback period (N_p) is defined as a simple technique that quickly determines the time period needed to recover the initial investment or the first cost. Conventional payback period assumes that interest rate is equal to zero, while discounted payback period utilizes the minimum acceptable rate of return. The focus of this section is on the conventional payback period. We are going to compute the conventional payback period for projects with a uniform net annual cash flow (NACF) and projects with non-uniform NACF.

Payback Period with Uniform Net Annual Cash Flows

If a project has uniform net annual cash flows, then we use the following equation to calculate payback period (N_p).

$$N_p = \frac{\text{First Cost}}{\text{Net Annual Cash Flow}}$$

$$N_p = \frac{P}{\text{Net Annual Cash Flow}}$$

Example 9.6

AvParts, an aircraft company, is considering purchasing a machine costing \$5,000,000 to produce wing parts from composite carbon fiber for airplanes. A uniform NACF of \$1,250,000 is expected for 10 years over the physical life of the asset with zero salvage value. What is the conventional payback period for this machine?

Utilizing the equation above, the payback period is computed as shown:

$$N_p = \frac{\$5,000,000}{\$1,250,000}$$

$$N_p = 4 \text{ years}$$

Payback Period with Non-Uniform Net Annual Cash Flows

If a project has non-uniform net annual cash flows, then we calculate the cumulative NACFs and use the following equation to compute the payback period (N_p).

$$N_p = (n - 1) + \frac{P - \text{The cumulative cash flow in } (n - 1) \text{ year}}{\text{Net Annual Cash Flow in year } n}$$

Where n is the number of years that the cumulative cash flow exceeds the first cost for the first time and P is the first cost.

Example 9.7

In Example 9.6, assume the aircraft company has non-uniform net annual cash flows, which are presented in Table 9.3. All other relevant information remains the same.

We first calculate the cumulative NACFs as shown in Table 9.4.

The cumulative net annual cash flows in Table 9.4 indicate that by year 4, the cumulative NACF exceeds the first cost for the first time. Therefore, $n = 4$. With the first cost of \$5 million, the payback period is calculated below.

Table 9.3 NACFs

Year	NACF
1	\$1,750,000
2	\$1,500,000
3	\$1,250,000
4	\$1,000,000
5	\$1,000,000
6	\$1,000,000
7	\$1,000,000
8	\$1,000,000
9	\$1,000,000
10	\$1,000,000

Table 9.4 Cumulative NACFs

Year	NACF	Cumulative NACF
1	\$1,750,000	\$1,750,000
2	\$1,500,000	\$3,250,000
3	\$1,250,000	\$4,500,000
4	\$1,000,000	\$5,500,000
5	\$1,000,000	\$6,500,000
6	\$1,000,000	\$7,500,000
7	\$1,000,000	\$8,500,000
8	\$1,000,000	\$9,500,000
9	\$1,000,000	\$10,500,000
10	\$1,000,000	\$11,500,000

$$N_p = (4 - 1) + \frac{\$5,000,000 - \$4,500,000}{\$1,000,000}$$

$$N_p = (3) + 0.5$$

$$N_p = 3.5 \text{ years}$$

Discounted Payback Period Analysis

By computing discounted payback period (N_{DP}), we eliminate the second shortcoming of the conventional payback period by assuming interest rate is greater than zero ($i > 0$) or considering time value of money in payback period analysis. To find the N_{DP} of a project, we set PW of all cash flows equal to zero and solve to the time period as shown:

$$PW = -P + \sum_{t=1}^{N_{DP}} \frac{NACF^t}{(1+i)^t} = 0$$

If NACFs are an annuity or a uniform series, then this equation can be simplified:

$$PW = -P + NACF (P/A, i, N_{DP}) = 0$$

Example 9.8

Using all relevant information in Example 9.6 for AvParts Aircraft Company and assuming a MARR of 10 percent, calculate the discounted payback period.

The N_{DP} is calculated as follows:

$$PW = -\$5,000,000 + \$1,250,000 (P/A, 10\%, N_{DP}) = 0$$

$$\$5,000,000 = \$1,250,000 (P/A, 10\%, N_{DP})$$

$$(P/A, 10\%, N_{DP}) = 4$$

We now go to Appendix A in the textbook for compound interest factor of 10 percent and, under column P/A , look for interest factor value. If we find the exact interest factor value in that column, then the corresponding time period under column n is the discounted payback period. If we do not find the exact interest factor value under column P/A , then we will interpolate between the two closest values. In this example, the interest factor value of 4 is in between five and six years and therefore we interpolate between five and six years as follows:

$$\frac{n}{5} = \frac{(P/A, i^*, N_{DP})}{3.7908}$$

$$\frac{5 + X\%}{6} = \frac{4}{4.33}$$

$$\frac{X\%}{1\%} = \frac{0.2092}{0.5645}$$

$$X = \frac{0.2092}{0.5645}$$

$$X = 0.37\%$$

Therefore, $N_{DP} = 5.37$ years

This number can be compared to the required payback period in a company policy to determine whether to consider the project for further economic valuation or to reject the project.

Using Spreadsheets for Break-even Analysis and Payback Periods

This section presents how to run an equation in Excel spreadsheets to calculate break-even value when interest rate is equal to zero ($i = 0$) or greater than zero ($i > 0$). Excel does not have a financial function to compute the break-even point. We can, however, insert the break-even equation into an Excel spreadsheet to compute the break-even value.

$$Q_{BE} = \frac{FC}{P - V}$$

We can also insert this equation into an Excel spreadsheet to compute break-even value.

$$Profit = Q \times (P - V) - FC$$

$$0 = Q_{BE} \times (P - V) - FC$$

To compute the break-even point of a single project where interest rate is greater than zero, we set the PW or AW of the project equal to zero and solve for the break-even value of the parameter. The AW technique is preferred when variable cost is expressed on an annual basis.

$$AW = 0$$

$$AW = -P(A/P, i, n) - AOC + (p - v) \times Q_{BE} = 0$$

$$P(A/P, i, n) + AOC = (p - v) \times Q_{BE}$$

$$Q_{BE} = \frac{P(A/P, i, n) + AOC}{(p - v)}$$

Where: P = first cost (capital P)
 p = price per unit (lower case p)
 v = variable cost per unit

Example 9.9

AvParts, an aircraft manufacturing firm, is considering purchasing equipment costing \$700,000 and with an expected eight-year life to produce wing strut fairings for single engine aircraft and sell them at a price of \$600 each. The variable cost per unit is \$250 and its annual operating cost is \$50,000 per year. Use Excel spreadsheets to determine the quantity of wing strut fairings AvParts should produce per year to break even. Assume AvParts uses a MARR of 9 percent.

Figure 9.8 Calculation of Break-even Quantity with Zero and Non-Zero Interest Rate Using Excel

	A	B	C	D
1				
2	First Cost	\$570,000		
3	AOC	\$50,000		
4	Life/years	6		
5	MARR	9%		
6	Variable cost per unit	\$250		
7	Price per unit	\$600		
8				
9	Break-even quantity	=((B2/B4)+B3)/(B7-B6)		
10	MARR = 0			
11				
12	Break-even quantity	505.90		
13	MARR = 9%			

	A	B	C	D
1				
2	First Cost	\$570,000		
3	AOC	\$50,000		
4	Life/years	6		
5	MARR	9%		
6	Variable cost per unit	\$250		
7	Price per unit	\$600		
8				
9	Break-even quantity	414.29		
10	MARR = 0			
11				
12	Break-even quantity	=PMT(B5,B4,-B2,0)+B3)/(B7-B6)		
13	MARR = 9%			

The Excel representation in Figure 9.8 shows the computation of break-even quantity. We plug the equation for break-even quantity where interest rate is equal to zero into cell B9, and plug the equation for break-even quantity where interest rate is greater than zero into B12 as shown in Figure 9.8. The solutions are provided in cell B9 and cell B12 for break-even quantities where interest rate is equal to zero and greater than zero respectively.

This section also presents how to run an equation in Excel spreadsheets to calculate conventional and discounted payback periods. For conventional payback periods, we simply plug in the equation as shown below into an Excel spreadsheet cell unit.

$$N_p = \frac{PV}{NACF}$$

To compute discounted payback period, we plug the following Excel function into an Excel spreadsheet cell unit.

$$= \text{NPER}(i\%, A, P, F)$$

Example 9.10

AvJets, an aircraft manufacturer of commercial, defense, and executive aircraft is considering purchasing equipment costing \$4,000,000 to produce wing parts from composite carbon fiber for airplanes. A uniform net annual cash flow of \$800,000 is expected for 10 years over the economic life of the equipment with salvage value of \$400,000. Assuming AvJets uses a MARR of 8 percent, compute both conventional and discounted payback periods.

Figure 9.9 shows the computation of payback period using Excel. We plug the equation for conventional payback period into cell B8 as shown in Figure 9.9. Salvage value is ignored because it occurs beyond the recovery period. We also plug the equation for discounted payback period into B11 as shown in Figure 9.9. The solutions are provided in cell B8 and cell B11 for conventional and discounted payback periods respectively.

Figure 9.9 Calculation of Conventional and Discounted Payback Periods with Zero and Non-Zero Interest Rate Using Excel

	A	B
1		
2	First Cost	\$4,000,000
3	AOC	\$800,000
4	Life/years	10
5	MARR	8%
6	Salvage Value	\$400,000
7		
8	Conventional Payback	=B2/B3
9	MARR = 0	
10		
11	Discounted Payback	6.11
12	MARR = 8%	

	A	B	C
1			
2	First Cost	\$4,000,000	
3	AOC	\$800,000	
4	Life/years	10	
5	MARR	8%	
6	Salvage Value	\$400,000	
7			
8	Conventional Payback	5	
9	MARR = 0		
10			
11	Discounted Payback	=NPER(B5,B3,-B2,B6)	
12	MARR = 8%		

Summary

This chapter presents two new techniques for professional engineers and practitioner decision makers, break-even analysis, and payback period analysis.

Break-even point is generally defined as a quantity of output, value of a parameter, or a decision variable that makes two relationships equal. For a single project, the annual usage rate (annual production or consumption units) must be equal to or greater than the break-even quantity to make the project economically acceptable.

Break-even analysis can be performed considering time value of money (opportunity cost of money) where interest rate is greater than zero ($i > 0\%$) or without it where interest rate is equal to zero ($i = 0\%$). The latter method is simple and used often. With interest rate equal to zero, the break-even point is equal to the project's fixed cost over the difference between price and variable cost per unit. To compute the break-even point of a single project where interest rate is greater than zero, we set the PW or AW of the project equal to zero and solve for the break-even value of the parameter. The AW technique is preferred when variable cost is expressed on an annual basis.

Break-even analysis may also be applied to two mutually exclusive alternatives. To calculate the break-even point between two alternatives, which is the point of indifference between two alternatives, we set the total costs of one alternative equal to the total costs of the other alternative and solve for break-even quantity.

Conventional payback period (N_p) is defined as a simple technique that quickly determines the time period needed to recover the initial investment. It has two major shortcomings. It ignores the cash flows beyond the recovery period and it ignores the time value of money.

By computing discounted the payback period (N_{DP}), we incorporate time value of money in payback period analysis. To find the N_{DP} of a project, we set PW of all cash flows equal to zero and solve the time period (N_{DP}).

Companies interested in recovering their investment quickly to finance other profitable projects often use payback period analysis.

Discussion Questions and Problems

- 9.1 At Av-Parts Aviation Support System (AASS), the main product is the sharklet wingtips for commercial airplanes. The fixed cost associated with manufacturing of sharklet wingtips is \$1,500 million. The average variable cost of sharklet wingtips is \$500,000 and AvParts sells sharklet wingtips at an average price of \$900,000. Calculate
 - a) the break-even point and
 - b) the profit if the company sells 4,000 units per year.
- 9.2 AvJets, the manufacturer of the AX960 aircraft, spent about \$15 billion at the beginning of the project for design, development, production, and testing, which are considered fixed costs. The variable cost per unit is about \$270 million. The average price is about \$320 million.
 - a) What is the break-even quantity?
 - b) What price should AvJets charge for each unit of AX960 to break even at 200 units?
 - c) At a price of \$320 million per unit, how much does the variable cost per unit have to be reduced to break even at 200 units?
- 9.3 DirectJet Airlines has frequent flights from Singapore to Hong Kong. Based on the following information, calculate the load factor and the break-even load factor, and determine whether or not this Singapore–Hong Kong route is economically justified.

ASM = 2,284 million
 RPM = 1,957 million
 Total revenue = \$388 million
 Total cost = \$365 million
- 9.4 EZjET Airlines is reviewing the flight route to North America from China. This route has a load factor of 82 percent. EZjET has a target break-even load factor of 75 percent. It has 45,000 million available seat miles and total revenue of \$5,400 million. Based on target break-even point, what is EZjET's cost per available seat mile (CASM)?
- 9.5 A commercial aircraft manufacturer has the following two alternatives to manufacture security approved cockpit doors to meet the FAA's new standards for commercial airplanes. The first alternative has a fixed cost of \$2,000,000 with variable cost of \$10,000 per unit. The second alternative has a fixed cost of \$1,200,000 with variable cost of \$20,000 per unit. What is the break-even quantity of the cockpit door that allows the aircraft manufacturer to be indifferent between these two alternatives?
- 9.6 At AvParts Manufacturing Company, the main product is the sharklet wingtips for commercial airplanes. The fixed cost associated with manufacturing of sharklet wingtips is \$150 million. The average variable cost of sharklet wingtips is \$50,000 and AvParts sells sharklet wingtips at an average price of \$90,000. Calculate the break-even quantity per year assuming a 10-year project with no salvage value and a MARR of 10 percent.
- 9.7 Majestic Air Company LLC manufactures evacuation slides, an inflatable slide used to evacuate an aircraft quickly. It is considering a more advanced and efficient machine to manufacture slides, with a first cost of

\$12 million and annual operating cost of \$500,000. The economic life of the machine is 10 years with a required rebuild cost of \$1,000,000 in four years, and salvage value of \$2,000,000. The machine requires six people to operate at the wage rate of \$50 per hour including fringe benefits. In an eight-hour shift per day, they produce eight units. Assuming a MARR of 8 percent, a sale price of \$15,000 per unit, what is the break-even quantity per year?

- 9.8 AvJets, a manufacturing company, designs and manufactures turbine engines for transport category aircraft. Advanced testing systems will ensure that their new signature program entitled “Cool-Start” does indeed work for every engine produced. AvJets has the following two cost-based testing systems. Assume a MARR of 10 percent, at what quantity per year will AvJets be indifferent in undertaking either of System A or System B?

	System A	System B
Robot and support equipment (first cost)	−\$1,200,000	−\$2,500,000
Annual operating cost (AOC)	−\$600,000	−\$500,000
Rebuild cost in year 5	−\$200,000	\$0
Salvage value	\$120,000	\$200,000
Estimated life, years	10	10
Labor wages per hour	\$50	\$30
People	6	10
Number of engine tested per hour	4	6

- 9.9 AvProp Aircraft Company produces prop-driven airplanes. It is considering a buy or make decision. AvProp can purchase propeller systems for its prop-driven airplanes with three-blade structure composite carbon fiber from Hartzell Inc. at a price of \$24,000 per unit. It can produce propeller systems in-house with equipment costing \$12 million, annual operating costs of \$300,000, a variable cost of \$12,000 per unit and an estimated salvage value of 8 percent of the end of a study period of 10 years. At what break-even quantity will AvProp be indifferent between making and buying propellers systems? Assume AvProp uses a MARR of 10 percent.
- 9.10 AvParts is considering purchasing a machine costing \$8,100,000 to produce wing parts from composite carbon fiber for airplanes. A uniform NACF of \$1,500,000 is expected for 10 years over the physical life of the asset with zero salvage value. What is the conventional payback period for this machine?
- 9.11 In problem 9.10, assume AvParts has non-uniform net annual cash flows as presented in the following table. All other relevant information remains the same. What is the conventional payback period for this machine?

Year	NACF
1	\$2,000,000
2	\$1,750,000
3	\$1,500,000
4	\$1,250,000
5	\$1,000,000
6	\$750,000
7	\$500,000
8	\$500,000
9	\$500,000
10	\$500,000

- 9.12** Using all relevant information in problem 9.10 for AvParts and assuming a MARR of 10 percent, calculate the discounted payback period.
- 9.13** AvParts is considering purchasing a machine costing \$15 million with an expected 10-year life to produce aircraft parts and sell them at a price of \$4,500 each. AvParts is expecting to sell 1,000 units per year with an annual operating cost of \$500,000 in year 1 and increases of \$50,000 per year starting the second year through the end of 10 years. AvParts uses a MARR of 12 percent and is expecting no salvage or residual value. What is the discounted payback period for this equipment?
- 9.14** Boeing, the manufacturer of the Dreamliner B787, spent about \$16 billion at the beginning of the project for design, development, production, and testing, which are considered fixed costs. The variable cost per unit is \$170 million. What price should Boeing charge per unit of B787 to break even at 200 units?
- 9.15** DirectJet is reviewing the flight route from Pittsburgh to San Francisco. This route has a load factor of 80 percent. DirectJet has a target break-even load factor of 70 percent. DirectJet has 56,000 million available seat miles and total revenue of \$5,376 million. Based on the target break-even point, what is DirectJet's cost per available seat mile (CASM)?
- 9.16** FAA requires slides on all commercial aircraft doors where the floor is 6 feet or more above the ground. Air Cruisers Company LLC manufactures evacuation slides, an inflatable slide used to evacuate an aircraft quickly. It is considering a more advanced and efficient machine to manufacture slides, with a first cost of \$10 million, and annual operating cost of \$800,000. The economic life of the machine is 10 years with a required rebuild cost of \$2,000,000 in five years, and salvage value of \$1,500,000. The machine requires five people to operate at the wage rate of \$60 per hour including fringe benefits. For an eight-hour shift per day, they produce 10 units. Assuming a MARR of 12 percent, a sale price of \$12,000 per unit, what is the break-even quantity per year?

- 9.17** Jet Aircraft is a recognized leader in general aviation. It manufactures an all-composite line of personal aircraft—the SR20, SR22 and the turbocharged SR22T. Jet currently produces 250 propeller systems in-house with a variable cost of \$15,000, using equipment costing \$15,000,000, with an annual operating cost of \$400,000 and an estimated salvage value of 10 percent of the end of a study period of 10 years. Jet is considering purchasing propeller systems for its prop-driven airplanes with a three-blade structure composite carbon fiber from Hartzell Inc. at a price of \$24,000 per unit. What is the break-even quantity at which Jet will be indifferent between making and buying propellers? Assume Jet uses a MARR of 10 percent.
- 9.18** BizJet Aircraft Company is the leading designer and manufacturer of light and midsize business jets. Jet is selling parts for its aircraft through BizJet part department. It is considering purchasing a machine costing \$2,500,000 with an expected 10-year life to produce right and left landing gear bulkheads and sell them at price of \$1,500 each. BizJet is expecting to sell 500 units per year with an annual operating cost of \$50,000 in year 1 and increases of \$20,000 per year starting the second year through the end of 10 years. BizJet uses a MARR of 9 percent and is expecting no salvage or residual value. What is the discounted payback period for this equipment?

10

Uncertainty, Risk Analysis, and Simulation

Failure saves lives. In the airline industry, every time a plane crashes the probability of the next crash is lowered by that. The Titanic saved lives because we're building bigger and better ships. So these people died, but we have effectively improved the safety of the system, and nothing failed in vain.

Nassim Nicholas Taleb, Author of *The Black Swan*



The aviation industry is vastly impacted by external shocks, regulatory reforms, and economic and financial uncertainties. Risk analysis forms one of the most fundamental aspects of business operation and management. The air transport industry is no exception; companies are exposed to many hazards, such as operational, financial, or investment, and to political risk. Keeping airline passengers safe while flying efficiently remains paramount, especially in the wake of recent terrorism. The ability to identify and react effectively to potential uncertainty is a major qualification to be a successful business manager. Risk management is an economic issue, as the management must balance the cost and the benefits associated with the risk mitigation stratagem. Without a proper risk management strategy, an airline would merely rely on fate to avoid financial or operations failures.

This chapter mainly focuses on explaining the various methods involved in predicting and analyzing the risk factors involved in a decision-making process and includes:

- ◆ Decision Making
- ◆ Probability and Business Decisions
 - Cumulative Probability Distribution
 - Joint Probability Distribution
- ◆ Measuring Risk and Reward

- Sample Space
- Expected Return or Mean
- Variance
- Standard Deviation
- Calculating Variance and Standard Deviation Using Spreadsheets
- ◆ Normal Distribution
 - Calculating Normal Distribution Probability Using Spreadsheets
 - Standard Normal Distribution
- ◆ Portfolios
 - Portfolio Beta
- ◆ Capital Asset Pricing Model (CAPM)
 - Security Market Line (SML)
- ◆ Sensitivity Analysis
 - Break-even Analysis
 - Monte Carlo Analysis (Simulation)
- ◆ Summary
- ◆ Discussion Questions and Problems

Decision Making

We believed we could develop a very efficient aircraft, even more efficient than what the competitors were offering, but after years of internal debate our final decision was that we could not take a step toward a larger aircraft because the business plan was not there for it.

Paulo Cesar Silva, Executive VP Commercial Aviation, Embraer

Decision making is the process of selecting between two or more different outcomes by setting goals, collecting information, and evaluating alternatives. We all make decisions of varying importance every day, but while some decisions are simple, others are more complicated. For example, when you decided to fly with DirectJet, your decision was facilitated by many previous decisions. You had to decide to fly with DirectJet, since you had a great experience with the airline earlier. Managerial decisions are made under different market conditions. Sometimes managers have little information or knowledge of conditions surrounding a decision. Conversely, decision makers may have almost perfect information and knowledge about the outcome of their decisions. Many managerial decisions are made without knowing in advance what the possible future outcomes will be. For instance, the decision to develop a new aircraft can be made without perfect advance knowledge of whether the manufacturer will reach the break-even point or whether there would be enough demand to justify a massive amount of investment.

When the outcomes are known and the values of the outcomes are certain, the decision makers know the precise outcome associated with each possible alternative course of action. A decision is made under conditions of risk when a single action may result in more than one potential outcome, but the relative probability of each outcome is known. Certainty implies all relevant information is available to the decision maker. An example of decision making under certainty includes investment in US government bonds. Every bond is backed by the full faith and credit of the United States government and the return is known with certainty.¹

John Maynard Keynes and Frank Knight developed the conceptual distinction between risk and uncertainty; it remains fundamentally correct today (Davidson, 2002). Uncertainty arises when there is more than one feasible possibility and it remains an uncertainty as long as the possibilities are not quantified. Once all the options are quantified, uncertainty is modified into risk. Risk can be mathematically measured as the probability of a particular event occurring as opposed to all the possible outcomes. In decision making under risk, probabilities of events are known precisely by the decision maker. Conversely, in decision making under uncertainty the likelihoods of outcomes are not known.

Probability and Business Decisions

Probability concepts are mathematical tools used by decision makers to identify the degree of risk associated with any project. Probability theory has many practical applications in inventory management, data analysis, revenue management, passenger no-show behavior, and aircraft maintenance and reliability. In finance, probability theory has extensive applications in portfolio management and risk analysis. Probability is particularly important in the areas of quality engineering and biomedical engineering, and in estimating the reliability component and system. Contemporary applications of probability theory run the gamut of business investment including portfolio theory, investment analysis, and capital budgeting. Knowledge of modern probability theory is vital to develop economic modeling and test the veracity through analysis of real world data. Probability and statistics could help to understand the Black–Scholes option pricing model (Black and Scholes, 1973) to develop pricing models for financial assets such as equities and bonds, and the Capital Asset Pricing Model (CAPM).

Putting this simply, the probability is the chance or likelihood of a given event occurring in the future (Olariu, 2013). When a particular outcome or a series of outcomes is defined, it is referred to as an event. Probability is a way of assigning every event a value, with the condition that the event made up of all possible results be given a value equal to one. For example, when a single die is thrown, there are six possible outcomes: {1, 2, 3, 4, 5, and 6}, and the probability of any one of them is 1/6. The probability that the experiment results in a successful outcome (A) is:

The probability of an event is the likelihood or the chance that the event will occur in the future as a result of an experiment.

$$P(A) = \frac{\text{Number of ways event A can occur}}{\text{Total number of outcomes}}$$

Mathematically, the probability of any event A [denoted as $P(A)$] is expressed as a number between 0 and 1.

$$0 \leq P(A) \leq 1$$

If $P(A) = 0$, event A will definitely not occur.

If $P(A) = 1$, event A will definitely occur.

If the goal for an investment is to produce a minimum 10 percent return every year on average, and the chances that the goal will not be met are to be assessed, the event is defined as average annual returns below 10 percent. Probability concepts

A probability is a real number between 0 and 1 that describes how likely it is that an event will occur.

are used to assess the chances that an event will take place. If a list of events is mutually exclusive, it means that only one of them can possibly take place. Exhaustive events refer to the need to incorporate all potential outcomes in the defined events.

Example 10.1

Airlines routinely overbook their flights, realizing that not all the passengers will show up for their flight; and predicting the number of no-shows has a significant impact on the revenue. MyJet tries to combat passenger no-shows by overbooking flights. The airline notices that one day 121 out of 125 passengers show up for their flight. The next day, 119 out of 125 passengers make the trip. The two relative frequencies are 121/125 and 119/125. However, summing results over many observations, you might find that the probability that a passenger makes the trip is closer to 0.96.

$$P(\text{Taking the flight}) = \frac{120}{125}$$

$$P(\text{Taking the flight}) = 96\%$$

$$P(\text{No-show}) = 100\% - 96\%$$

$$P(\text{No-show}) = 4\%$$

Weather conditions, mechanical issues, airspace congestion, crew scheduling, and other issues can cause airlines to experience delays or cancel a flight, which in turn affects the number of no-shows. The knowledge of probability helps the revenue managers to minimize the cost associated with the possibility that a flight leaves with empty seats. For example, if on a flight between Los Angeles and New York an airline sells 125 tickets, and at the time of departure only 100 passengers show up then:

$$P(\text{No-show}) = \frac{125 - 100}{125}$$

$$P(\text{No-show}) = \frac{25}{125}$$

$$P(\text{No-show}) = 20\%$$

Thus, assume you pick a random flight on that route, the probability that a passenger does not show up for this flight is expected to be 20 percent. To obtain the optimum booking, the airline needs to overbook the flight:

$$125 = \text{Booking} \times (1 - \text{no-show rate})$$

$$125 = \text{Booking} \times (1 - 0.20)$$

$$\text{Booking} = 156$$

$$\text{Overbooking} = 156 - 125$$

$$\text{Overbooking} = 31$$

Cumulative Probability Distribution

The Cumulative Distribution Function (CDF) valued at ' x ', is the probability that a real-valued random variable X will take a value less than or equal to x . In other words,

$$f(x) = P(X \leq x)$$

To better understand cumulative distribution, the probability of filling seats in a particular fare class in a flight can be analyzed.

Example 10.2

Table 10.1 represents the booking pattern for DirectJet Airlines. The revenue manager wishes to determine the cumulative probabilities of filling the allotted number of seats in the \$300 fare class (Class Y). Based on historical data, the mean number of bookings was found to be 10 and the standard deviation was found to be 3.

In order to determine the probability of filling a seat, the inverse cumulative probability needs to be found, since, the probability of a higher number of seats being filled will genuinely be low. Table 10.1 shows the progression in cumulative and inverse cumulative probabilities for different numbers of tickets booked.

Figure 10.1 is a representation of the cumulative probability distribution of the number of bookings presented in Table 10.1. In this graph, the probability depicts the probability of filling equal to or less than a certain number of seats. For example, the probability of filling 7 seats or less is 0.159.

$$(X \leq 7) = 0.159$$

$$(X \geq 7) = 0.841$$

$$P(X \leq 7) \text{ or } P(X \geq 7) = 100\%$$

$$P(X \leq 7) + P(X \geq 7) = 100\%$$

Figure 10.2 is a depiction of the inverse cumulative probability distribution for the number of bookings, based on the given historical data. The inverse cumulative probability denotes the probability of filling up equal to or greater than a certain number of seats. For example, the probability of filling seven seats or more is 0.841.

Joint Probability Distribution

Probability distributions can also be applied to groups of random variables which gives rise to joint probability distributions. The most common type of

Table 10.1 Cumulative Probability Distribution

Mean	Standard Deviation	
10	3	
No. of Bookings	Normal Cumulative Probability	Inverse Cumulative Probability
1	0.001	0.999
2	0.004	0.996
3	0.010	0.990
4	0.023	0.977
5	0.048	0.952
6	0.091	0.909
7	0.159	0.841
8	0.252	0.748
9	0.369	0.631
10	0.500	0.500
11	0.631	0.369
12	0.748	0.252
13	0.841	0.159
14	0.909	0.091
15	0.952	0.048
16	0.977	0.023
17	0.990	0.010
18	0.996	0.004
19	0.999	0.001
20	1.000	0.000

joint probability distribution is a two-dimensional distribution (i.e., only two random variables) but higher dimensions (more than two variables) are also possible.

Because all random variables are divided into discrete and continuous random variables, there exist both discrete and continuous joint probability distributions. These distributions are similar to one-variable distributions but vary with respect to complexity of application.

Essentially, joint probability distributions describe situations where both outcomes represented by random variables occur.

Figure 10.1 Normal Cumulative Distribution (Probability of filling equal or less than ...)

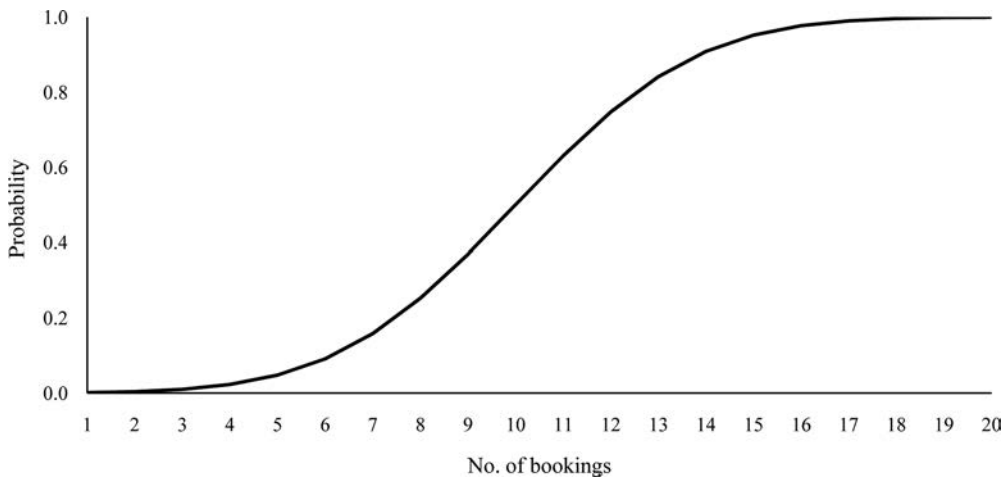
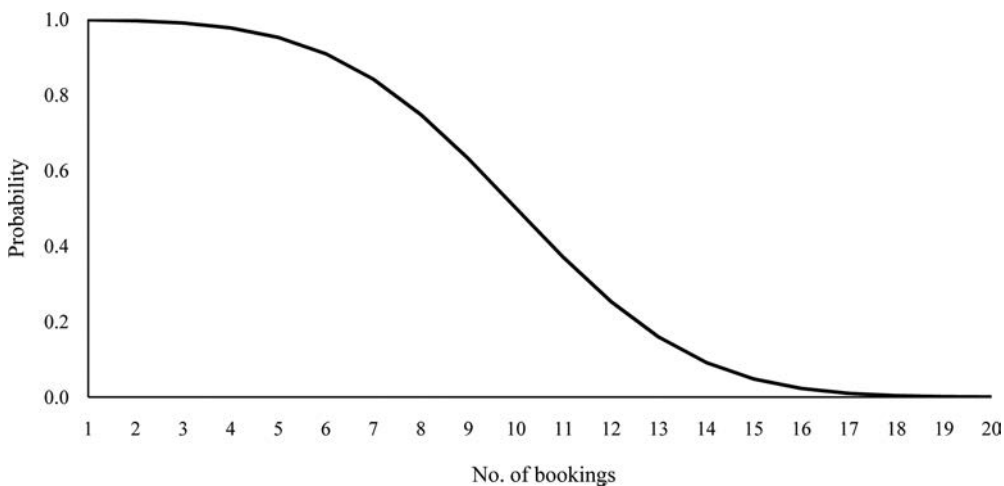


Figure 10.2 Inverse Cumulative Distribution (Probability of filling equal or more than ...)



Joint probability distributions are defined in the form below:

$$P(A \text{ and } B) = P(A) \times P(B) = P(A \cap B)$$

Where the above represents the probability that events **A** and **B** occur at the same time. For example, the probability that a card is an ace and black;

$$P(\text{Ace and Black}) = P(\text{Ace}) \cap P(\text{Black})$$

$$P(A \cap B) = \frac{4}{52} \times \frac{13}{52}$$

$$P(A \cap B) = \frac{1}{52}$$

Example 10.3

Flight delays cost airlines and passengers billions of dollars annually. To identify the major factors that influence flight departure delays, and to develop a strategic departure delay prediction model, a flight dispatcher estimates the probability that two flights arrive late at the terminal. Assume these two events are independent and the probabilities of late arrivals are given as below:

$$\begin{aligned}P(FL-1_{LAR}) &= 0.025 \\P(FL-2_{LAR}) &= 0.035\end{aligned}$$

The probability that the two flights arrive late is:

$$\begin{aligned}P(FL-1_{LAR}) \cap P(FL-2_{LAR}) &= P(FL-1_{LAR}) \times P(FL-2_{LAR}) \\P(FL-1_{LAR}) \cap P(FL-2_{LAR}) &= 875 \times 10^{-6}\end{aligned}$$

Measuring Risk and Reward

Risk in traditional terms measures the deviation of the actual outcome from its expected result. Generally, the concept of risk may have negative connotations, but a risk can also have positive results (Gray and Hamilton, 2006). For example, the risk of an investment measures the chance that the investment's actual return will be different from what is expected. An example of risk is drawing an ace from a deck of playing cards. Before we draw, we know in advance what the odds are for each possible outcome.

Risk refers to the dispersion or deviation of returns from the average return of such experiment.

Investors purchase financial assets such as shares of stock primarily to earn a positive rate of return on their investments. Investors like high expected returns but dislike high volatility. Measuring risk and reward, the actual future, however, is uncertain and unpredictable; investors cannot assert with certainty the rate of return their investments will realize.

Managing risk under certainty is not very difficult, you just need to balance up your investment to the odds of it paying off. Managing uncertainty is more difficult. The general assumption is that individuals base their decisions on what they expect to happen and their instinct as to what occurs will be close to what they expect to happen. When evaluating potential financial investments, these two dimensions of the decision-making process are called "Expected Return" and "Risk."

Example 10.4

By using simple probability techniques, we can estimate the likelihood of different events' occurrence in the future. Assume two cards are drawn from an ordinary deck of 52 playing cards without replacement. What is the probability that both cards are from the hearts set?

- ♦ There are 13 hearts out of 52 cards total.
So, $P(\text{drawing heart on first draw}) = 13/52$

- ♦ After the first draw, there are 12 hearts out of 51 total.
So, $P(\text{drawing heart on second draw}) = 12/51$

Since the cards are drawn without replacement,

$$P(\text{both hearts}) = P(\text{drawing heart on first draw}) \times P(\text{drawing heart on second draw})$$

$$P(H \cap H) = (13/52) \times (12/51)$$

$$P(H \cap H) = \frac{156}{2,652}$$

Hence, the probability of drawing two hearts from a deck of cards, without replacement, is

$$P(H \cap H) = \frac{1}{17}$$

Sample Space

In probability theory, the sample space of an experiment is the set of all possible outcomes for that experiment. Consider the experiment of tossing a dice, where the typical sample space is $\{1, 2, 3, 4, 5, 6\}$. The concept of a sample space is also relevant for experiments where the outcomes are non-numerical. When determining a sample space, be sure to consider all possibilities. Suppose you reach into your pocket and pull out a coin, and you toss the coin three times. In this case the sample space is:

$$SS = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$SS = \left\{ \frac{1}{8}, \frac{1}{8}, \dots, \frac{1}{8} \right\}$$

The sum of the probabilities of the distinct outcomes within this sample space is:

$$Sum = \frac{1}{8} + \frac{1}{8} + \dots + \frac{1}{8} = 1$$

Example 10.5

In mathematics, a prime number is defined as a number which can be divided only by 1, or itself. The sample space for choosing a prime number less than 30 at random is given by:

$$SS = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$$

Example 10.6

A family has four children. Using B to stand for boy and G to stand for girl, provide the sample space for this experiment. What is the probability that all the children are boys?

$$SS = \{GGGG, GGGB, GGBB, GBBB, BBBB\}$$

$$P(B \cap B \cap B \cap B) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 6.25\%$$

Expected Return or Mean

Expected return is the amount of return one would anticipate receiving on an investment that has different potential rates of return.

An amount of return from any investment can be measured by subtracting the amount invested from the final amount realized. For example, if you invested \$100 at the beginning of the year and got a return of \$10 as dividend at the end of the year and finally you sold your investments at the end of the year for \$105, then the above returns can be presented as follows:

$$\text{Return} = \$10 \text{ (dividend)} + (\$105 - \$100)$$

$$\text{Total return} = \$15$$

$$\text{Average return} = \frac{\$15}{\$100}$$

$$\text{Average return} = 15\%$$

Although the Expected Return is an estimate of an investor's expectations of the future, it can be estimated using either ex ante (forward looking) or ex post (historical) data, generally called the average or mean.

Expected return is one of the fundamental concepts in engineering economics, in a sense crucial in understanding any investment option. The expected rate of return uses probabilities to weigh the various return outcomes that an investment may produce. The term mean is used to describe the central tendency of any data set. The mean or the average is calculated by adding up all the numbers and then dividing it by the number of observations.

$$\bar{X} = \sum_{i=1}^n \frac{X_1 + X_2 + \dots + X_n}{n}$$

The expected return represents the mean of possible future returns. In simple terms, expected return is the sum of each of the possible outcomes, multiplied by its probability.

$$E(\bar{R}) = \sum_{i=1}^N P_i \times R_i$$

Where:

$E(\bar{R})$ = Expected return

N = Number of possible outcomes

P_i = Probability of outcome i

R_i = Return on outcome i

Example 10.7

When an investor has historical data, he can examine how the investment has performed in the past and use that as an indication of how it may perform in the future. You are to start a new investment in three different mutual funds with varying risk. The first mutual fund has a 25 percent probability of providing a 10 percent rate of return. The second mutual fund has a 50 percent probability of earning an 8 percent rate of return, and the last one a 25 percent probability of providing a 12 percent rate of return. The expected rate of return is:

$$E(\bar{R}) = (0.25) \times (10\%) + (0.50) \times (8\%) + (0.25) \times (12\%)$$

$$E(\bar{R}) = 9.5\%$$

Positive return alone does not determine the quality of an investment. The concept that risk and return are positively correlated is one of the most central theories of finance (Danfy, 1975). An investment with a high return most probably entails a high amount of risk as well. Risk factors vary from one type of investment to another. For example, the risk of investing in low grade securities (junk bonds) can be high, but higher returns are required to compensate for the higher risk.

Example 10.8

From its hub in Pittsburgh, DirectJet Airlines serves four different destinations. The airline has noticed that inbound flights are delayed at their point of departure. Determine the expected number of flight delays if the historical data shows: 10 percent of the time DirectJet will have 20 delays, 25 percent of the time it will have 10 delays, 35 percent of the time it will have seven delays, and the remaining time it will have three delays.

$$\begin{aligned} \text{Expected number of delays for January} &= (10\% \times 20) + (25\% \times 10) + (35\% \times 7) \\ &\quad + (30\% \times 3) \\ \text{Expected number of delays} &= 2.0 + 2.5 + 2.45 + 0.9 \\ \text{Expected number of delays} &= 7.85 \text{ flights} \end{aligned}$$

Example 10.9

A major part of statistical quality control and statistical process control deals with the concepts of probability. A company produces electronic chips by four different processes. Assume that an analyst writes a report on the company and, based on the research, he assigns the following probabilities to the defective items:

Table 10.2 Probabilities Assigned to Defective Items

Process	Probability of Defective Items	Items Produced
A	0.05	10,000
B	0.02	25,000
C	0.01	15,000
D	0.04	20,000

The quality controller's expected number of defective items is

$$E(D) = (0.05) \times (10,000) + (0.02) \times (25,000) + (0.01) \times (15,000) + (0.04) \times (20,000)$$

$$E(D) = 1,950$$

Variance (σ^2)

The variance measures variation of selected observations from the mean, and it is symbolized by σ^2 (pronounced "sigma squared"). A small variance denotes that numbers in the set are distributed around the mean value, and consequently a variance of zero indicates that all the values are identical with the mean. When we have historical data, we can study how the observations have varied in the past and use that as an indication of how they may vary in the future. Variation has been studied by scientists, engineers, and statisticians to evaluate the performance of projects.

The concept of variance has been also used extensively by engineering economists. Variance of the actual return from the previously calculated expected return denotes the risk involved in that particular decision (Hood et al., 2001). The mathematical formula to calculate the variance is given by:

$$\sigma^2 = \frac{\sum_{i=1}^N [X_i - E(\bar{X})]^2}{N}$$

Where:

$E(\bar{X})$ = Mean (can also be denoted as μ)

N = Number of observations

X_i = Value of observation i

Example 10.10

A random sample of 10 flights reported the following number of passenger no-shows. The mean and the variance can be calculated as shown in Table 10.3.

Use the "AVERAGE" function in Excel to find the mean of a set of numbers. Enter the range of numbers (no-shows) in your Excel spreadsheet.

Standard Deviation (σ)

In economics and finance, standard deviation and variance are the most commonly used terms to denote risk associated with a project. Standard deviation measures how spread out the values in a data set are around the mean, and can be found by

taking the square root of variance (Bernhard, 1967). The standard deviation, denoted by Greek letter " σ " (sigma), is calculated by taking the square root of the variance.

$$SD(R) = \sigma = \sqrt{\sigma^2}$$

$$SD(R) = \sqrt{Var(R)}$$



Table 10.3 Mean and Variance

Day	No-shows (X)	$(X-\mu)$	$(X-\mu)^2$
1	1	-1	1
2	2	0	0
3	6	4	16
4	1	-1	1
5	1	-1	1
6	1	-1	1
7	2	0	0
8	2	0	0
9	1	-1	1
10	3	1	1
Sum	20		22
Mean (μ)	2.0		
Variance (σ^2)			2.2

Example 10.11

DirectJet Airlines, on a domestic flight, in the past 10 days, has found the following number of no-show passengers (Table 10.4). First, we calculate the mean or expected value. In this case, we are expecting to have four no-shows on average on each flight. Second, we subtract the mean from each observation. Third, we square each and every deviation. Finally, the variance is the sum of the last row in this table divided by 10 (i.e., a number of observations—days). Once expected return is known, Variance, which is a measure of risk, can be calculated.

To calculate standard deviation from variance, just take the square root. In our example, the variance was 6.4, and therefore the standard deviation is 2.5. The standard deviation is expressed in the same units as the mean is, whereas the variance is expressed in squared units, in this case the number of no-show passengers. For looking at a distribution, you can use either standard deviation or variance as long as you are clear about what you are intending to achieve. Variance and standard deviation can also be calculated using Excel functions. An illustration of how that can be done is provided below.

Calculating Variance and Standard Deviation Using Spreadsheets**Step 1 (Figure 10.3):**

Enter the data into the Excel spreadsheet as in Figure 10.3 in the cells B2 to B11.

Table 10.4 Variance and Standard Deviation

Day	No-shows (X)	(X- μ)	(X- μ) ²
1	5	1	1
2	2	-2	4
3	7	3	9
4	1	-3	9
5	0	-4	16
6	8	4	16
7	2	-2	4
8	6	2	4
9	5	1	1
10	4	0	0
Sum	40.0		64
Mean (μ)	4.0		
Variance (σ^2)			6.4
Std. deviation (σ)			2.5

Step 2 (Figure 10.3):

Select cell B12 and enter the following formula to calculate statistical variance of a given population in Microsoft Excel: **=VAR.P(B2:B11)**.

Figure 10.3 Data and Formula Entry for Variance Calculation

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D
1	Day	No shows (X)		
2	1	5		
3	2	2		
4	3	7		
5	4	1		
6	5	0		
7	6	8		
8	7	2		
9	8	6		
10	9	5		
11	10	4		
12	Variance (σ^2)	=VAR.P(B2:B11)		
13				

The formula bar at the top shows the active formula: **=VAR.P(B2:B11)**. A tooltip for the VAR.P function is visible at the bottom, showing the syntax: VAR.P(number1, [number2], ...).

Note: Use the “=VAR.P” function if you are dealing with the entire population or the “=VAR.S” function if you are dealing with the sample.

Step 3 (Figure 10.4):

Hit Enter to obtain the result.

Figure 10.4 Result Obtained for Variance Using Excel Formula

B12 ✕ ✓ <i>fx</i> =VAR.P(B2:B11)				
	A	B	C	D
1	Day	No shows (X)		
2	1	5		
3	2	2		
4	3	7		
5	4	1		
6	5	0		
7	6	8		
8	7	2		
9	8	6		
10	9	5		
11	10	4		
12	Variance (σ^2)	6.4		

The formula used to determine standard deviation is “=STDEV.P” which denotes standard deviation of a given population.

Note: Use the “=STDEV.P” function if you are dealing with the entire population or the “=STDEV.S” function if you are dealing with the sample.

Follow the same steps as stated above, but replace the formula in Step 2 with =STDEV.P(B2:B11)

The procedure and the result obtained are shown in Figures 10.5 and 10.6 respectively.

Table 10.5 gives another example, which considers the returns for a certain portfolio consisting of varied probabilities for each return. The variance and standard deviation in such a case can be calculated as explained further below.

Therefore, Variance = 0.0127, which, in turn, means Standard Deviation = 0.1126 or 11.26%.

The measure of risk involved in deciding on investing in this asset is 11.26 percent.

This can be represented as a normal distribution, as shown in Figure 10.7. Sixty-eight percent of the returns will be between one standard deviation less than the mean and one standard deviation more than the mean. This is clearly depicted in the graph which is a normal probability distribution for the returns (Tijms, 2004). The mean return of 10 percent has a 50 percent probability of occurring.

Figure 10.5 Calculation of Standard Deviation Using Excel Formula

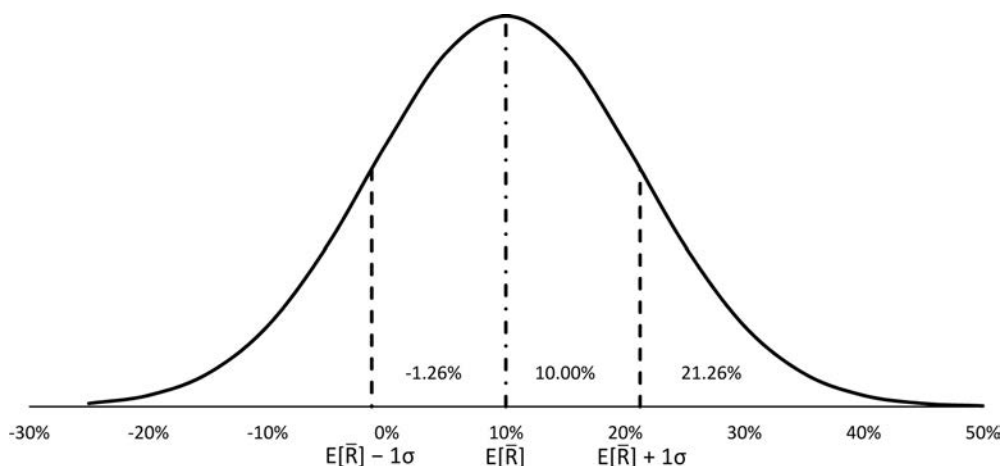
	A	B	C	D
1	Day	No shows (X)		
2	1	5		
3	2	2		
4	3	7		
5	4	1		
6	5	0		
7	6	8		
8	7	2		
9	8	6		
10	9	5		
11	10	4		
12	Std. Deviation (σ)	=STDEV.P(B2:B11)		
13		STDEV.P(number1, [number2], ...)		

Figure 10.6 Result Obtained for Standard Deviation Using Excel Formula

	A	B	C	D
1	Day	No shows (X)		
2	1	5		
3	2	2		
4	3	7		
5	4	1		
6	5	0		
7	6	8		
8	7	2		
9	8	6		
10	9	5		
11	10	4		
12	Std. Deviation (σ)	2.53		

Table 10.5 Calculation of Portfolio Returns Variance

R_i	p_i	$E[\bar{R}] = (R_i)(p_i)$	$(R_i - E[\bar{R}])^2 p_i$
-10%	0.1	-1.00%	0.0040
-5%	0.2	-1.00%	0.0045
18%	0.4	7.20%	0.0026
19%	0.2	3.80%	0.0016
10%	0.1	1.00%	0.0000
<i>Sum</i>	<i>1</i>	<i>10.00%</i>	<i>0.0127</i>

Figure 10.7 Normal Distribution for Returns

Normal Distribution

The normal distribution is the most widely and frequently used distribution in statistics. The shape of the normal distribution resembles that of a bell which is why sometimes it is referred to as bell curve (Lehmann, 1997). A normal distribution is characterized by two parameters:

μ = Mean

σ = Standard Deviation

The smaller the standard deviation, the more concentrated the data is. The normal distribution curve is symmetrical, centered around its mean, and the mean, mode, and the median are the same.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The distribution is symmetrical in that it divides data into two equal halves at the mean, so that 50 percent of the data fall below the mean and 50 percent above the mean, and the entire area under the normal curve contains 100 percent of the data.

About 68 percent of all cases fall within one standard deviation of the mean, that is

$$\diamond \quad \mu \pm 1\sigma = 68\%$$

About 95 percent of cases lie within two standard deviations of the mean, that is

$$\diamond \quad \mu \pm 2\sigma = 95\%$$

And finally, about 99 percent of cases lie within three standard deviations of the mean, that is

♦ $\mu \pm 3\sigma = 99\%$

Example 10.12

In general, engine life is measured in terms of two things: flight hours and flight cycles.² Assume the life of a jet engine is normally distributed with a mean of 30,000 hours and standard deviation of 2,000 hours. Construct a normal distribution interval for this engine:

- ♦ 68%: $30,000 \pm 2,000$ (28,000 – 32,000)
- ♦ 95%: $30,000 \pm 2 \times 2,000$ (26,000 – 34,000)
- ♦ 99%: $30,000 \pm 3 \times 2,000$ (24,000 – 36,000)

Figure 10.9 Normal Distribution ($\pm 1\sigma$)

$\mu \pm 1\sigma = 68\%$

Figure 10.8 Normal Distribution

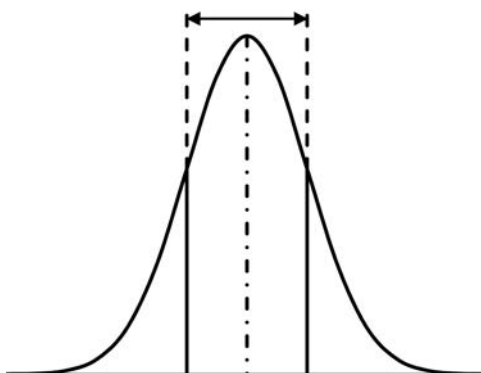
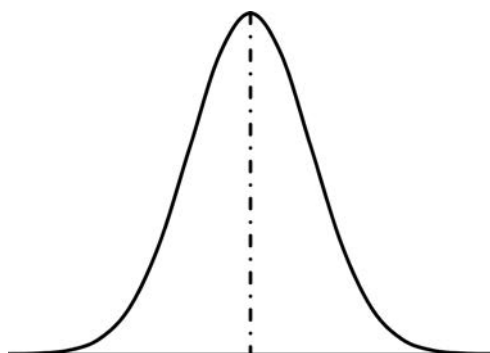


Figure 10.10 Normal Distribution ($\pm 2\sigma$)

$\mu \pm 2\sigma = 95\%$

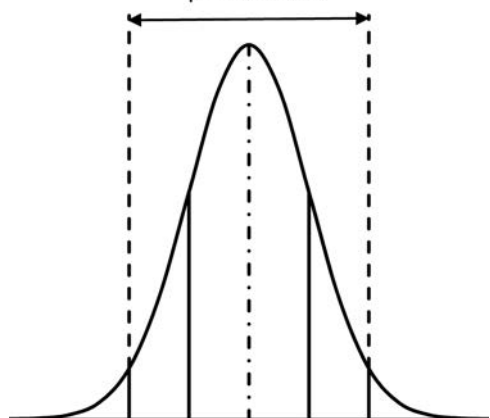
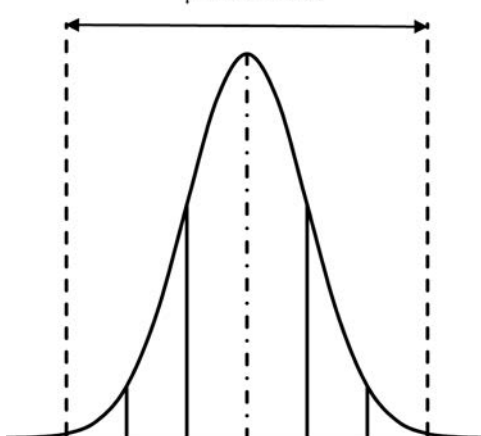


Figure 10.11 Normal Distribution ($\pm 3\sigma$)

$\mu \pm 3\sigma = 99\%$



Example 10.13

The total number of passengers for a particular flight at a given time of the day fluctuates day by day. Airlines have no way of knowing exactly how many passengers book a given flight. Based on many empirical studies of actual airline passenger records, airlines may choose normal distribution functions to estimate the future number of passengers for a given flight. Airlines regularly estimate the number of no-show passengers in order to minimize the number of empty seats by overbooking the aircraft. Assume the average number of no-show passengers is distributed normally with a mean of 10 and standard deviation of 2 passengers. Let us apply the characteristics of the normal distribution to this problem:

- ♦ $10 \pm 1 \times 2 = 68\%$
- ♦ $10 \pm 2 \times 2 = 95\%$
- ♦ $10 \pm 3 \times 2 = 99\%$

Calculating Normal Distribution Probability Using Spreadsheets

The normal distribution for the scenarios presented above can be found using the Excel function “=NORM.DIST”. The parameters of the function are the following:

=NORM.DIST(x, Mean, Standard Deviation, “True” for cumulative or “False” for discrete). An illustration is presented in Figure 10.12.

Step 1:

Enter mean, cell B1.

Step 2:

Enter standard deviation, cell B2.

Step 3:

In cells A4 to A14, enter the values 0 to 20 with increments of 2.

Step 4:

In cell B4, enter the formula: =NORM.DIST(A4, \$B\$1, \$B\$2, TRUE)

Note: make sure you “freeze” mean and standard deviation values by typing the dollar sign for both column and row or by simply pressing the F4 button.

Step 5:

Drag the above formula until cell B14 to obtain results.

The above representation can then be plotted into a graph that forms the shape of a traditional normal distribution, which was described previously.

Standard Normal Distribution

Because μ and σ can have infinitely different values, there are infinitely many normal distributions. To make the comparison among different distributions possible, the normal distributions can be rescaled into a standard form by a simple

A standard normal distribution is a normal distribution with a mean of 0 and a standard deviation of 1.

Figure 10.12 Normal Distribution Using Excel Function

PMT		✕ ✓ f_x		=NORM.DIST(A4,\$B\$1,\$B\$2,TRUE)		
	A	B	C	D	E	F
1	Mean	10				
2	Std. Deviation	6.32				
3						
4	0	=NORM.DIST(A4,\$B\$1,\$B\$2,TRUE)				
5	2	NORM.DIST(x, mean, standard_dev, cumulative)				
6	4	0.1714				
7	6	0.2635				
8	8	0.3759				
9	10	0.5000				
10	12	0.6241				
11	14	0.7365				
12	16	0.8286				
13	18	0.8970				
14	20	0.9431				

linear transformation. Standardizing a normal distribution facilitates the calculation of areas under various regions of the curve. If X is a normal random variable with mean μ_x and σ_x , therefore, the standard normal variable is measured by:

$$Z = \frac{x - \mu_x}{\sigma_x}$$

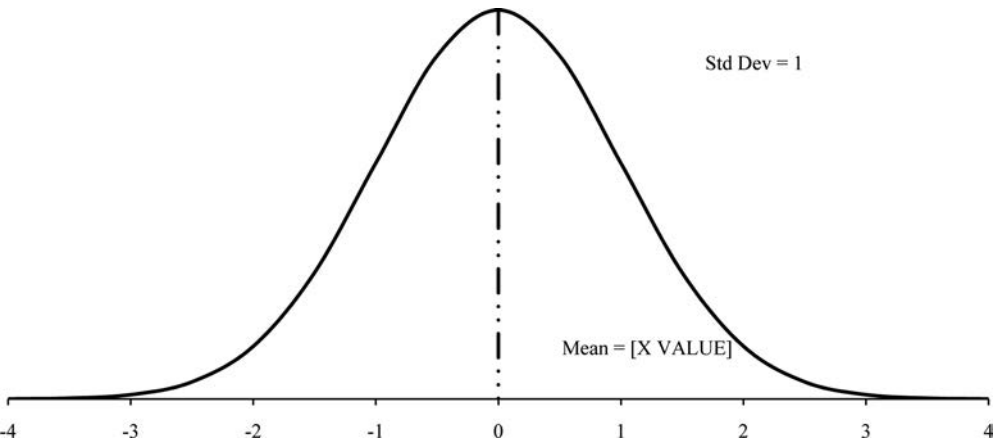
If all the values in a distribution are transformed to Z-scores, then the distribution will have a mean of 0 and a standard deviation of 1. The notation $N(\mu, \sigma^2)$ means normally distributed with mean μ and variance σ^2 .

Hence, a normal distribution with a $\mu=0$, and $\sigma=1$ is called a standard normal distribution. The main application of a standard normal distribution is that it allows the comparison of two or more normal distributions which may be of different scales. The standard normal distribution ensures that all the normal distributions are converted into normal distributions with a mean of zero and standard deviation of 1. A standard normal distribution is depicted in Figure 10.13.

Example 10.14

DirectJet Airlines charges five different ticket prices on a given flight with an aircraft with a hundred seats capacity. You believe you will only sell 10 percent of your seats at \$1,800, 12 percent at \$1,500, 22 percent at \$1,200, 24 percent at \$900, and finally 32 percent at \$600. For this flight, multiply the ticket price number by the probability of selling that seat at the specified price. Make the calculations for each ticket price,

Figure 10.13 Standard Normal Distribution



and add the numbers together to get your expected revenue from each seat; multiply that number by 100 to get the estimated total revenue from this flight.

Hence the expected value from the sale of each ticket is \$1,032.

The variance can be calculated by finding the variance measures and then summing them up.

Table 10.6 Probability Profile for Different Ticket Prices

Ticket Price	Probability
\$1,800	0.10
\$1,500	0.12
\$1,200	0.22
\$900	0.24
\$600	0.32
<i>Sum</i>	<i>1.00</i>

Table 10.7 Calculation of Expected Value of the Ticket

Ticket Price	Probability	Price × Probability
\$1,800	0.10	180.00
\$1,500	0.12	180.00
\$1,200	0.22	264.00
\$900	0.24	216.00
\$600	0.32	192.00
<i>Sum</i>	<i>1.00</i>	<i>1032.00</i>

Table 10.8 Calculation of Variance

Ticket Price	Probability	(Value)×(Probability)	Variance Measures
\$1,800	0.10	180.00	58,982.40
\$1,500	0.12	180.00	26,282.88
\$1,200	0.22	264.00	6,209.28
\$900	0.24	216.00	4,181.76
\$600	0.32	192.00	59,719.68
<i>Sum</i>	<i>1.00</i>	<i>1032.00</i>	<i>155,376.00</i>

Variance = 155,376

Hence,

$$\text{Standard Deviation} = \sqrt{155,376}$$

$$\text{Standard Deviation} = 394.18$$

From the resulting normal distribution, the risk associated with selling the tickets at various prices can be found by converting the normal distribution into a standard normal distribution. For instance, the risk associated with selling the tickets at \$1,400 can be found as follows:

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{1400 - 1032}{394.18}$$

$$Z = 0.93$$

In the Standard Normal Probabilities Table, a Z-value of 0.93 corresponds to 0.8238 probability, so the risk associated with the sale of the tickets at \$1,400 is found to be 82.38 percent. This can be clearly visualized from Figure 10.14, which provides a normal distribution for the ticket prices. Sixty-eight percent of the time, the tickets will be sold at a price that is between one standard deviation above (\$1,426.18) and one standard deviation below (\$637.82) the mean which is the expected value of \$1,032.

A normal distribution can be used as a means of comparison when different sets of data have the same mean but different standard deviations. Consider the example depicted in Figure 10.15; the mean of all the three sets of data is 2.0 and the standard deviations are 1.9, 1.5, and 0.9. The smaller the standard deviation, the narrower the normal distribution curve.

Example 10.15

According to DirectJet's passenger booking report, about 10 percent of the ticketed passengers do not show up for their flight. Find the probability that five or more of the 100 ticketed passengers do not show for their flight. Assume the number of no-show passengers follows a normal distribution.

Figure 10.14 Normal Distribution of Ticket Prices

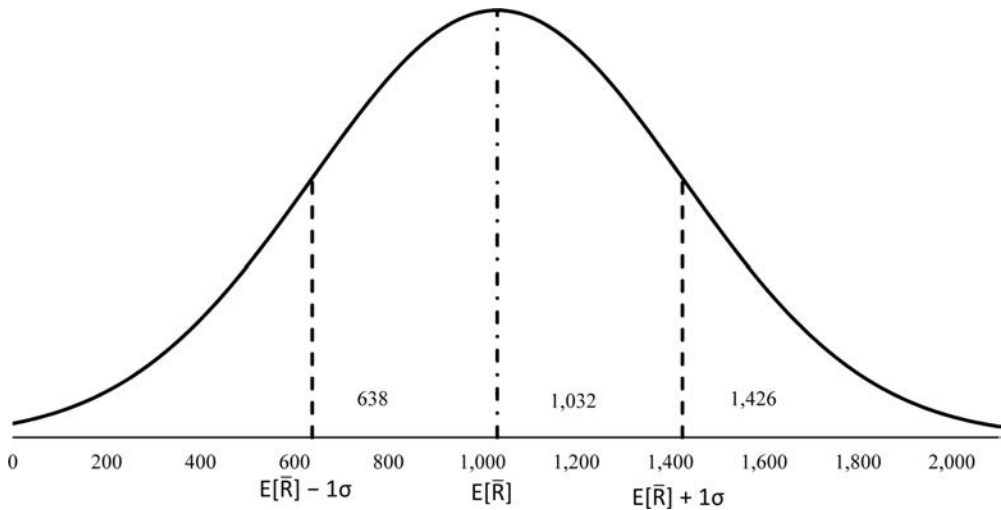
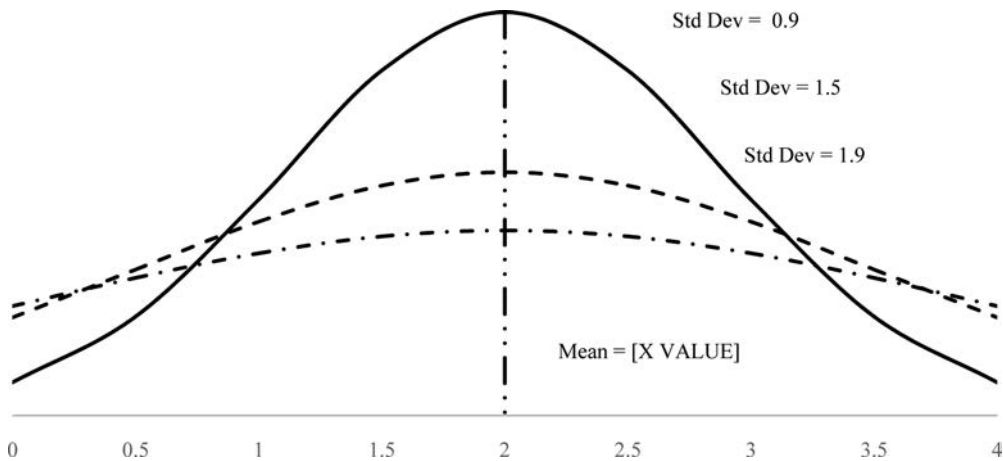


Figure 10.15 Normal Distribution with Varying Standard Deviations



Solution:

$$\begin{aligned}\mu &= n \times P \\ \mu &= 100 \times 10\% = 10 \text{ passengers} \\ \sigma &= \sqrt{n \times P \times q} \\ \sigma &= \sqrt{100 \times 0.10 \times 0.90} \\ \sigma &= \sqrt{9} \\ \sigma &= 3\end{aligned}$$

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{5 - 10}{3} = -1.67$$

$$P(Z \leq -1.67) = 0.0475$$

$$P(Z \leq -1.67) = 4.75\%$$

$$P(Z \geq -1.67) = 1 - 0.0475$$

$$P(Z \geq -1.67) = 0.9525$$

$$P(Z \geq -1.67) = 95.25\%$$

Hence, the probability that five or more of the 100 ticketed passengers do not show up for the flight is 95.25 percent.

Portfolios

A portfolio is a collection of financial assets (stocks, bonds, bank deposits) or physical assets (aircraft, inventory, equipment, plants). Investors can construct portfolios to balance their financial objectives and strategies. From an economic point of view, a portfolio is constructed to stabilize the risk of non-performance of various units of investment. Rational investors select portfolios by evaluating returns as well as risk. Investors may be willing to assume higher risk if they are adequately compensated by a higher expected return. If you own more than one security, undoubtedly, you have an investment portfolio. The GE Capital Aviation Services (GECAS) portfolio contains a fleet of over 1,800 narrow-body and wide-body passenger aircraft, cargo aircraft, and regional and turboprop aircraft from manufacturers such as Boeing, Airbus, Embraer, and Bombardier. About 245 airline customers finance these aircraft through leases. If GECAS invests in only one type of aircraft, the return will depend solely on that aircraft; if something goes wrong with that aircraft, GECAS's entire return will be severely affected. By diversifying the fleet, GECAS can substantially reduce the risk of the single type aircraft (security). Therefore, if an investor owns shares in the Boeing Company, Pratt and Whitney, and debt instruments such as bonds in Airbus Group, one collectively refers to these as the investor's portfolio.

Airbus designed the A350 to replace the older generation of A340s, A342s, and A343s. Airbus knew that they would face a gap in their product portfolio, the wide-body short-to-medium-range airliner, so the new aircraft was launched to provide a full menu of aircraft to the airline customers.

A product portfolio varies significantly with the overall strategy of corporations.

Aviation Snippets

There has been considerable interest in aircraft portfolios since the second quarter of this year, with money still pouring into the aircraft finance sector. At the moment there is no shortage of takers when portfolios are on offer.

Flightglobal, October 2015

Portfolio Beta

As we mentioned above, a portfolio is a specific combination of financial securities, usually defined by portfolio weights that sum up to 1.

$$w = \{w_1, w_2, \dots, w_n\}$$

$$\sum_{i=1}^n w_i = 1$$

$$w_i = \frac{n_i \times P_i}{n_1 \times P_1 + \dots + n_m \times P_m}$$

A beta is a degree of volatility of a security or a portfolio in comparison to the market as a whole. When beta is positive, the stock price tends to move in the same direction as the market, and the magnitude of beta tells by how much. A beta of 2.00 relative to the market implies that if the market's return increases by 10 percent the portfolio is expected to increase by 20 percent. The portfolio beta is a weighted average of the betas of the stocks within the portfolio. Hence, the beta of the market is by definition equal to 1.0, as the market is being compared to itself. Individual stock betas are very important when selecting a portfolio of assets. Generally, a beta less than 1 indicates that the investment is less volatile than the market, whereas a beta greater than 1 indicates that the investment is more volatile than the market.

Calculating the weighted average beta of a portfolio allows the investor to measure the overall risk of the portfolio. The use of a weighted average accounts for the fact that different amounts are invested in each stock, so the betas of the stocks that are owned in larger quantities will have a greater effect on the overall portfolio beta than those of the stocks owned in smaller quantities:

$$\beta_{\text{Portfolio}} = w_1 \times \beta_1 + w_2 \times \beta_2 + \dots + w_n \times \beta_n$$

Beta measures the responsiveness of a security to movements in the market portfolio (Ross et al., 2009). It is calculated using the formula:

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma^2(R_M)}$$

Where:

$\text{Cov}(R_i, R_M)$ = Covariance between the return on asset i and the return on market portfolio

$\sigma^2(R_M)$ = Variance of the market.

Thus, beta can be considered a value that relates the expected return and the return on the market, implying that expected return on a security is linearly related to its beta.

Most investors use beta calculated by a third party: online brokerages give investors extensive data on a stock's beta value, and some free financial news and analysis websites also show current beta measurements.

Example 10.16

Your investment account of \$500,000 consists of three stocks: 400 shares of stock A, 2,000 shares of stock B, and 1,500 shares of stock C. Your portfolio is summarized by

Table 10.9 Investment Summary

Asset	Shares	Price per Share	Investment	Portfolio Weight
A	400	\$500	\$200,000	40%
B	2,000	\$75	\$150,000	30%
C	1,500	\$100	\$150,000	30%
Total			\$500,000	

Table 10.10 Calculation of Portfolio Beta

Projects	Beta	Percentage of Portfolio	Beta times % of Portfolio
A	0.35	40%	0.140
B	0.50	30%	0.150
C	0.76	30%	0.228

the following weights. Assume you have invested in three different assets. Let's assume Stock A has a 0.35 beta and represents 40 percent of your portfolio. Stock B has a beta of 0.5 and is 30 percent of your portfolio. Stock C has a 0.76 beta and is 30 percent of your portfolio. First, we will multiply each individual stock's beta with the respective weight in the portfolio.

We should add up the last column to determine our portfolio's beta. In this case, the portfolio's beta is 0.518.

Capital Asset Pricing Model (CAPM)

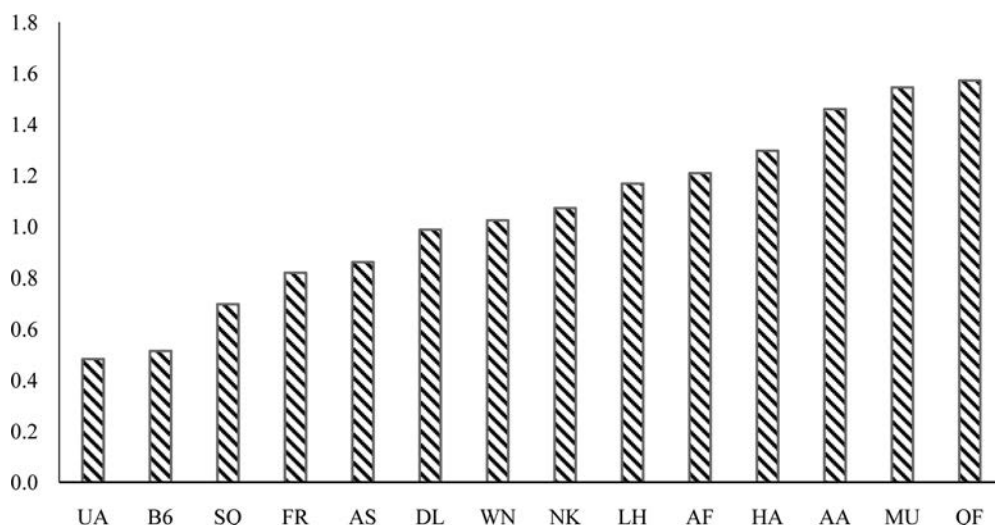
The Capital Asset Pricing Model (CAPM) is a model that describes the relation between risk and expected return, and is one of the most significant innovations in portfolio theory. The CAPM model is based on the relationship between an investment's expected return and the average market return. If this expected return does not satisfy the required return, then the investment should not be undertaken. An extension of expected returns is the CAPM, which argues that expected return should be positively related to risk. That is, a risky asset will be purchased only if it promises a high return which compensates for the risk. Building on the earlier work of Harry Markowitz (1952) on modern portfolio theory, William F. Sharpe (1964), and Jan Mossin (1966) independently introduced the CAPM model, which is represented by the following equation:

$$R_{RE} = R_{RF} + \beta(R_M - R_{RF})$$

Where:

R_{RE} = Required rate of return

R_{RF} = Risk-free rate of return

Figure 10.16 Beta for Selected World Airlines as of March 2016

R_M = Market rate of return

β = Beta: sensitivity of the return on the investment to the market return

The risk-free rate (R_{RF}) is generally associated with the returns obtained on government securities such as Treasury bills because their returns are the most certain. $R_M - R_{RF}$ is referred to as “Market Premium,” which is the difference between the expected return on the market and the risk-free rate. The beta for selected world airlines is presented in Figure 10.16.

Example 10.17

As a financial analyst, you could use CAPM to decide what rate of return you should require for a particular investment. If investment in DirectJet is riskier than the market, naturally you require a higher return. Assume your company is planning to invest \$100 million in DirectJet equity, and assume the following information is provided:

$$R_{RF} = 2\%$$

$$R_M = 10\%$$

$$\beta = 1.5$$

By using CAPM, we calculate that you should demand the following rate of return to invest in DirectJet:

$$R_{RE} = R_{RF} + \beta(R_M - R_{RF})$$

$$R_{RE} = 2\% + 1.5(10\% - 2\%)$$

$$R_{RE} = 14\%$$

The best way to come up with market risk premium is the use of historical data. Once all these parameters are known, the expected return on a security can be estimated using CAPM (Markowitz, 1952).

The concept of CAPM can be illustrated better in the following example. Let us presume that the risk-free rate equals 4 percent, the beta of the stock is 1.5 and the expected market return over the period equals 14 percent. Then, as per the aforementioned formula, the stock is expected to produce returns equivalent to:

$$\begin{aligned} R_{RE} &= R_{RF} + \beta \times (R_M - R_{RF}) \\ R_{RE} &= 4\% + 1.5 \times (14\% - 4\%) \\ R_{RE} &= 19.00\% \end{aligned}$$

Example 10.18

Assume a risk free rate of 2.00 percent and a market required return of 9.00 percent. DirectJet has a beta of 4, and it offers a return of 17 percent at the moment. Is DirectJet fairly priced according to the CAPM?

Solution:

$$\begin{aligned} R_{RE} &= 2\% + 4 \times (9\% - 4\%) \\ \text{DirectJet's } R_{RE} &= 22.00\% \end{aligned}$$

This means that the risk-based appropriate return should be 22 percent. The return of 17 percent offered by DirectJet is less than the required return. Therefore, DirectJet is not fairly priced according to CAPM as the current return does not adequately compensate for the risk undertaken by an investor.

Example 10.19

You own \$200,000 of DirectJet Airline's stock that has a beta of 3.0. You also own \$300,000 of EZjET Airlines (beta = 1.5) and \$300,000 of FunJet (beta = 0.4). Assume that the market return will be 14 percent and the risk-free rate is 6 percent.

- a. What is the market risk premium?
 - b. What is the required rate of return on each stock?
 - c. What is the required rate of return on the portfolio?
- a. Market risk premium = $14\% - 6\% = 8\%$
 - b. $R_{RE(\text{DirectJet})} = 6\% + 3 \times (14\% - 6\%) = 30\%$
 $R_{RE(\text{EZjET})} = 6\% + 1.5 \times (14\% - 6\%) = 18\%$
 $R_{RE(\text{FunJet})} = 6\% + 0.4 \times (14\% - 6\%) = 9.2\%$
 - c. Total value = $\$200,000 + \$300,000 + \$300,000 = \$800,000$
 DirectJet weight = $\$200,000 / \$800,000 = 25\%$
 EZjET weight = $\$300,000 / \$800,000 = 37.5\%$
 FunJet weight = $\$300,000 / \$800,000 = 37.5\%$
 Portfolio beta = $0.25 \times 3.00 + 0.375 \times 1.5 + 0.375 \times 0.4 = 1.4625$
 Required rate of return on the portfolio = $6\% + 1.4625 \times (14\% - 6\%) = 17.7\%$

Security Market Line (SML)

Capital asset pricing can also be presented graphically. According to the CAPM model, R_{RE} can be plotted as a function of systematic risk, β . The security market line (SML) plots

the expected return of stocks on the y-axis, against beta on the x-axis. The SML may be an effective tool when looking at portfolio composition at different market returns and risk (Scheinkman and Xiong, 2003).

In Figure 10.17 expected returns are plotted against the portfolio betas. This graph of a security market line assumes a market return of 14 percent and a risk-free rate of 4 percent. The slope of the SML is equal to the market risk premium, and the expected return with a beta of 0 is equal to the risk-free rate, while the expected return with a beta of 1 has a relative risk equal to the market (14 percent).

The security market line (SML) is a graphical representation of the capital asset pricing model with beta on the **x-axis** and expected return on the **y-axis**.

Example 10.20

Use the security market line to determine the required rate of return for the Atlas Avionics stock. The Atlas Avionics has a beta of 1.25, the required return in the market is 12.00 percent, and the risk-free rate of return is 2.00 percent.

Solution:

$$R_{RE} = 2\% + 1.25 \times (12\% - 2\%)$$

$$R_{RE} = 14.50\%$$

An investor with a low risk tolerance would choose an investment at the beginning of the security market line. An investor with a higher risk tolerance would thus choose an investment higher along the security market line. If the risk premium required by investors was to change, the slope of the SML would change as well.

Figure 10.17 Security Market Line (SML)

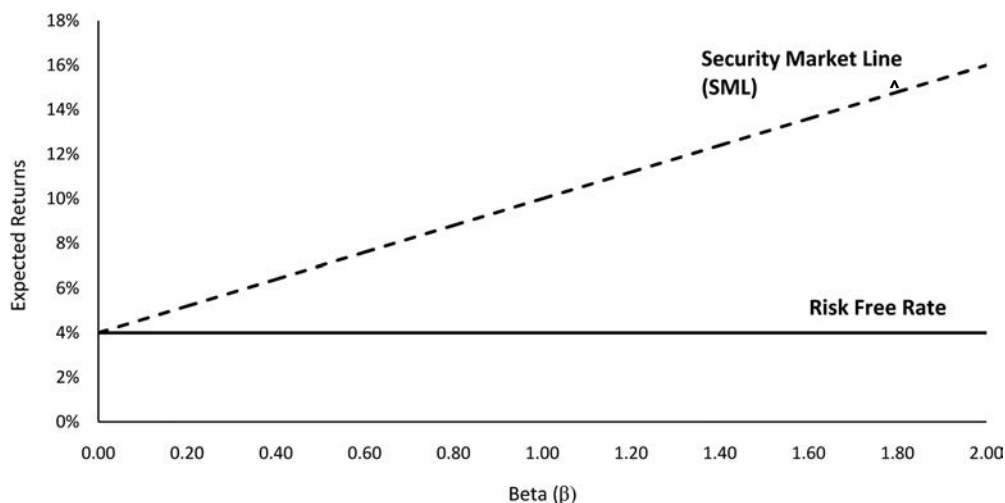
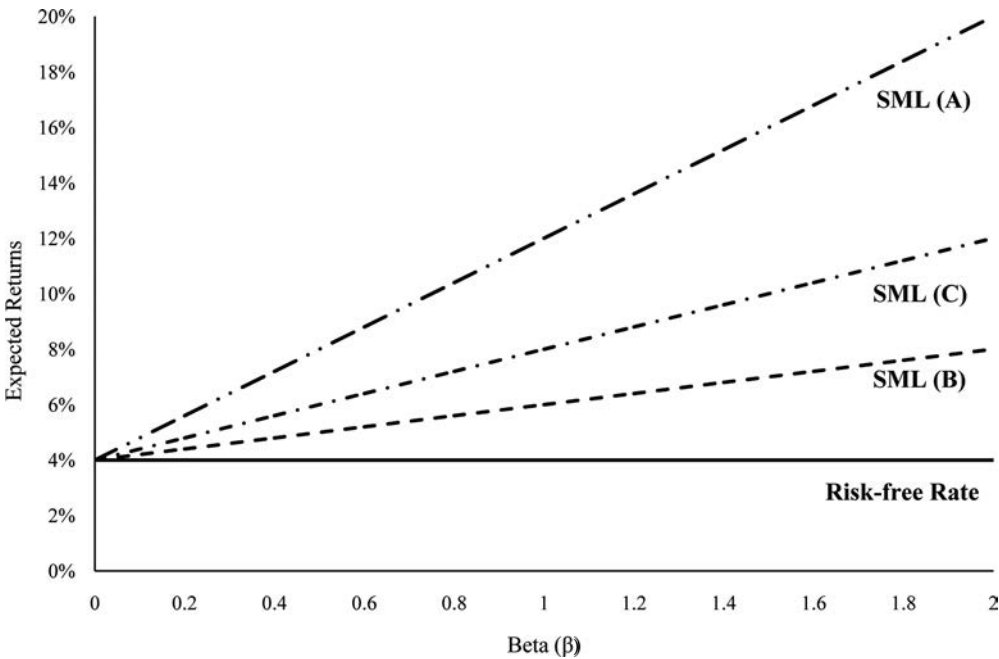


Table 10.11 Required Rate of Return under Different Market Premium

Required Rates of Return			Beta
Portfolio A (12%)	Portfolio B (6%)	Portfolio C (8%)	
4.00%	4.00%	4.00%	0.0
5.60%	4.40%	4.80%	0.2
7.20%	4.80%	5.60%	0.4
8.80%	5.20%	6.40%	0.6
10.40%	5.60%	7.20%	0.8
12.00%	6.00%	8.00%	1.0
13.60%	6.40%	8.80%	1.2
15.20%	6.80%	9.60%	1.4
16.80%	7.20%	10.40%	1.6
18.40%	7.60%	11.20%	1.8
20.00%	8.00%	12.00%	2.0

Figure 10.18 Shifts in Security Market Lines of Three Different Portfolios



Example 10.21

Consider three different portfolios of risky assets, airline companies (A), aircraft companies (B), and leasing companies (C), as shown in Table 10.11. These industries may have been presented to potential investors by three different SMLs. Each

portfolio is combined with the risk-free asset to create three capital allocation lines, SML (A), SML (B), and SML (C). The risk-free rate is assumed to be 4 percent and the market return for Portfolio A is 12 percent, for Portfolio B 6 percent, and for Portfolio C 8 percent. It can be clearly seen from Figure 10.18 that the portfolio invested in airline companies is superior to the other two portfolios because it has a greater expected return for any given level of risk. As a result, an investor will choose the portfolio that lies on the security market line for Portfolio A. The combination of the risk-free asset and the risky Portfolio A that is selected for an investor depends on the investor's degree of risk aversion.

Sensitivity Analysis

Sensitivity analysis is used to determine how “sensitive” a variable is to changes in the value of other variables. For example, sensitivity analysis can be used to examine the degree of uncertainty in a cost/benefit analysis (CBA) and how that affects study results. Sensitivity analysis provides a way to show how project results would be affected, and how responsive or sensitive those results would be to changes in the values of other factors.

Sensitivity analysis can be broadly classified into two different types, namely:

- ♦ Break-even Analysis
- ♦ Monte Carlo Analysis

Break-even Analysis

The foundation of this type of analysis is to select one variable and change its value, while holding the values of other variables constant, and observe the amount of change in the results. This is frequently applied to discount rates. Break-even analysis identifies how great a policy impact must be for its benefits to equal its costs, that is, to break even. Breaking even results in a net benefit of \$0 considering all the existing parameters. If exceeding the break-even point is feasible, the benefits are likely to outweigh the costs, ensuring a profit. Also, if reaching the break-even point is not feasible, the costs are likely to exceed the benefits which will result in a loss.

Break-even analysis is extensively used in aviation as well. One of the most important parameters considered in the operation of an airline is the Break-even Load Factor. It indicates the minimum load factor an airline needs to maintain in order to just about break even. Anything below this results in a loss for the airline, and certain market changes such as increase in fuel

Aviation Snippets

Boeing slashed 787 production costs by about two-thirds in the fourth quarter compared to the third quarter, but fell short of its target of breaking even on a unit cost basis by the end of the year. Overall deferred production costs for the 787 program rose to \$28.5 billion by the end of the fourth quarter, a \$201 million increase compared to the end of September. Boeing delivered 34 787s in the fourth quarter, implying that each was delivered at a real loss of \$5.91 million per aircraft on average.

Flightglobal, January 2016

prices could raise this break-even load factor pushing the airline further towards increasing its current load factor.

A similar type of analysis is also done by aircraft manufacturers to ensure that the costs they incur in the development of an aircraft are adequately met upon its sale. It is thus important to determine the break-even price that needs to be set on the sale.

Example 10.22

Air-Ship International, a startup aircraft manufacturer in Latin America, has invested close to \$20 billion in the development of a wide-body aircraft, the ASI-333. The aircraft is priced at \$350 million. Calculate the break-even number of aircraft, assuming the variable cost of producing each aircraft is about \$250 million:

$$\text{Profit} = \text{Total Revenue} - \text{Total cost}$$

$$\text{At break-even point, Profit} = \$0$$

$$\text{Therefore, Total Revenue} = \text{Total Cost}$$

$$\text{Price} \times \text{Quantity} = \text{Fixed costs} + (\text{Variable cost} \times \text{Quantity})$$

$$P \times Q = FC + (VC \times Q)$$

Which implies,

$$Q_{BE} = \frac{FC}{P - VC}$$

To calculate the break-even number of ASI-333, we just substitute the numbers in the above equation:

$$Q_{BE} = \frac{\$20,000 \text{ million}}{(\$350 - \$250) \text{ million}}$$

$$Q_{BE} = 200 \text{ Aircraft}$$

The simple break-even representation for this situation is depicted in Figure 10.19. The break-even quantity in the above example is 200 aircraft.

Scenario 1:

Now, consider a situation where the fixed cost goes up by \$5 billion. Figure 10.20 demonstrates how the break-even point changes due to this change in the fixed cost. An increase in the fixed cost results in an increase in the quantity required to break even. As seen from the equation derived earlier, fixed cost and quantity are directly related. This increase in the fixed cost by \$5 billion forces the break-even quantity to be increased to 250 aircraft.

Scenario 2:

Another possible situation could be an increase in the variable cost of production. The changes in break-even quantity for various scenarios of variable costs are shown in Figure 10.21. An increase in variable cost implies that the total cost line increasingly becomes steeper. From the equation for quantity, it can be seen that an increase in variable cost results in lowering the denominator which, in turn, increases the required quantity to break even. An increase in variable cost by \$15 million to \$265 million

Figure 10.19 Break-even Point Based on Initial Assumptions

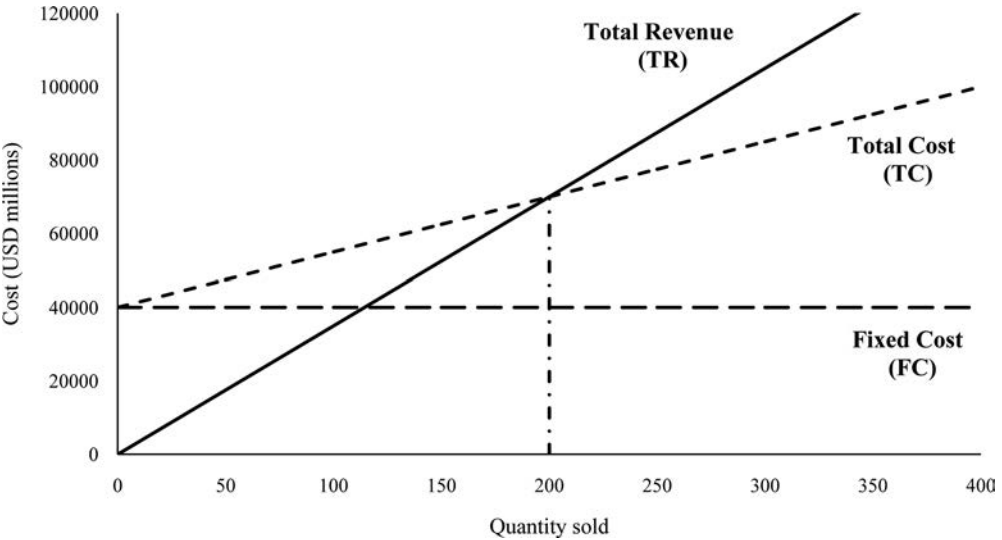
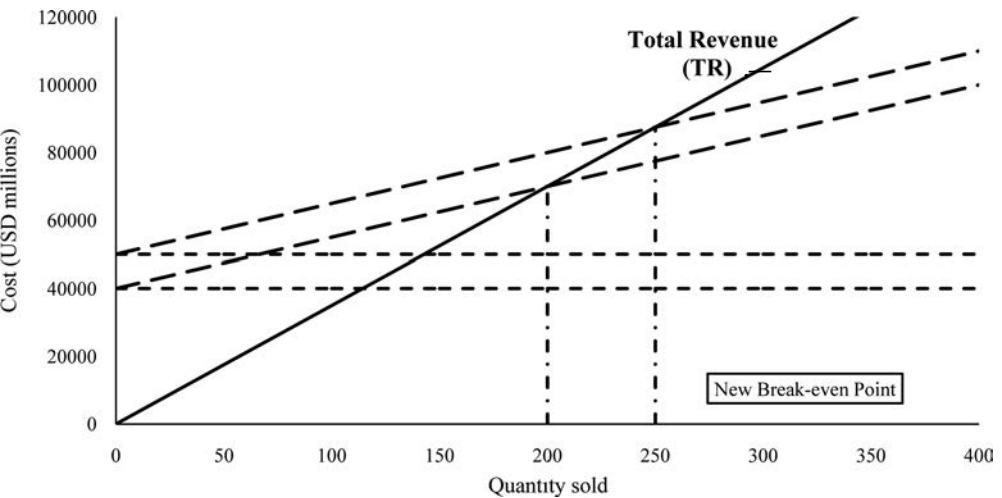


Figure 10.20 Sensitivity Analysis Scenario 1—Increase in the Fixed Cost



makes the required quantity 235 aircraft to break even. An increase in variable cost by \$35 million, making the variable cost \$285 million results in a required quantity of 308 aircraft to break even.

Scenario 3:

The final possible alteration could arise from an increase in the market price of the aircraft. An analysis based on those changes is represented in Figure 10.22. An increase in the market price only alters the total revenue line, keeping everything else the same. As the price increases, the line becomes steeper. From the equation for break-even

Figure 10.21 Sensitivity Analysis Scenario 2—Increase in the Variable Cost

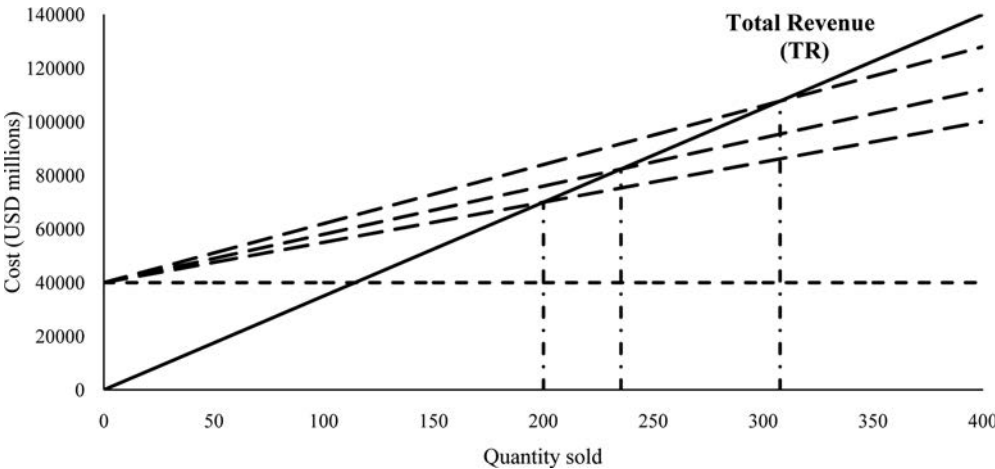
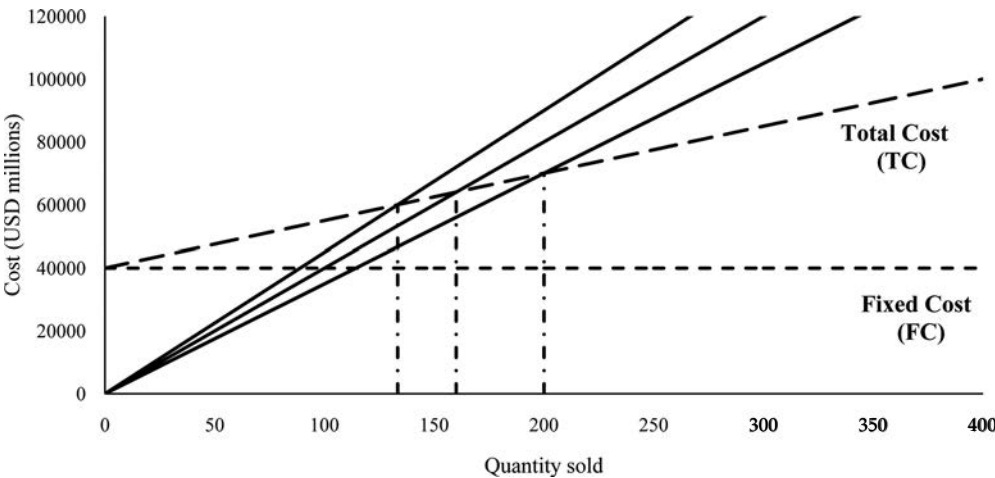


Figure 10.22 Sensitivity Analysis Scenario 3—Increase in the Market Price



quantity, it can be seen that price and quantity are inversely related. Hence an increase in price by \$25 million lowers the required break-even quantity to 160 aircraft. An increase in price by \$50 million makes the required break-even quantity as low as 133 aircraft.

Monte Carlo Analysis (Simulation)

Simulation can be a valuable tool to improve the understanding of underlying mechanisms that control the behavior of a system. However, using simulation to make predictions of the future behavior of a system can be difficult. The main reason behind this is the fact that, for most real-world systems, at least some of the controlling parameters, processes, and events are often stochastic, uncertain, and/or poorly understood. The primary objective of many simulations is to identify and quantify the risks associated with a particular option, plan, or design.

Probabilistic simulation is the process of explicitly representing these uncertainties by specifying inputs as probability distributions. If the inputs describing a system are uncertain, the prediction of future performance is necessarily uncertain. That is, the result of any analysis based on inputs represented by probability distributions is itself a probability distribution.³

In order to compute the probability distribution of predicted performance, it is necessary to translate the input uncertainties into uncertainties in the results. One common technique for translating the uncertainty into various aspects of a system to obtain the predicted performance is the Monte Carlo simulation. Monte Carlo simulation furnishes the decision-maker with a range of possible outcomes and the probabilities that they will occur for any choice of action. It shows the extreme possibilities—the outcomes of going for broke and of the most conservative decision—along with all possible consequences for middle-of-the-road decisions.

In Monte Carlo simulation, the entire system is simulated a large number of times. Each simulation is equally likely, and is referred to as a realization of the system. For each realization, all of the uncertain parameters are sampled. The system is then simulated through time (given the particular set of input parameters) such that the performance of the system can be computed. This results in a large number of separate and independent outputs, each representing a possible “future” for the system (i.e., one possible path the system may follow through time). The results of the independent system realizations are assembled into probability distributions of possible outcomes.

The Monte Carlo simulation is based on the fact that each of the inputs which are to be varied in the numerous iterations is randomly assigned a value every single time. These inputs are randomized between the specified levels or distributions defined for them.

An example of how this randomization can be performed is the use of the “=RANDBETWEEN(bottom, top)” function in Excel.

If you enter the following function in cell A1,

=RANDBETWEEN(0,30)

it will return a random number in that specified range, every single time (Figure 10.23).

As seen above, the function, when executed, returns a random number between zero and 30. Every time it is executed it gives a different number and upon execution thousands of times, the probability of the occurrence of different numbers can be plotted as a probability distribution, which attempts at quantifying that uncertainty.

Figure 10.23 Generating a Random Number in a Given Range Using Excel Formula

	A	B	C
1	=RANDBETWEEN(0,30)		
2			
3			

	A	B	C
1	7		
2			
3			

A good example which could be considered to show the implementation of the simulation is a basic cost estimation for a development project such as an airplane construction. Consider the following data about the cost for the project represented in Table 10.12.

The output obtained for a possible budget to be chosen from the results of the Monte Carlo simulation is shown in Table 10.13.

Graphical representations from which various conclusions can be drawn in regards to the probability of all possible choices for the given budget are shown in Figure 10.24, Figure 10.25, and Figure 10.26.

Table 10.12 Components of Cost of the Project (All in USD millions)

Cost Elements	Base Case	Minimum	Most Likely	Maximum	Minimum	Most Likely	Maximum
Rent	\$2,000	90%	100%	125%	\$1,800	\$2,000	\$2,500
Set-up cost	\$5,000	90%	100%	125%	\$4,500	\$5,000	\$6,250
Raw materials	\$4,000	90%	100%	125%	\$3,600	\$4,000	\$5,000
Labor cost	\$2,000	90%	100%	125%	\$1,800	\$2,000	\$2,500
IT	\$1,000	90%	100%	125%	\$900	\$1,000	\$1,250
Vehicles	\$500	90%	100%	125%	\$450	\$500	\$625
Marketing	\$1,500	90%	100%	125%	\$1,350	\$1,500	\$1,875
Other general overhead	\$2,500	90%	100%	125%	\$2,250	\$2,500	\$3,125
Totals	\$18,500						

Figure 10.24 Probability Density Distribution

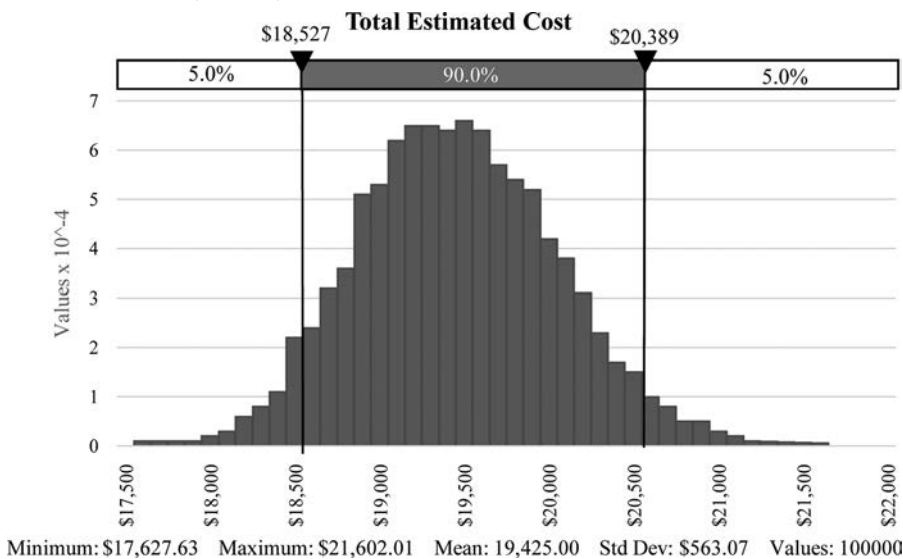
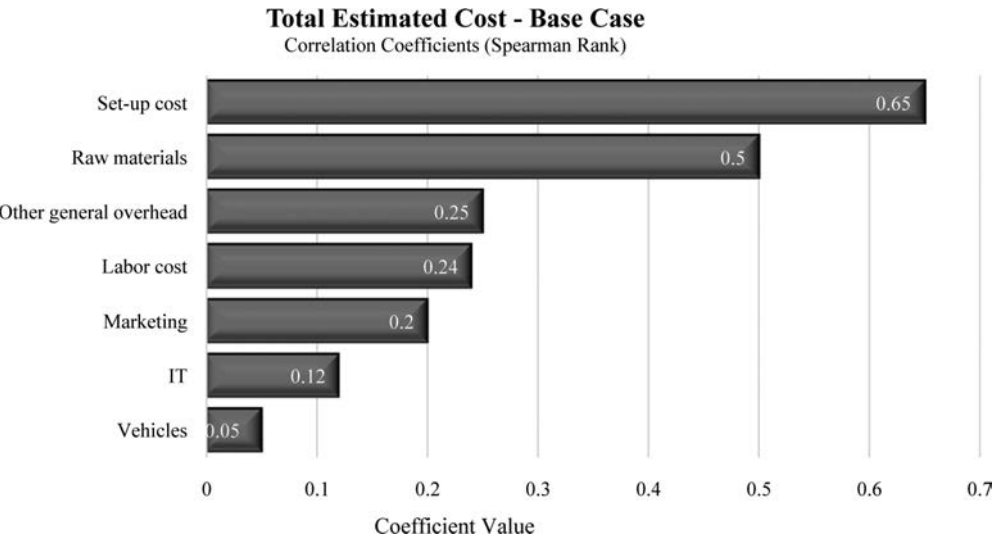


Table 10.13 Summary of the Analysis

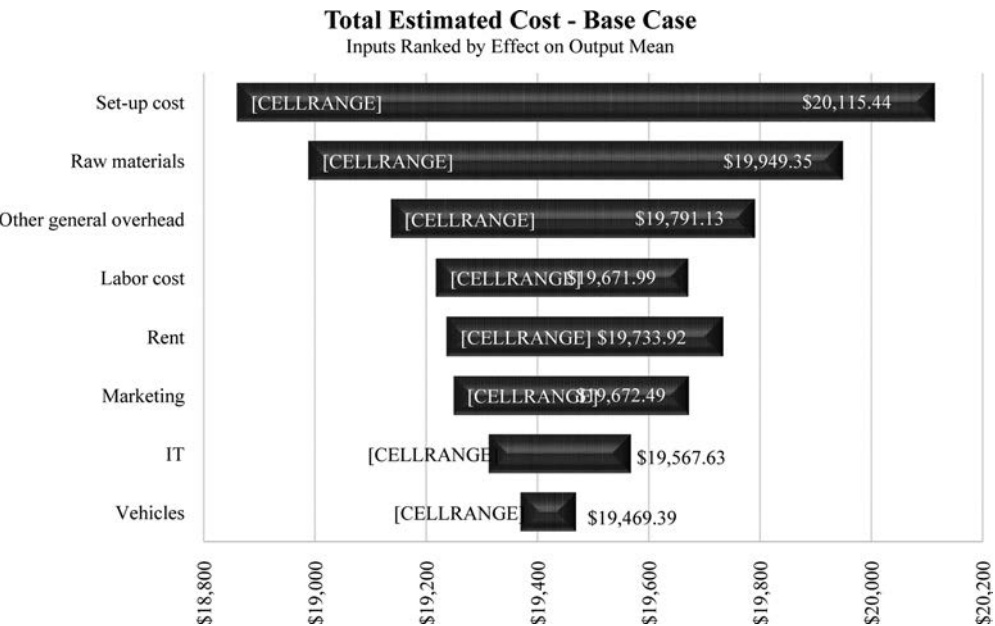
Probability of meeting base case value	4.30%
Total budget required for 95.0% confidence	\$20,389
Contingency required for 95.0% confidence	\$1,889

Figure 10.25 Correlation Coefficients



Source: Licensed by @RISK Course Version, Embry Riddle Aeronautical University

Figure 10.26 Inputs Ranked by Effect on Output



Source: Licensed by @RISK Course Version, Embry Riddle Aeronautical University

Summary

This chapter covers the essentials of risk and returns. Every organization is faced with various risk elements while doing business. Risk management is an economic issue, as the management must balance the cost and the benefits associated with the risk reduction strategy. Businesses experience tremendous challenges throughout the course of their operations. They face the possibility of inadequate profits, change in government regulations and tax policies, restrictions related to international trade, intense competition, market volatility, changes in global economy, and many others. To successfully cope with these uncertainties, firms need to know the risk associated with each of the possible outcomes. At this point, an organization or an individual may decide on the return required to adequately compensate for taking the risk. Risk is traditionally associated with probability calculation and this suggests that an event can be predicted and controlled. A common measure of return volatility is the standard deviation of investment returns. Standard deviation is the square root of the variance of the annual rate of return on investment. Uncertainty, however, is not capable of measurement and deals with possibilities incapable of calculation which are based on guesswork and judgement (Gray and Hamilton, 2006). An investment portfolio is a collection of securities that together provide an investor with an attractive tradeoff between risk and return. In this chapter, we introduce the students to the capital asset pricing model (CAPM) and the security market line (SML) that describe the relation between the risk and the expected return. The concept of the CAPM and the SML is explained throughout the chapter, including the use of expected return, risk-free rate of return, and the risk of investment. This chapter also discusses sensitivity analysis, used to determine how “sensitive” a variable is to changes in the value of other variables, and reflects on the two most common sensitivity analysis practices—the break-even analysis and the Monte Carlo simulation.

Discussion Questions and Problems

- 10.1 What is the security market line?
- 10.2 A stock has an expected return of 18 percent, the risk-free rate is 6 percent, and the market risk premium is 6 percent. What must the beta of this stock be?
- 10.3 A stock has an expected return of 12 percent and a beta of 0.75, and the expected return on the market is 14 percent. What must the risk-free rate be?
- 10.4 A die is thrown once. What is the probability of getting a one?
- 10.5 The time that it takes to board an aircraft is normally distributed with a mean of 40 minutes and a standard deviation of 15 minutes.
 - a. What is the probability that a flight can board all of the passengers in less than 20 minutes?
 - b. By using Excel spreadsheets draw a normal distribution curve on which this mean and standard deviation are correctly indicated.
 - c. What is the chance that the flight spends more than an hour at the gate to board all passengers?
 - d. How often will a flight spend between 30 and 45 minutes at the gate for taking off?

- 10.6** A stock has a beta of 0.75, the expected return on the market is 12 percent, and the risk-free rate is 6 percent. What must the expected return on this stock be?
- 10.7** Suppose the risk free rate is 10 percent. The expected return on the market is 15 percent. Assuming DirectJet equity has a beta of 0.8, what is its expected return based on the CAPM? If another equity has an expected return of 20 percent, what must its beta be?
- 10.8** What is the capital asset pricing model (CAPM)?
- 10.9** Suppose we had the following investments:

Investment	Amount	Expected Return	Beta
Project A	\$20,000.0	8	0.7
Project B	\$25,000.0	12	0.8
Project C	\$40,000.0	15	1.1
Project D	\$30,000.0	18	1.6

- a. What is the expected return on this portfolio?
 - b. What is the beta of this portfolio?
- 10.10** You own a portfolio that has \$5,800 invested in DirectJet and \$7,800 invested in EZjET equities. If the expected returns on these equities are 6 percent and 8 percent respectively, what is the expected return on the portfolio?
- 10.11** Go to Yahoo! Finance and download the ending monthly equity prices for the Boeing Company for the last 58 months. Use the adjusted closing price, which adjusts for dividend payments and stock splits. Next, download the ending value of the Dow Jones Transportation Average (DJTA) over the same period. For the historical risk-free rate, download the historical three-month Treasury bill secondary market rate.
- a. What are the monthly returns, average monthly returns, and standard deviations for Boeing equity, the three-month Treasury bill and the Dow Jones Transportation Average (DJTA) for this period?
 - b. How do you interpret the weights for the two assets in this case? Explain.
- 10.12** Your portfolio is comprised of investment in two different jet engine manufacturers. RR Stock has a beta of 1.40 and Stock PW has a beta of 0.9. Suppose $R_{RF} = 2\%$ and $R_M = 12\%$.
- (a) According to the CAPM, what are the expected returns for each stock?
 - (b) What is the expected return of an equally weighted portfolio of these two stocks?
 - (c) What is the beta of an equally weighted portfolio of these two stocks?
 - (d) How can you use your answer to part (c) to answer part (b)?
- 10.13** EZjET airline estimates that with probability of 0.5 a passenger who purchased a ticket won't take the designated flight. Suppose on their flight from Philadelphia, PA to Phoenix, AZ the airplane can carry 100 passengers and the airline has sold 102 tickets.

- a) Calculate the probability that exactly 101 passengers show up to take the flight (four digits after decimal point).
 - b) Estimate the probability that exactly 102 passengers show up to take the flight (four digits after decimal point).
 - c) Determine the probability that no more than 100 passengers show up to take the flight so the EZjET doesn't have to compensate some people and ask them to leave the plane (three digits after decimal point).
- 10.14** DirectJet regularly overbooks the flight to balance the spillage cost against spoilage cost. On their flight between Charleroi and Bergamo, the probability of a passenger arriving for a booked flight is 0.8. DJA booked 225 passengers and there are 195 seats available on the flight. Assume that data is normally distributed and find the probability that more booked passengers arrive than there are seats available.
- 10.15** The TSA rule requires flight schools and flight instructors to provide security awareness training (SAT) to employees and take a final exam. The average score on SAT test was 76 with a standard deviation of 5. To get an A, a student must have a score of 90 or higher; a B is 80–90; a C is 70–80; and a D is 60–70. What is the probability that a randomly selected student scored between a 75 and 99? Assume the variable is normally distributed.
- 10.16** The variance of Stock A is 0.0016, the variance of the market is 0.0049 and the covariance between the two is 0.0026. What is the correlation coefficient?
- 10.17** The portfolio's required rate of return is 17 percent. The risk-free rate, $R_{RF} = 7\%$ and the return on the market, $R_M = 14\%$. What is Stock D's estimated beta?
- 10.18** Given the following data for Jet Leasing Company (JLC), calculate the stock variance and standard deviation. The expected return based on the data is 14 percent.

Aircraft Type	Probability	Return	Expected Return (%)
Regional	10%	0.2	0.02
Narrow-body	40%	0.1	0.04
Wide-body	40%	0.14	0.056
Cargo	10%	0.24	0.024

Notes

- 1 Decision theory and all related concepts are further discussed in extensive detail in Chapter 15.
- 2 Flight cycles are the number of takeoffs and landings that a particular aircraft performs.
- 3 GoldSim Technology Group, 2006, <http://www.goldsim.com>.

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11

Analysis of Depreciation, Replacement, and Abandonment

Singapore-based low-cost carrier Tiger Airways Holdings forecast a 12.8 million SGD (\$8.93 million) second quarter 2016 net loss, down 93% from a \$143 million loss in the year-ago period. Revenue rose 12.8% to \$117 million while expenses increased 2.4% to \$125 million, partly due to higher maintenance charges, higher aircraft rentals, aircraft depreciation and appreciation of the US dollar against the Singapore dollar.

Air Transport World, October 23, 2015



Practically all the assets that a company owns either appreciate or depreciate in value. Depreciation refers to the systematic and rational process of distributing the cost of an asset over the life of the asset. Most types of tangible property such as plants and equipment, buildings, machinery, vehicles, and furniture are depreciable. Since the value of a physical asset systematically diminishes with time, the process of predicting the rate of this depreciation is considered critically important. The absence of depreciation expense in an income statement would inflate the income of the company. From the tax point of view and official financial statements, depreciation methods are constrained by legal requirements. For internal purposes, depreciation can even be applied over the economic life of an asset. Economic life of an asset differs from the physical life, in the sense that the economic life ends when the cost of maintaining an asset increases to an extent that it is no longer economical. For example, the potential life of an aircraft could be endless, but an airline may replace its old aircraft with new aircraft every six years. Older aircraft are more expensive to operate, but newer aircraft have a higher purchase price and annual depreciation expense. Many airlines such as Flydubai, Emirates, and Singapore Airlines do not keep their aircraft more than six or seven years. Flydubai operates a single-model fleet of 50 Boeing 737–800 Next-Generation aircraft, being one of the youngest fleets in the skies with an average age of 3.1 years.¹ There are many methods

which can be used to determine the rate of depreciation and each method produces a different pattern of expenses over time. This section will focus on some of the major depreciation techniques.

- ♦ Introduction
- ♦ Depreciation Methodologies
 - Straight Line Depreciation (SL) Method
 - Declining Balance Depreciation (DB)
 - Modified Accelerated Cost Recovery System (MACRS)
 - Sum-of-the-Years Digits (SYD) Method
- ♦ Comparison of Different Approaches to Depreciation
- ♦ Using Spreadsheets for Depreciation Computations
- ♦ Basics of a Replacement Decision
- ♦ Impairment
- ♦ Abandonment
- ♦ Summary
- ♦ Discussion Questions and Problems

Introduction

Depreciation can be broadly explained as systematic non-cash deductions for allocating the cost of long-lived assets over the useful life of the assets. A long-term asset is an asset with a useful life of more than one year, and is acquired for use in the business, and not intended for resale to customers. Each time a company prepares the financial statements for a fiscal year, a depreciation expense is recorded to allocate a portion of the cost of the buildings, machines, or equipment that had been purchased in the current year. The airline industry is characterized by high fixed costs; aircraft, plant, and equipment are some of the largest asset categories for any airline. More importantly, depreciation expenses help to reduce the amount of tax that an airline must pay. The purpose of recording depreciation as an expense is to spread the initial price of the asset over its useful life.

Depreciation thus becomes an extremely important tool with varied financial consequences because the amount to be used as a depreciation

Depreciation is an annual income tax deduction that allows you to recover the cost or other basis of certain property over the time you use the property. It is an allowance for the wear and tear, deterioration, or obsolescence of the property.

United States of America, Internal Revenue System (IRS)

Aviation Snippets

An aircraft depreciation expense is derived by estimating both the useful life and the residual value or the fair market value of the aircraft at the end of its estimated useful life. Useful life is a period over which an asset is expected to be available for use by a company.

International Accounting Standards. IAS 16

expense is entirely dependent on the method of choice to calculate it. Although internally any particular method can be used, there are certain specifications when it comes to government agencies and depreciation expenses.

In order for a taxpayer to be allowed a depreciation deduction for a property, the property must meet all the following requirements:²

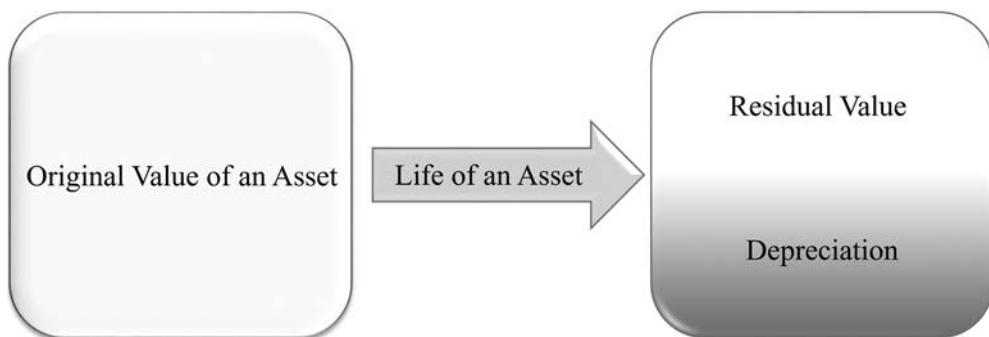
- ♦ The business entity must own the property.³ But under capital lease, the lessee is able to calculate and record depreciation expense for the recognized amount of the asset. A capital lease is a long-term lease agreement considered to have the economic characteristics of asset ownership.
- ♦ The business entity may also depreciate any capital improvements for property the taxpayer leases.
- ♦ The company must use the property in business or in an income-producing activity. If the company uses a property for business and for personal purposes, the taxpayer can only deduct depreciation based only on the business use of that property.
- ♦ The property must have a quantifiable useful life of more than one year.⁴

Depreciation Methodologies

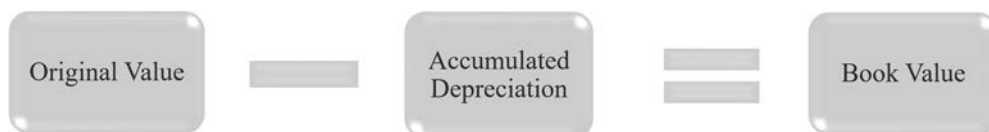
Depreciating assets generate tax deductions and defer tax payments. Let D denote the annual depreciation expense, and T_c the company's tax rate. In this case, the company saves $(D \times T_c)$ from the taxes it needs to pay. For a company with the annual depreciation expense of \$1 million and a tax rate of 35 percent, the projected tax shield will be \$350,000.

The book value of a physical asset is its original cost minus the related accumulated depreciation.

The total depreciation amount charged over an asset's entire useful life (i.e., depreciable amount) is the same irrespective of the choice of depreciation method. The adoption of a particular depreciation method does, however, affect the amount of depreciation expense charged in each year of an asset's life. Depreciation benefits business in several different ways. Companies such as airlines or aircraft manufacturers can get tax benefits for buying plants, equipment, and other physical assets.



To help you understand the concept, let's look at an example of depreciation expense and its impact on the after-tax income. In 2017, DirectJet purchases an Airbus A320NEO for \$75,000,000 that is expected to last for 10 years, and during the same year earns \$10,000,000 profit. Since the aircraft is expected to last 10 years, DirectJet can take the cost of the aircraft and divide it by 10 $\left(\frac{\$75,000,000}{10} = \$7,500,000 \text{ per year} \right)$. Instead of realizing a one-time expense, the airline can subtract \$7,500,000 each year for the next 10 years, reporting taxable income of $\$10,000,000 - \$7,500,000 = \$2,500,000$.



Depreciation affects the company's profit and loss statement as well as the balance sheet. Generally, an aircraft depreciation expense is derived by estimating both the residual value and the useful life of the aircraft. The residual (or salvage) value is the amount an airline can receive from selling the aircraft at the end of the useful life. Usually, the value of a commercial aircraft depreciates based on an estimate of 25 years of useful-life expectancy, and capitalized maintenance allowance for wear and tear over the period leading up to the next overhaul. Aircraft components such as engines, spare parts, and undercarriage or landing gear depreciate at different rates. Although a number of different depreciation methods exist, we shall discuss the following:

- ♦ Straight Line Depreciation (SL)
- ♦ Declining Balance Depreciation (DB)
- ♦ Modified Accelerated Cost Recovery System (MACRS)
- ♦ Sum-of-the-Years Digits (SYD)

Straight Line Depreciation (SL) Method

Depreciation is a term used for tax and accounting purposes that describes the method that a company uses to account for the declining value of its fixed assets. Straight Line depreciation is likely to be the most common method of matching an asset's cost to the accounting periods during which it is in service. Under the Straight Line method of depreciation, each full accounting year will be allocated the same amount or percentage of an asset cost, such that the value either becomes zero at the end of its useful life or reaches a previously set salvage value. Straight Line depreciation method is appropriate where economic benefits from an asset are expected to be realized evenly over its useful life. It is also convenient to use SL depreciation when no reliable estimate can be made regarding the pattern of economic benefits expected to be derived over an asset's useful life.

$$\text{Depreciation Expense} = \frac{\text{Original Cost of Asset} - \text{Salvage Value}}{\text{Useful Life}}$$

Original cost of an asset includes all costs to bring that asset or associated project into service. For example, original cost of a new factory would consist of:

- ♦ Contract fees
- ♦ Cost of permits
- ♦ Engineering expenses
- ♦ Equipment cost
- ♦ Insurance against accidents during the construction process
- ♦ Legal charges
- ♦ Supplies and materials
- ♦ Taxes related to the project

Example 11.1

DirectJet Airlines purchased a Boeing 737 which was valued at \$75 million at the time of purchase. Assume that the aircraft has an estimated salvage value of \$0.00, and a useful life of twenty years. DirectJet calculates the annual Straight Line depreciation for the aircraft as:

$$\text{Annual depreciation expense} = \frac{\$75,000,000 - \$0.00}{20}$$

$$\text{Annual depreciation expense} = \$3,750,000$$

Table 11.1 Annual Depreciation, Straight Line Method

Age	Depreciation Expense	Book Value
0	\$0	\$75,000,000
1	\$3,750,000	\$71,250,000
2	\$3,750,000	\$67,500,000
3	\$3,750,000	\$63,750,000
4	\$3,750,000	\$60,000,000
5	\$3,750,000	\$56,250,000
6	\$3,750,000	\$52,500,000
7	\$3,750,000	\$48,750,000
8	\$3,750,000	\$45,000,000
9	\$3,750,000	\$41,250,000
10	\$3,750,000	\$37,500,000
11	\$3,750,000	\$33,750,000
12	\$3,750,000	\$30,000,000
13	\$3,750,000	\$26,250,000
14	\$3,750,000	\$22,500,000
15	\$3,750,000	\$18,750,000
16	\$3,750,000	\$15,000,000
17	\$3,750,000	\$11,250,000
18	\$3,750,000	\$7,500,000
19	\$3,750,000	\$3,750,000
20	\$3,750,000	\$0

Example 11.2

In the above example, if the potential salvage value of the aircraft is \$10 million, then the depreciation schedule changes as follows:

$$\text{Annual depreciation expense} = \frac{\$75,000,000 - \$10,000,000}{20}$$

$$\text{Annual depreciation expense} = \$3,250,000$$

The above examples demonstrate that the higher the residual value, the lower the depreciation. Conversely, lower residual value means higher depreciation. Higher amount of depreciation would cause the profit (or EBIT) to be lower, thus, lowering the earnings per share (EPS).⁵

For example, on January 1, 1999, Southwest Airlines extended the depreciable lives of its Boeing 737 airplanes from 20 years to 23 years. As a result, depreciation

Table 11.2 Straight Line Example with Salvage Value

Age	Depreciation Expense	Value
0	\$0	\$75,000,000
1	\$3,250,000	\$71,750,000
2	\$3,250,000	\$68,500,000
3	\$3,250,000	\$65,250,000
4	\$3,250,000	\$62,000,000
5	\$3,250,000	\$58,750,000
6	\$3,250,000	\$55,500,000
7	\$3,250,000	\$52,250,000
8	\$3,250,000	\$49,000,000
9	\$3,250,000	\$45,750,000
10	\$3,250,000	\$42,500,000
11	\$3,250,000	\$39,250,000
12	\$3,250,000	\$36,000,000
13	\$3,250,000	\$32,750,000
14	\$3,250,000	\$29,500,000
15	\$3,250,000	\$26,250,000
16	\$3,250,000	\$23,000,000
17	\$3,250,000	\$19,750,000
18	\$3,250,000	\$16,500,000
19	\$3,250,000	\$13,250,000
20	\$3,250,000	\$10,000,000

expense for the three and nine months ended September 1999 was reduced by \$6.4 million and \$19.3 million, respectively.⁶

Declining Balance Depreciation (DB)

Declining Balance method charges depreciation at a higher rate in the earlier years of an asset, and reduces depreciation as the life of the asset progresses. Declining Balance is a type of accelerated depreciation technique which is more applicable for assets that are extremely useful in the first years of service and the level of service keeps reducing as time goes on. Depreciation under Declining Balance method may be calculated as follows:

$$\text{Depreciation per annum} = \text{Net Book Value} \times \text{Depreciation Rate (\%)}$$

Where:

- ♦ *Net Book Value* is the asset's net value at the start of an accounting period. It is calculated by deducting the accumulated (total) depreciation from the cost of the fixed asset (historical book value).
- ♦ *Rate of Depreciation* is defined according to the estimated pattern of an asset's use over its life term. For example, for Straight Line depreciation rate of 10 percent, Double Declining Balance rate will be $2 \times 10\% = 20\%$.

$$\text{Rate of Depreciation} = \text{Accelerator} \times \text{Straight Line Rate}$$

The accelerator can be double, 150 percent and so on depending on the type of an asset and its usability. Typically, the choice of accelerator is the prerogative of a manager.

Example 11.3

EZJET fleet manager has decided to buy a narrow-body commercial aircraft (A320NEO) for \$75,000,000 and plans to use the aircraft for 20 years. The salvage value at the end of the useful life is estimated to be \$10 million. Straight Line depreciation rate equals 5 percent and the accelerator is 200 percent (2). Because of rapid obsolescence, the airline is planning to depreciate the aircraft under Double Declining Balance method. How much depreciation expense should be charged each year (Table 11.3)?

For the last year, even though the actual depreciation expense should be \$1,013,139, we record only an expense of \$131,388, so that the salvage value of \$10 million can be achieved.

Another type of declining balance could be the 150 percent Declining Balance method. In this, technique, the accelerator used is 1.50. However, on applying the 150 percent Declining Balance method, it is observed that the value at the end of the 20 years would be \$15,772,332, and so the salvage value of \$10 million is understated. Hence, the salvage value should be \$16 million (Table 11.4).

To achieve the salvage value of \$16 million, the actual depreciation expense for year 20 is replaced by \$1,051,170.

Table 11.3 Annual Depreciation, DDB Depreciation Method

Age	Depreciation Expense	Book Value
0	\$0	\$75,000,000
1	\$7,500,000	\$67,500,000
2	\$6,750,000	\$60,750,000
3	\$6,075,000	\$54,675,000
4	\$5,467,500	\$49,207,500
5	\$4,920,750	\$44,286,750
6	\$4,428,675	\$39,858,075
7	\$3,985,808	\$35,872,268
8	\$3,587,227	\$32,285,041
9	\$3,228,504	\$29,056,537
10	\$2,905,654	\$26,150,883
11	\$2,615,088	\$23,535,795
12	\$2,353,579	\$21,182,215
13	\$2,118,222	\$19,063,994
14	\$1,906,399	\$17,157,594
15	\$1,715,759	\$15,441,835
16	\$1,544,183	\$13,897,651
17	\$1,389,765	\$12,507,886
18	\$1,250,789	\$11,257,098
19	\$1,125,710	\$10,131,388
20	\$131,388	\$10,000,000

Modified Accelerated Cost Recovery System (MACRS)

MACRS is the depreciation technique which is used to calculate taxes in the United States of America. The Internal Revenue Service Reform Act of 1986 changed the way assets were depreciated, and the Modified Accelerated Cost Recovery System (MACRS) has been used since 1986.⁷ Under MACRS, fixed assets are assigned to a specific asset class. The Internal Revenue Service has published a complete set of depreciation tables for each of these classes (detailed explanation of all the asset classes is provided in the appendix). A business determines its tax depreciation based on the information in the asset class table. The resulting depreciation is included in the company's income tax return as part of the derivation of taxable income.

It is important to note that internally a company is permitted to use any of the depreciation techniques available. But for tax deduction purposes, every company, depending on specific asset classes, is forced to use the MACRS technique under US law.

Table 11.4 Annual Depreciation, DDB Method (150%)

Age	Depreciation Expense	Book Value
0	\$0	\$75,000,000
1	\$5,625,000	\$69,375,000
2	\$5,203,125	\$64,171,875
3	\$4,812,891	\$59,358,984
4	\$4,451,924	\$54,907,061
5	\$4,118,030	\$50,789,031
6	\$3,809,177	\$46,979,854
7	\$3,523,489	\$43,456,365
8	\$3,259,227	\$40,197,137
9	\$3,014,785	\$37,182,352
10	\$2,788,676	\$34,393,676
11	\$2,579,526	\$31,814,150
12	\$2,386,061	\$29,428,089
13	\$2,207,107	\$27,220,982
14	\$2,041,574	\$25,179,408
15	\$1,888,456	\$23,290,953
16	\$1,746,821	\$21,544,131
17	\$1,615,810	\$19,928,321
18	\$1,494,624	\$18,433,697
19	\$1,382,527	\$17,051,170
20	\$1,051,170	\$16,000,000

Table 11.5 shows the various rates of depreciation for different useful years or asset classes.

Under MACRS, assets are always completely depreciated, meaning there is no salvage value. Depreciating an asset fully allows for further tax deduction, and hence is preferable. MACRS depreciation is calculated using tables that the Internal Revenue Service (IRS) created for each asset class.⁸

Example 11.4

The example of DirectJet purchasing a Boeing 737 valued at \$75 million can be extended to using the MACRS technique as well. The original cost of an asset is multiplied with the respective year depreciation rate under the relevant asset class. The depreciation schedule assuming a 20-year useful life is shown in Table 11.5.

The depreciation rates provided in Table 11.5 follow the half-year convention, which is that you only get one-half year of depreciation in the first year of ownership

Table 11.5 MACRS Depreciation Rates

Year	3-year	5-year	7-year	10-year	15-year	20-year
1	33.33%	20.00%	14.29%	10.00%	5.00%	3.75%
2	44.450	32.000	24.490	18.000	9.500	7.219
3	14.810*	19.200	17.490	14.400	8.550	6.677
4	7.410	11.520	12.490	11.520	7.700	6.177
5		11.520	8.930*	9.220	6.930	5.713
6		5.760	8.920	7.370	6.230	5.285
7			8.930	6.550*	5.900*	4.888
8			4.460	6.550	5.900	4.522
9				6.560	5.910	4.462
10				6.550	5.900	4.461
11				3.280	5.910	4.462
12					5.900	4.461
13					5.910	4.462
14					5.900	4.461
15					5.910	4.462
16					2.950	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

* switch to Straight Line method as it will provide a higher depreciation amount

regardless of when you purchased the asset during the year. That is the reason why depreciation is distributed over 21 years considering two halves in the first and 21st year and making the useful life of the asset equal to 20 years. Also, it should be noted that the aircraft has been fully depreciated making the salvage value zero at the end of its useful life.

Sum-of-the-Years Digits (SYD) Method

Many assets, such as automobiles and computers, lose a substantial portion of their value early in their lifetime. Assume an asset depreciates from original price V to salvage value S in N years. Under this method, depreciation is computed by multiplying the book value (V) by the remaining periods of useful life at the start of the period divided by the sum of the digits in the original useful life. For an asset with a five-year useful life:

- ♦ $N = 5$
- ♦ The sum of years is $1 + 2 + 3 + 4 + 5 = 15$
- ♦ The depreciation in the first year would be $(\frac{5}{5+4+3+2+1} = \frac{5}{15} = 33.33\%)$ of depreciable value $(V - S)$. In this methodology the amount of annual depreciation progressively declines as the asset ages.

$$\text{First Year} = \frac{N}{\text{Sum-of-Years}} \times (V - S)$$

$$\text{Second Year} = \frac{N - 1}{\text{Sum-of-Years}} \times (V - S)$$

$$\text{Final Year} = \frac{1}{\text{Sum-of-Years}} \times (V - S)$$

Table 11.6 MACRS Depreciation Expense Calculation

Age	Depreciation Expense	Value
1	\$2,812,500	\$72,187,500
2	\$5,414,250	\$66,773,250
3	\$5,007,750	\$61,765,500
4	\$4,632,750	\$57,132,750
5	\$4,284,750	\$52,848,000
6	\$3,963,750	\$48,884,250
7	\$3,666,000	\$45,218,250
8	\$3,391,500	\$41,826,750
9	\$3,346,500	\$38,480,250
10	\$3,345,750	\$35,134,500
11	\$3,346,500	\$31,788,000
12	\$3,345,750	\$28,442,250
13	\$3,346,500	\$25,095,750
14	\$3,345,750	\$21,750,000
15	\$3,346,500	\$18,403,500
16	\$3,345,750	\$15,057,750
17	\$3,346,500	\$11,711,250
18	\$3,345,750	\$8,365,500
19	\$3,346,500	\$5,019,000
20	\$3,345,750	\$1,673,250
21	\$1,673,250	\$0

Example 11.5

Using the Sum-of-the-Years digits method of depreciation, find the amount of depreciation to be charged each year on a flight simulator purchased by a flight training school. The simulator has a cost of \$1,000,000, estimated life of five years, and a trade-in/scrap value of \$100,000. Hence, in this case the depreciable base is:

$$\$1,000,000 - \$100,000 = \$900,000$$

Table 11.7 Sum-of-the-Years Digits Depreciation Method

Year	Book Value (V)	V – S	Depreciation Rate	Depreciation Expense	Accumulated Depreciation	Book Value Year End
1	\$1,000,000	\$900,000	0.33	\$300,000	\$300,000	\$700,000
2	\$700,000	\$900,000	0.27	\$240,000	\$540,000	\$460,000
3	\$460,000	\$900,000	0.20	\$180,000	\$720,000	\$280,000
4	\$280,000	\$900,000	0.13	\$120,000	\$840,000	\$160,000
5	\$160,000	\$900,000	0.07	\$60,000	\$900,000	\$100,000

Depreciation expense at the end of the:

- ♦ First year: $\$900,000 \times (5/15) = \$300,000$
 - Book value at the end of the first year: $\$1,000,000 - \$300,000 = \$700,000$
- ♦ Second year: $\$700,000 \times (4/15) = \$240,000$
 - Book value at the end of the second year: $\$700,000 - \$240,000 = \$460,000$
- ♦ Third year: $\$460,000 \times (3/15) = \$180,000$
 - Book value at the end of the third year: $\$460,000 - \$180,000 = \$280,000$
- ♦ Forth year: $\$280,000 \times (2/15) = \$120,000$
 - Book value at the end of the fourth year: $\$280,000 - \$120,000 = \$160,000$
- ♦ Fifth year: $\$160,000 \times (1/15) = \$60,000$
 - Book value at the end of the fifth year: $\$160,000 - \$60,000 = \$100,000$

Comparison of Different Approaches to Depreciation

Consider the example of an asset with an initial cost of \$75,000,000 and a salvage value of \$0, with a useful life of 20 years. All four of the previously discussed depreciation approaches can be applied to the asset, and the outcomes in current value compared.

It can be clearly seen that the amount of depreciation recorded every year is different for all four approaches, thereby leading to changes in current value of the

Table 11.8 Comparison of Different Approaches to Depreciation (SL and MACRS)

Year	Straight Line Depreciation	Current Value (Straight Line)	MACRS (20-year rate)	MACRS Depreciation	Current Value (MACRS)
0	\$0	\$75,000,000	0%	\$0	\$75,000,000
1	\$3,750,000	\$71,250,000	3.75%	\$2,812,500	\$72,187,500
2	\$3,750,000	\$67,500,000	7.22%	\$5,414,250	\$66,773,250
3	\$3,750,000	\$63,750,000	6.68%	\$5,007,750	\$61,765,500
4	\$3,750,000	\$60,000,000	6.18%	\$4,632,750	\$57,132,750
5	\$3,750,000	\$56,250,000	5.71%	\$4,284,750	\$52,848,000
6	\$3,750,000	\$52,500,000	5.29%	\$3,963,750	\$48,884,250
7	\$3,750,000	\$48,750,000	4.89%	\$3,666,000	\$45,218,250
8	\$3,750,000	\$45,000,000	4.52%	\$3,391,500	\$41,826,750
9	\$3,750,000	\$41,250,000	4.46%	\$3,346,500	\$38,480,250
10	\$3,750,000	\$37,500,000	4.46%	\$3,345,750	\$35,134,500
11	\$3,750,000	\$33,750,000	4.46%	\$3,346,500	\$31,788,000
12	\$3,750,000	\$30,000,000	4.46%	\$3,345,750	\$28,442,250
13	\$3,750,000	\$26,250,000	4.46%	\$3,346,500	\$25,095,750
14	\$3,750,000	\$22,500,000	4.46%	\$3,345,750	\$21,750,000
15	\$3,750,000	\$18,750,000	4.46%	\$3,346,500	\$18,403,500
16	\$3,750,000	\$15,000,000	4.46%	\$3,345,750	\$15,057,750
17	\$3,750,000	\$11,250,000	4.46%	\$3,346,500	\$11,711,250
18	\$3,750,000	\$7,500,000	4.46%	\$3,345,750	\$8,365,500
19	\$3,750,000	\$3,750,000	4.46%	\$3,346,500	\$5,019,000
20	\$3,750,000	\$0	4.46%	\$3,345,750	\$1,673,250
21			2.23%	\$1,673,250	\$0

asset over its useful life of 20 years. A notable distinction is that under MACRS the total number of years becomes 21 years due to the half-year assumptions in year 1 and year 21 (Table 11.8). Also, the salvage value in the case of Double Declining Balance method is \$10,000,000 (Table 11.9).

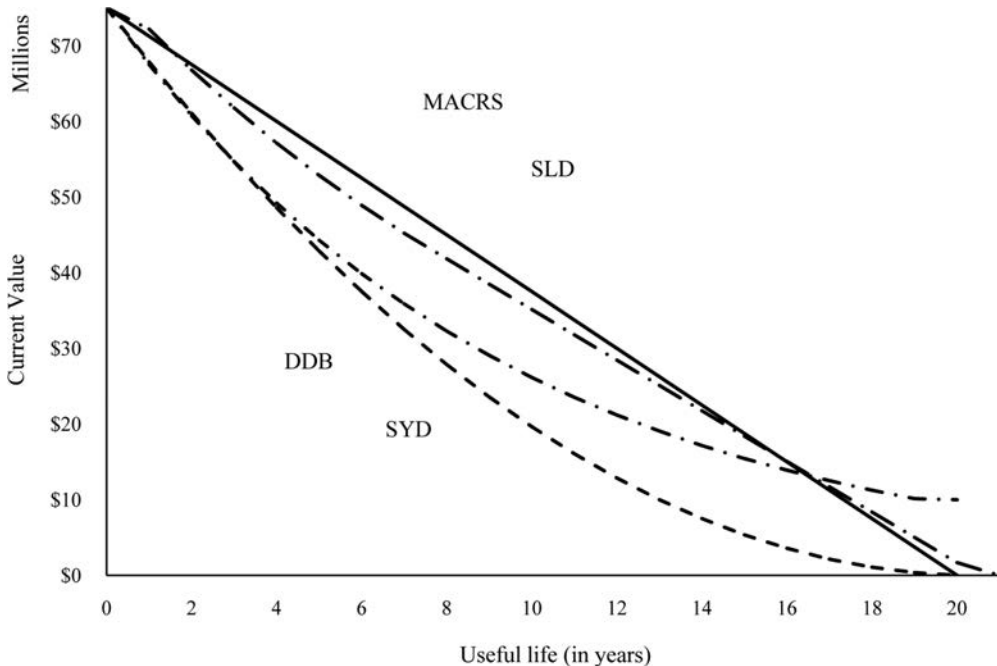
An illustration of the variation in current value is presented in Figure 11.1.

The Sum-of-the-Years digits method and the Double Declining Balance method are seen to be closely correlated in the first four years and then begin to diverge. MACRS and Straight Line are fairly close throughout the life of the asset, as well. This is the reason why in cases when the Straight Line method offers a higher depreciation, the MACRS rate after that point for certain asset classes is substituted with the Straight Line rate.

Table 11.9 Comparison of Different Approaches to Depreciation (SYD and DDB)

Year	Sum-of-the-Years Digits Depreciation	Current Value (SYD)	DDB Depreciation	Current Value (DDB)
0	\$0	\$75,000,000	\$0	\$75,000,000
1	\$7,142,857	\$67,857,143	\$7,500,000	\$67,500,000
2	\$6,785,714	\$61,071,429	\$6,750,000	\$60,750,000
3	\$6,428,571	\$54,642,857	\$6,075,000	\$54,675,000
4	\$6,071,429	\$48,571,429	\$5,467,500	\$49,207,500
5	\$5,714,286	\$42,857,143	\$4,920,750	\$44,286,750
6	\$5,357,143	\$37,500,000	\$4,428,675	\$39,858,075
7	\$5,000,000	\$32,500,000	\$3,985,808	\$35,872,268
8	\$4,642,857	\$27,857,143	\$3,587,227	\$32,285,041
9	\$4,285,714	\$23,571,429	\$3,228,504	\$29,056,537
10	\$3,928,571	\$19,642,857	\$2,905,654	\$26,150,883
11	\$3,571,429	\$16,071,429	\$2,615,088	\$23,535,795
12	\$3,214,286	\$12,857,143	\$2,353,579	\$21,182,215
13	\$2,857,143	\$10,000,000	\$2,118,222	\$19,063,994
14	\$2,500,000	\$7,500,000	\$1,906,399	\$17,157,594
15	\$2,142,857	\$5,357,143	\$1,715,759	\$15,441,835
16	\$1,785,714	\$3,571,429	\$1,544,183	\$13,897,651
17	\$1,428,571	\$2,142,857	\$1,389,765	\$12,507,886
18	\$1,071,429	\$1,071,429	\$1,250,789	\$11,257,098
19	\$714,286	\$357,143	\$1,125,710	\$10,131,388
20	\$357,143	\$(0.00)	\$131,388	\$10,000,000

As a percentage of airline revenue, 6–8 percent is typically spent on aircraft depreciation, 3 percent on leasing and 2 percent on interest. In the US, operators normally use the MACRS policy to depreciate aircraft. This could be different in other parts of the world. The carriers Qatar Airways and Emirates Airlines, both based in the Middle East, are required to depreciate their airplanes using only a Straight Line depreciation method for a specified duration. Lufthansa Group amended its depreciation policy in March 2014, which, the company said, will affect its operating result from 2014 onwards. Lufthansa extended the depreciation period for its aircraft from 12 to 20 years, reducing their residual book value from 15 percent to 5 percent of the purchase price. The changes to its depreciation policy provided a EUR351 million benefit in 2014. In 2012, the Australian and International Pilots Association (AIPA) called for an acceleration of depreciation rates for aircraft in Australia. It took 10 years for Australian airlines to write off a new aircraft. While a reduction to a five-year

Figure 11.1 Comparison of Different Depreciation Methods

write-off could at least match the rate available to Air New Zealand, it would fall short of matching the investment environment of the major international airlines flooding the Australian market. For example, Singapore Airlines could depreciate its aircraft over three years. In January 2016, Tiger Air reported that changes in accounting estimates for maintenance provisions and aircraft depreciation policy resulted in a SGD3.8 million (USD2.7 million) increase in aircraft depreciation.

Using Spreadsheets for Depreciation Computations

This section covers the usage of Excel spreadsheets for computing depreciation of various types. Let us assume the following scenario and compute the depreciation values according to the various methods explained earlier.

DirectJet has bought a Boeing 777 aircraft for \$7,000,000 and plans to use it for a period of 10 years following which it expects to sell the aircraft for a salvage value of \$200,000.

- ♦ Compute the depreciation for each year using the **Straight Line** method on a spreadsheet.
 - The Straight Line method depreciation can be calculated on an Excel spreadsheet using the function **SLN ()**. This function requires three parameters: **Cost**, **Salvage Value**, and **Life**. Cost is the amount paid to buy the asset, Salvage Value is the value of the asset after depreciation and Life is the useful life of the asset in years.

- The syntax of this function is **=SLN(COST, SALVAGE, LIFE)**
- Look at the example in Figure 11.2 and enter the data on your spreadsheet.

Figure 11.2 Calculating Straight Line Depreciation Using Excel Spreadsheets

	A	B
1	Initial Value	\$7,000,000
2	Salvage Value	\$200,000
3	Number of Years	10
4	Depreciation Amount Each Year	=SLN(B1,B2,B3)

	A	B
1	Initial Value	\$7,000,000
2	Salvage Value	\$200,000
3	Number of Years	10
4	Depreciation Amount Each Year	\$680,000

- The following table (Figure 11.3) can be generated by subtracting the depreciation amount from the current value of each year.

Figure 11.3 Calculating Annual Asset Value Change under Straight Line Depreciation Using Excel Spreadsheets

	A	B	C
1	Initial Value	\$7,000,000	
2	Salvage Value	\$200,000	
3	Number of Years	10	
4	Depreciation Amount Each Year	\$680,000	
5			
6	Year	Depreciation Amount	Current Value
7	0	0	\$7,000,000
8	1	\$680,000	\$6,320,000
9	2	\$680,000	\$5,640,000
10	3	\$680,000	\$4,960,000
11	4	\$680,000	\$4,280,000
12	5	\$680,000	\$3,600,000
13	6	\$680,000	\$2,920,000
14	7	\$680,000	\$2,240,000
15	8	\$680,000	\$1,560,000
16	9	\$680,000	\$880,000
17	10	\$680,000	\$200,000

- ◆ Compute the depreciation for each year using the **Declining Balance** method, assuming that the aircraft was bought in the month of April.
 - The Declining Balance method depreciation can be calculated on Excel spreadsheets (Figures 11.4 and 11.5) using the function **DB ()**. This function requires five parameters: *Cost, Salvage Value, Life, Period, and Month*. The former three have been explained in the previous example. Period refers to the year of depreciation, while month refers to the number of months remaining in the year of purchase.
 - The syntax of this function is **=DB (COST, SALVAGE, LIFE, PERIOD, MONTH)**, Month is optional and if unmentioned, it is set at 12 by default.
 - The DB function is entered into the cell to calculate the annual depreciation in the depreciation amount column. The depreciation amount for the last year must be slightly adjusted to account for the salvage value of \$200,000.

Figure 11.4 Calculating Declining Balance Depreciation Using Excel Spreadsheets

	A	B	C
1	Initial Value	\$7,000,000	
2	Salvage Value	\$200,000	
3	Number of Years	10	
4	Depreciation Amount Each Year	\$680,000	
5			
6	Year	Depreciation Amount	Current Value
7	0	0	\$7,000,000
8	1	=DB(\$B\$1,\$B\$2,\$B\$3,A8)	

- Also, **Double Declining Balance (DDB)** method could be used to calculate the depreciation value for each year (Figures 11.6 and 11.7).
- In this case, instead of the *Month* parameter, there is a parameter called *Factor*, which represents depreciation rate. If unmentioned, a default rate of 200 percent is used. Generally, any rate can be used as the factor. To use a 150 percent depreciation rate, enter 1.5 as factor.
- The previous spreadsheet example can be used to demonstrate this method. While keeping all the initial information constant, the formula **=DDB(COST, SALVAGE, LIFE, PERIOD, FACTOR)** is used.
- This formula provides the annual depreciation amount using the DDB method. Results are displayed in the Excel snapshot in Figure 11.7.

Figure 11.5 Calculating Annual Asset Value Change under Declining Balance Depreciation Using Excel Spreadsheets

	A	B	C
1	Initial Value	\$7,000,000	
2	Salvage Value	\$200,000	
3	Number of Years	10	
4	Depreciation Amount Each Year	\$680,000	
5			
6	Year	Depreciation Amount	Current Value
7	0	0	\$7,000,000
8	1	\$2,093,000	\$4,907,000
9	2	\$1,467,193	\$3,439,807
10	3	\$1,028,502	\$2,411,305
11	4	\$720,980	\$1,690,325
12	5	\$505,407	\$1,184,918
13	6	\$354,290	\$830,627
14	7	\$248,358	\$582,270
15	8	\$174,099	\$408,171
16	9	\$122,043	\$286,128
17	10	\$85,552	\$200,576

Figure 11.6 Calculating Double Declining Balance Depreciation Using Excel Spreadsheets

	A	B	
1	Initial Value	\$7,000,000	
2	Salvage Value	\$200,000	
3	Number of Years	10	
4			
5	Year	Depreciation Amount	Cu
6	0	0	\$
7	1	=DDB(\$B\$1,\$B\$2,\$B\$3,A7)	

- The salvage value in this case can only be as low as \$751,000 approximately. Hence, to obtain a salvage value of \$800,000, the depreciation amount for year 10 is adjusted to \$139,524 (Figure 11.7).

- ♦ Compute the depreciation amount for each year using the **MACRS** method.

Figure 11.7 Calculating Annual Asset Value Change under Double Declining Balance Depreciation Using Excel Spreadsheets

	A	B	C
1	Initial Value	\$7,000,000	
2	Salvage Value	\$200,000	
3	Number of Years	10	
4			
5	Year	Depreciation Amount	Current Value
6	0	0	\$7,000,000
7	1	\$1,400,000	\$5,600,000
8	2	\$1,120,000	\$4,480,000
9	3	\$896,000	\$3,584,000
10	4	\$716,800	\$2,867,200
11	5	\$573,440	\$2,293,760
12	6	\$458,752	\$1,835,008
13	7	\$367,002	\$1,468,006
14	8	\$293,601	\$1,174,405
15	9	\$234,881	\$939,524
16	10	\$139,524	\$800,000

- Calculating depreciation using MACRS method on a spreadsheet is a simple task which does not require any specific functions. The depreciation rates are to be found in the appropriate MACRS table. Here, we are going to use the 10-year rates.
 - Look at the example in Figure 11.8 and enter the information related to Year and MACRS rate on your spreadsheet.
 - However, for years 7, 8, 9, and 10 Straight Line depreciation is to be used as it results in higher depreciation amounts. Hence in cell D13 enter the formula $=B13/4$. This gives the Straight Line depreciation value per year for a period of four years without any salvage value.
 - Copy the value in cell D13 into cells D14, D15, and D16. This fully depreciates the value of the asset.
 - Depreciation for years 1 to 6 involves the half-life concept as per MACRS.
- ♦ Compute the depreciation amount for each year using the **Sum-of-the-Years Digits** method.
- Calculating depreciation using Sum-of-the-Years Digits method is almost the same as the Declining Balance method, except that in this case the parameter *Month* is not used.

Figure 11.8 Calculating Depreciation using MACRS Method

	A	B	C	D	E
1	Initial Value	\$7,000,000			
2	Salvage Value	\$200,000			
3	Number of Years	10			
4					
5	Year	Initial Value	MACRS Rate	Depreciation Amount	Current Value
6	1	\$7,000,000	10%	\$700,000	\$6,300,000
7	2	\$6,300,000	18%	\$1,134,000	\$5,166,000
8	3	\$5,166,000	14.40%	\$743,904	\$4,422,096
9	4	\$4,422,096	11.52%	\$509,425	\$3,912,671
10	5	\$3,912,671	9.22%	\$360,748	\$3,551,922
11	6	\$3,551,922	7.37%	\$261,777	\$3,290,146
12	7	\$3,290,146	6.55%	\$822,536.41	\$2,467,609
13	8	\$2,467,609	6.55%	\$822,536.41	\$1,645,073
14	9	\$1,645,073	6.56%	\$822,536.41	\$822,536
15	10	\$822,536	6.55%	\$822,536.41	\$0
16	11	\$0	3.28%	\$0.00	\$0

Figure 11.9 Calculating Depreciation Using Sum-of-the-Years Digits Method

	A	B	C
1	Initial Value	\$7,000,000	
2	Salvage Value	\$200,000	
3	Number of Years	10	
4			
5	Year	Depreciation Amount	Current Value
6	1	=SYD(\$B\$1,\$B\$2,\$B\$3,A6)	
7	2	SYD(cost, salvage, life, per)	,650,909

- The syntax of this function is =SYD(COST, SALVAGE, LIFE, PERIOD).
- This formula returns the depreciation amount per year based on the Sum-of-the-Years Digits method and generates the Excel snapshot shown in Figure 11.10. The correct salvage value at the end of year 10 is automatically obtained.

Figure 11.10 Calculating Annual Asset Value Change under Sum-of-the-Years Digits Depreciation Method Using Excel Spreadsheets

	A	B	C
1	Initial Value	\$7,000,000	
2	Salvage Value	\$200,000	
3	Number of Years	10	
4			
5	Year	Depreciation Amount	Current Value
6	1	\$1,236,364	\$5,763,636
7	2	\$1,112,727	\$4,650,909
8	3	\$989,091	\$3,661,818
9	4	\$865,455	\$2,796,364
10	5	\$741,818	\$2,054,545
11	6	\$618,182	\$1,436,364
12	7	\$494,545	\$941,818
13	8	\$370,909	\$570,909
14	9	\$247,273	\$323,636
15	10	\$123,636	\$200,000

Basics of a Replacement Decision

An asset replacement decision is always considered challenging. Numerous factors ought to be taken into account, including purchase price, financing options, ownership costs, operating costs, and salvage value. These factors make the replacement decision an economic decision. If an asset is kept for too long, its salvage value decreases and it becomes increasingly expensive to operate. Alternatively, if an asset is replaced too soon, it is too costly from an investment perspective. This is particularly true in the case of an asset such as aircraft, because the operating cost of an aircraft grows extremely high as the age of the aircraft increases. As the fleet ages, managers need to identify when the useful life of an aircraft is to be over. The typical lifespan of a Boeing 737 is about 25 to 30 years. Finding the optimum time to replace an aircraft has

Aviation Snippets

Delta retires its last DC-9, the oldest plane in U.S. fleet

Delta Air Lines is retiring its last DC-9, the oldest passenger plane in the fleet of the big U.S. airlines. The final passenger flight was scheduled for Minneapolis to Atlanta on Monday evening. McDonnell Douglas delivered the first DC-9s in 1965, and eventually built 976 of them. Most airlines retired the DC-9s by the 1990s. But instead of retiring them, Northwest in 1995 refurbished their interiors to squeeze more flying out of them. But federal rules don't limit how many years a plane can fly, only how many takeoffs and landings.

*St. Louis Post-Dispatch,
January 4, 2014*

always been a focus of many studies carried out in the past. Assets may be replaced because of:

◆ *Technological developments*

Technological advancements have been the major factor in improving the efficiency of air transport, and affecting the growth of the air transport industry. Significant progress in aircraft design and fuel efficiency has been achieved since the beginning of the commercial aviation industry. Today, Boeing 787's great fuel efficiency and range flexibility enables airlines to expand operations and profitably open new routes as well as optimize fleet and network performance.⁹ Comparably, A380's advanced technology has resulted in about 20 percent lower seat-mile costs, while its range is 10 percent greater than that of the Boeing 787.¹⁰

◆ *An asset is at the end of its economic life*

Retirement is part of the aircraft life cycle, and the main reasons behind a decision to retire an old plane are typically financial. Older designs are more expensive to maintain, and such airplanes are generally less fuel efficient. An increase in fuel prices may have encouraged some airlines to replace the existing fleet with a more fuel-efficient one, the International Bureau of Aviation has said. Conversely, lower fuel prices may very well compensate for the increased maintenance costs, in which case management will consider continuing to use their older fleet.¹¹ For example, a pair of nearly 50-year-old Vietnam-era OV-10 Bronco turbo-propeller planes has been brought out of retirement and into service. The estimated cost of flying more modern jets (like the F-15 or F-35) is \$45,000 per hour of flight, while the OV-10 Bronco could cost less than \$5,000 per flight hour.

◆ *Assets do not satisfy current needs and growing demand*

An increase in demand in a certain market could force an airline to switch to a bigger aircraft and this may typically result in replacement. Also, some airlines, such as Southwest, have been using only one type of aircraft for all their operations. When Southwest acquired AirTran, it had to unload the entire fleet of Boeing 717 aircraft inherited from acquired AirTran, as it did not match Southwest's policy of operating Boeing 737s only. Flying a single-model fleet allows Southwest Airlines to manage its training and maintenance needs more efficiently and reduce the operating expenses as the smaller Boeing 717 is more expensive to fly on a per-seat basis than Boeing 737.¹²

◆ *Deterioration*

Most aircraft suffer a deterioration of their physical condition toward the end of their service life due to high utilization, corrosion, and wear and tear. Drastic deterioration of aircraft would also require the airlines to opt for replacement of their assets. For example, American Airlines had to replace most of the MD 80s it owned since they were in a very poor operating condition due to extensive deterioration after many years of service and expensive heavy checks. The MD-80 was built by McDonnell Douglas, bought by Boeing in 1997.

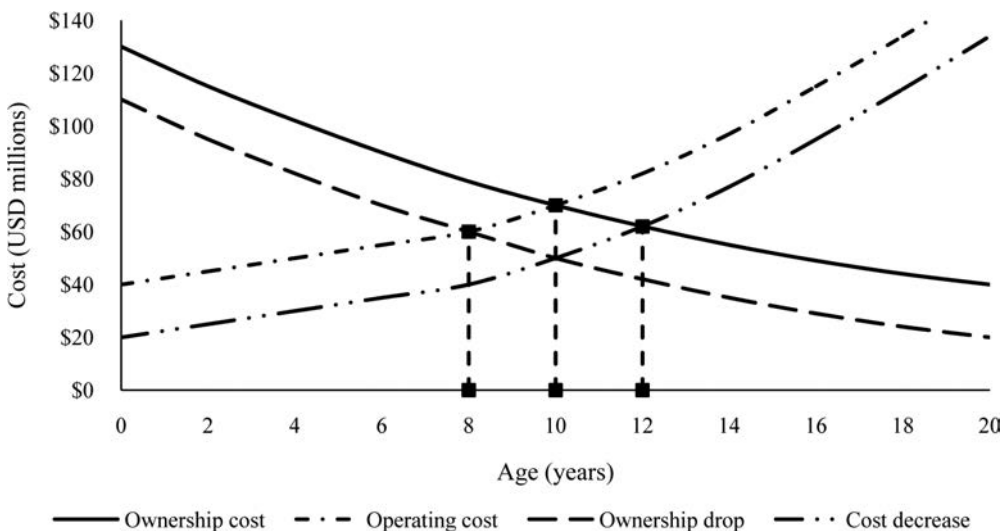
Consider the following three scenarios where the ownership costs of an aircraft are compared against its operating costs. Figure 11.11 provides the graphical representation of all three scenarios.

In the first scenario, ownership cost and operating cost intersect at the point of 10 years that indicates the asset's replacement life, as the operating cost overshadows the ownership cost beyond this point.

Table 11.10 Ownership Cost vs Operating Cost

Age (Years)	Ownership Cost	Operating Cost	Ownership Drop	Cost Decrease
0	\$130	\$40	\$110	\$20
2	\$115	\$45	\$95	\$25
4	\$102	\$50	\$82	\$30
6	\$90	\$55	\$70	\$35
8	\$79	\$60	\$60	\$41
10	\$70	\$70	\$50	\$50
12	\$62	\$82	\$42	\$62
14	\$55	\$97	\$35	\$77
16	\$49	\$115	\$29	\$95
18	\$44	\$134	\$24	\$114
20	\$40	\$154	\$20	\$134

Figure 11.11 Possible Changes in Economic Service Life



The second scenario, involving decrease in ownership cost, causes this replacement life to drop to eight years because operating cost now takes over the ownership cost much earlier than before.

The third possible scenario is a drop in operating cost which may be due to a fall in fuel prices. Replacement life now increases to 12 years, because it takes longer for the operating cost to overshadow the ownership cost.

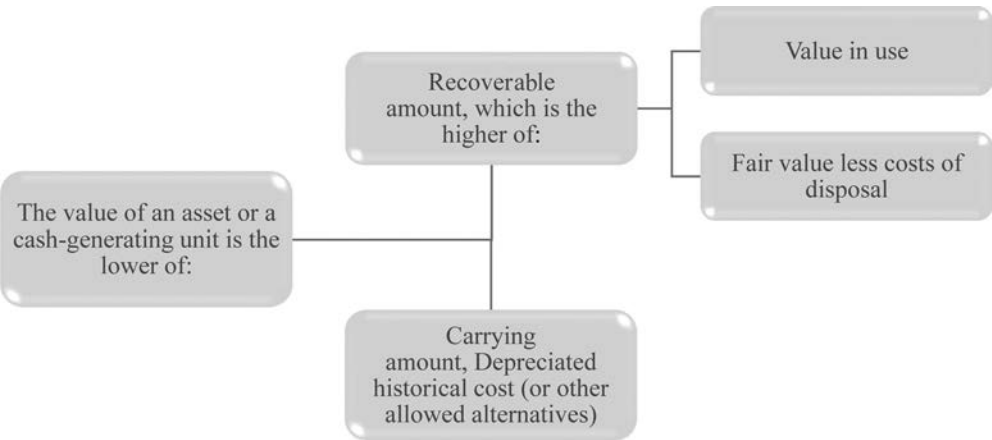
Impairment

Aircraft values have fallen significantly for many older aircraft over the last two to three years. New factors such as the introduction of the NEO (New Engine Option) into the Airbus A320 family, the fastest selling ever commercial aircraft, and the Boeing 737-Max once delivered in the next 5–6 years may over time affect prices on existing aircraft...

Aviation Finance, 2016

The term impairment is usually related to a fixed asset that has a market value which has decreased significantly and is recognized as a loss in the income statement. Recoverable amount is the value of the benefits that can be obtained from a fixed asset. Economic benefits are obtained either by selling the asset or by using the asset. Recoverable amount equals the higher of fair value less costs to sell and the value in use (net present value of future cash flows from the asset). Carrying amount refers to the current value of the asset found by subtracting the accumulated value of depreciation from the purchase value of the asset. If the carrying amount exceeds the recoverable amount, an impairment expense is recognized in the period. If the carrying amount is less than the recoverable amount, no impairment is recognized. The impairment loss is the number by which the carrying amount of the asset exceeds its recoverable amount (IAS 36.6, IAS 36.8). The process of measuring impairment is illustrated in Figure 11.12.

Figure 11.12 Impairment Measurement under IAS 36



Assume that in January 2010 DirectJet purchased a Boeing 737–800 for \$75 million. Its estimated useful life at that date was 20 years. The airline uses Straight Line depreciation. In 2013, Boeing embarked on a plan to introduce a more efficient B737, called B737 MAX. B737 MAX delivers 8 percent lower operating costs than its main competitor. The new 737 MAX Advanced Technology Winglet increases fuel efficiency by 1.8 percent. DirectJet estimates that it can sell the old aircraft for \$70 million but it has to incur costs of \$10,000,000. Alternatively, if DirectJet continues to use it, the present value of the net cash flows will help in generating \$62 million. The basic rule is to recognize impairment if the carrying amount exceeds the recoverable amount.

First, we need to determine the carrying amount. The aircraft has a cost of \$75 million, has been in service for three years so far and has a useful life of 20 years. This means that accumulated depreciation is $\$75/20 \times 3$ or \$11.25 million. Thus, carrying amount is \$75 million minus \$11.25 million or \$63.75 million.

Second, we need to determine the recoverable amount. Recoverable amount is the higher of fair value less costs to sell and value in use. Fair value less costs to sell in this scenario is \$70 million minus \$10 million or \$60 million. Value in use is the present value of future cash flows which amounts to \$62 million. Recoverable amount is the higher of \$60 million and \$62 million. Hence, recoverable amount is \$62 million.

Carrying amount is \$63.75 million while recoverable amount is \$62 million. An impairment loss of \$1.75 million is to be recognized.

Example 11.6

A regional airline has purchased an aircraft tug for \$300,000. The tug will be depreciated under the MACRS as a five-year property (for a five-year property, the MACRS percentages are 20, 32, 19.2, 11.52, 11.52, and 5.76 percent). The tug's salvage value is expected to be \$50,000 less each year. The operating and maintenance costs are expected to be \$30,000 per year. Also, the net present value of future cash flows the aircraft tug will help in generating is \$135,000. The airline decides to sell the tug at the end of two years. Compute the impairment loss incurred by the airline.

Depreciation for year 1 is calculated as $(20\% \times \$300,000) = \$60,000$

Depreciation schedule for the aircraft tug is as given in Table 11.11:

Table 11.11 MACRS Depreciation Schedule

Year	Depreciation Expense	Accumulated Depreciation	Value
1	\$60,000	\$60,000	\$240,000
2	\$96,000	\$156,000	\$144,000
3	\$57,600	\$213,600	\$86,400
4	\$34,560	\$248,160	\$51,840
5	\$34,560	\$282,720	\$17,280
6	\$17,280	\$300,000	\$0

The first step is to determine the carrying amount. The aircraft tug costs \$300,000 and the accumulated depreciation at the end of year 2 is \$156,000. Thus, the carrying amount is \$300,000 minus \$156,000, which is \$144,000.

Next, the recoverable amount needs to be determined. Fair value less costs to sell in this scenario is \$200,000 (i.e., \$300,000 – \$100,000) minus \$60,000, which is \$140,000. Value in use is the present value of future cash flows that amounts to \$135,000. Recoverable amount is the higher of \$140,000 and \$135,000. Hence, recoverable amount is \$140,000.

Carrying amount is \$144,000, while recoverable amount is \$140,000. An impairment loss of \$4,000 is to be recognized.

Abandonment

Abandoned or unclaimed aircraft means an aircraft that has not been utilized, flown or worked on for a period of 13 consecutive months when stored inside a hangar or on a tie-down, and/or has not been maintained in flyable condition.

Ginnow and Milorad (1985)

The voluntary, intentional and complete surrender of rights to a certain property/asset without the naming of a recipient as owner is known as abandonment. Acts that imply abandonment include continuous lack of use and/or maintenance and failure to pay property taxes. If there is an entity or individual with a prior interest or claim to that property/asset, it is possible that the property/asset will revert to their ownership. If no owner is apparent, the property/asset will be given to the state. Abandoning a property/asset does not remove the obligations associated with the property unless explicitly accepted by the new owner.

Some of the more common acts of abandonment include abandonment of mines and abandonment of oil and natural gas wells. Throughout the world there are numerous fields of dangerous, exposed holes from mining operations that were abandoned soon after they stopped yielding minerals or coal. Economic events such as a decrease in the price of minerals or an unforeseen disaster could lead to the bankruptcy of the mining company, making it unable to complete the mine closure for financial reasons. For decades, old abandoned wells have leaked oil, natural gas and brine into soil and drinking water, and also posed an explosion risk. Increased danger of contamination or explosion from abandoned wells has led to more aggressive efforts to find and plug old wells. Pennsylvania, the birthplace of the US oil industry, is turning to citizen volunteers, scientists, drillers and archivists to find hundreds of thousands of abandoned wells. It is one of several proving grounds where new technology could lead the way for other states to locate and plug abandoned wells.¹³ Abandonment could also arise from a financial standpoint. It results from the

Aviation Snippets

A Mexican airline abandoned three 727s in three different locations—one in Venezuela, one in New Mexico and one in Florida, since they went out of business and couldn't afford to move them.

ABC News, December 2015

overall expected expenses being greater than the overall expected revenues in the long run. This would mean that bearing the already realized sunk costs would be cheaper than continuing operations and compounding the losses for a longer period of time.

A very appropriate example for this would be the impact of the 2008 financial crisis on the construction industry in Spain. Building construction, prior to the collapse of the economy, contributed twice as big a share of GDP in Spain as in other large European countries. The crisis caused this industry to rapidly shrink and a side effect of that was the abandonment of thousands of homes, thought to be relinquishing their value. People and construction companies felt that abandonment would incur fewer losses than completion and non-sale of the houses. Another country which faced similar issues during the financial crisis was UAE. The striking outcome was the mass abandonment of luxurious cars, which mostly belonged to expatriates who were fleeing the country after being unable to pay their debt or mortgages. The strict Sharia Law forced people to leave the country to cope with the fear of imprisonment, since failure to pay debt is a serious punishable offence.¹⁴

The aviation industry has had a fair share of abandonment issues as well. Common reasons for aircraft abandonment include aging, damage, lower demand during economic recession, and increasing economic expenses. Abandonment could be short-term (where an aircraft will be brought back into operation when it is considered profitable again) or long-term (where it is believed that profitability can never be recovered to previous levels). During the financial crisis of 2008, many airlines were reported to have abandoned aircraft due to financial constraints they faced. There have also been cases when entire airports have been abandoned due to increasing financial burdens. For example, Don Quijote Airport in the town of Ciudad Real, nearly 150 miles from Madrid, was meant to be a symbol of Spain's economic boom. Instead, the project denounced the country's glaring financial woes, when it was ultimately completely shut down in 2012.¹⁵

To meet market conditions in case of a catastrophic drop in demand, airlines abandon aircraft in order to boost their load factors. Abandoned aircraft and plane graveyards exist across the world, from the plane boneyard of Davis-Monthan Air Force Base,¹⁶ Arizona, in US to Rinkaby military airfield in Sweden. The largest graveyard for commercial passenger jets in the United States is Mojave Air and Space Port, California. Most of the airplanes that sit in desert graveyards today date from the Vietnam era or later. They are divided into four categories, depending on their future prospects:

- ♦ Roughly 1,000 aircraft are preserved for possibly flying them again.
- ♦ About 2,000 aircraft are maintained for spare parts.
- ♦ About 3,000 aircraft are kept in near ready-to-fly condition.
- ♦ Around 4,000 aircraft are destined for future use in museums.

Aviation Snippets

A decommissioned 747-200 is stationed at the entrance to Arlanda airport in Stockholm, Sweden. It was abandoned in 2002 after its owner went bankrupt. In 2008 it was turned into the Jumbo Stay Hotel with 79 beds across 27 rooms

ABC News, Dec. 2015

Example 11.7

Jet Aviation Company (JAC) is planning to abandon its only landing gear manufacturing company, which is currently in operation, in June 2018. The factory, acquired 15 years ago for \$100 million, was initially assigned a 20-year estimated useful life and \$5 million salvage value. As a result of the commitment to a plan to abandon the factory, JAC has reduced the factory's estimated remaining useful life from six years to one year and will account for the change in estimate in accordance with the policy. Thus, the factory's carrying value of \$15.7 million at June 30, 2018 will be depreciated over the next year, such that its carrying value will equal \$1 million.

Example 11.8

DirectJet purchased an old DC-9 for \$36,000,000, and recognized \$3,000,000 of depreciation per year over the following 12 years. After 10 years, the aircraft is not only fully depreciated but, because of the very high oil price, it is ready for retirement. DirectJet gives away the aircraft for free, and records the following entry.

	Debit	Credit
Aircraft		\$36,000,000
Accumulated depreciation	\$30,000,000	
Loss on aircraft disposal	\$6,000,000	

Now assume DirectJet disposes of its \$36,000,000 aircraft, but does so after six years, and sells it for \$22,000,000 in cash. In this case, it has already recorded \$18,000,000 of depreciation expense. The entry is:

	Debit	Credit
Aircraft		\$36,000,000
Cash	\$22,000,000	
Accumulated depreciation	\$18,000,000	
Gain on aircraft disposal		\$4,000,000

If DirectJet Airlines had sold the aircraft for \$15,000,000 instead of \$22,000,000, then there would be a loss of \$3,000,000 on the sale. The entry would be:

	Debit	Credit
Aircraft		\$36,000,000
Cash	\$15,000,000	
Accumulated depreciation	\$18,000,000	
Loss on aircraft disposal	\$3,000,000	

The asset write-offs should be recorded as soon after the disposal of an asset as possible, or the accumulated depreciation and balance sheet would be overstated.

Summary

Depreciation is a methodology to allocate the entire cost of depreciable assets to the operating expense for a series of periods. Generally, the depreciation expense is derived by initially estimating both the useful life and the perceived fair market value of the asset at the end of its estimated useful life (residual value). Furthermore, depreciation is the tax deduction given to recover the cost of investments over the tax life of the project. Some international airlines have a certain degree of flexibility in deciding on the effective useful life of their aircraft and their residual value. By decreasing the estimated residual value and reducing the estimated useful life of an asset, the company would prospectively record a higher depreciation expense on its income statement. The most common methods of depreciation include: Straight Line (SL) method, Modified Accelerated Cost Recovery System (MACRS), Sum-of-the-Years Digits (SYD), method and Declining Balance (DB) method. Under the Straight Line method, the annual depreciation amount is the total depreciation divided by the recovery period in years. The Straight Line method leaves a company exposed to a much greater probability of an unexpected asset impairment loss, especially with assets that may lose value quickly due to technical obsolescence rather than physical depreciation. In this case, it may be difficult to estimate fair values, and this method may not be desirable. The Modified Accelerated Cost Recovery System lets businesses deduct more in the early years of the recovery period or tax life of the property than Straight Line methods.¹⁷ Under MACRS, fixed assets are assigned to a specific asset class. The Internal Revenue Service has published a complete set of depreciation tables for each of these classes. A business determines its tax depreciation based on the information in the asset class table. The resulting depreciation is included in the company's income tax return as part of the derivation of taxable income. Sum-of-the-Years Digits method is used for the assets that lose a substantial portion of their value early in their lifetime. Under this method, depreciation is computed by multiplying the book value by the remaining periods of useful life at the start of the period divided by the sum of the digits in the original useful life. Declining Balance method charges depreciation at a higher rate in the earlier years of an asset, and reduces depreciation as the life of the asset progresses. Declining Balance is a type of accelerated depreciation technique which is more applicable for assets that are extremely useful in the first years of service and the level of service keeps reducing as time goes on.

It is important to note that internally a company is permitted to use any of the depreciation techniques available. However, for tax deduction purposes, every company is forced to use the MACRS technique under US law.

In addition to various depreciation techniques, this chapter also discusses the basics of asset replacement decision (affected by many factors such as purchase price, financing options, ownership costs, operating costs, and salvage value), impairment (replacement of a fixed asset that is recognized as a loss in the income statement due to a significant drop in its market value), and abandonment (voluntary, intentional and complete surrender of rights to a certain property/asset without the naming of a recipient as owner).

Discussion Questions and Problems

- 11.1** Explain the concept and the rationale of the depreciation process.
- 11.2** Define the salvage value or residual cost.
- 11.3** Could any company suffer from loss of residual value?
- 11.4** What is the difference between market value and residual value?
- 11.5** Identify the factors involved in the depreciation process.
- 11.6** Flight Services, a provider of maintenance services to commercial airlines, purchased a piece of equipment for \$1,000,000 with an estimated useful life of three years and a residual value of \$100,000. The company commonly uses Straight Line depreciation. Complete the table below.

Year	Year-beginning Book Value	Depreciation Amount	Accumulated Depreciation	Year-end Value
2017	\$1,000,000			
2018				
2019				\$100,000

- 11.7** Jet Aviation has purchased a tug for \$1,000,000, a machine that Jet Aviation estimates will have a useful life of five years and a residual value of \$46,565:
- Jet Aviation uses DDB depreciation. Complete the amortization table.
 - Prepare the adjusting journal entry to record the 2017 depreciation expense.
- 11.8** In January 2017, Jet Aviation purchases a push truck for \$1,000,000 that Jet Aviation estimates will have a useful life of five years and a residual value of \$100,000. Jet Aviation uses Sum-of-the-Years (SYD) depreciation. Complete the table below.

Year	Year-beginning Book Value	SYD	Depreciation Expense	Accumulated Depreciation	Year-end Book Value
2017					
2018					
2019					
2020					
2021					

Sum-of-the-Years denominator $[(n(n+1))/2] = (5 \times 6)/2 = 15$

$\$900,000 \times 5/15 = \$300,000$ first 12 months' depreciation

$\$900,000 \times 4/15 = \$240,000$ second 12 months' depreciation

$\$900,000 \times 3/15 = \$180,000$ third 12 months' depreciation
 $\$900,000 \times 2/15 = \$120,000$ fourth 12 months' depreciation
 $\$900,000 \times 1/15 = \$60,000$ last 12 months' depreciation

Depreciation expense:

2017: $(\$900,000 \times 5/15) = \$300,000$
 2018: $(\$900,000 \times 4/15) = \$240,000$
 2019: $(\$900,000 \times 3/15) = \$180,000$
 2020: $(\$900,000 \times 2/15) = \$120,000$
 2021: $(\$900,000 \times 1/15) = \$60,000$

- 11.9** DirectJet purchased an aircraft simulator to train their new pilots for \$800,000. The simulator is expected to last for seven years and at the end of the seven years have a salvage value of \$80,000. Use the Straight Line method of depreciation to calculate the annual depreciation.
- 11.10** Air Service Avionic is planning to add a new product to its line. To manufacture this product, the company needs to buy a new machine at a \$484,000 cost with an expected five-year life and a \$15,700 salvage value. All sales are for cash, and all costs are out of pocket except for depreciation on the new machine. Additional information includes the following:
- Expected annual sales of new product equal \$1,870,000
 - Expected annual costs of new product and direct materials are \$455,000
 - Direct labor cost is \$673,000
 - Overhead excluding Straight Line depreciation on new machine is \$337,000
 - Selling and administrative expenses are \$157,000
 - Income taxes are 34 percent
- Compute Straight Line depreciation for each year of this new machine's life.
- 11.11** What is accelerated depreciation using the units of production (units of service) method?
- 11.12** Given the following information regarding DirectJet Airline's purchase of equipment on January 1, 2015:

Cost	\$120,000,000
Salvage (residual) value	\$ 2,000,000
Estimated life	5 years
Estimated units of production	1,000 units

complete the following depreciation schedules:

A. Assume DirectJet uses the Double Declining Balance method:

Year	Calculations	Depreciation Expense	Accumulated Depreciation	End-of-year Book Value
2016	_____	_____	_____	_____
2017	_____	_____	_____	_____

B. Assume DirectJet uses the 150 percent Declining Balance method ($1.5 \times$ Straight Line rate):

Year	Calculations	Depreciation Expense	Accumulated Depreciation	End-of-year Book Value
2016	_____	_____	_____	_____
2017	_____	_____	_____	_____

C. Assume DirectJet uses the Sum-of-the-Years Digits method:

Year	Calculations	Depreciation Expense	Accumulated Depreciation	End-of-year Book Value
2016	_____	_____	_____	_____
2017	_____	_____	_____	_____

D. Assume DirectJet uses the Straight Line method:

Year	Calculations	Depreciation Expense	Accumulated Depreciation	End-of-year Book Value
2016	_____	_____	_____	_____
2017	_____	_____	_____	_____

11.13 Assume that an aircraft has a historical cost of \$50,000,000, with an estimated useful life of 25 years, and was 60 percent depreciated when it was discovered that the hull had structural problems. The estimated cost of maintenance and overhaul is \$10,000,000. The estimated current cost to replace the aircraft is \$750,000,000. The aircraft is still going to be used. Calculate the impairment value of the aircraft.

11.14 EZJET Airways recently hired a new manager for its struggling finance division. The manager has been given the responsibility for operating the airline more profitably. A fixed asset having a useful life of three years is purchased on January 1, 2017. The cost of the asset is \$5,000,000 whereas its residual value is expected to be \$1,250,000. Calculate the depreciation expense for the years ending December 30, 2017 and December 30, 2018.

Appendix

Table 11.A1 Double Declining Balance Depreciation Method

Asset Class	Depreciation Period	Description	Depreciation Method
3-year property	3 years	Tractor units for over-the-road use, race horses over two years old when placed in service, any other horse over 12 years old when placed in service, qualified rent-to-own property	200% Declining Balance
5-year property	5 years	Automobiles, taxis, buses, trucks, computers and peripheral equipment, office equipment, any property used in research and experimentation, breeding cattle and dairy cattle, appliances, etc.	200% Declining Balance
7-year property	7 years	Office furniture and fixtures, agricultural machinery and equipment, any property not designated as being in another class, natural gas gathering lines	200% Declining Balance
10-year property	10 years	Vessels, barges, tugs, single-purpose agricultural or horticultural structures, trees/ vines bearing fruits or nuts, qualified small electric meter and smart electric grid systems	200% Declining Balance
15-year property	15 years	Certain land improvements (such as shrubbery, fences, roads, sidewalks and bridges), retail motor fuels outlets, municipal wastewater treatment plants, clearing and grading land improvements for gas utility property, electric transmission property, natural gas distribution lines	150% Declining Balance
20-year property	20 years	Farm buildings (other than those noted under 10-year property), municipal sewers not categorized as 25-year property, the initial clearing and grading of land for electric utility transmission and distribution plants	150% Declining Balance
Residential rental property	27.5 years	Any building or structure where 80% or more of its gross rental income is from dwelling units	Straight Line
Nonresidential real property	39 years	An office building, store, or warehouse that is not residential property or has a class life of less than 27.5 years	Straight Line

Notes

- 1 *Gulf Digital News*, November 9, 2015.
- 2 The United States of America, Internal Revenue Service, IRS Pub 946.
- 3 Ibid.
- 4 Ibid.
- 5 EPS is also known as net income per share.
- 6 Overview of Airline Industry Depreciation Policies. Center for Financial Research and Analysis. December 1999.
- 7 The United States of America, Internal Revenue Service, IRS Pub 946.
- 8 Internal Revenue Service, Publication 946 (2014).
- 9 Boeing, Company Information. September 2015.
- 10 Eaton's Aerospace Case History Airbus A380 Aircraft. September 2010.
- 11 How Does An Airline Decide A Plane Is Too Old To Fly? *International Business Times*, March 24, 2015.
- 12 Southwest to Unload AirTran's Boeing 717s with Delta Leases. *Bloomberg Business*, May 12, 2012.
- 13 Hoover, S. Special Report: Uncovering Abandoned Oil and Natural Gas Wells, July 2015.
- 14 Dubai Expats Abandon Cars at Airport Car Park, *The Sunday Times*, February 2009.
- 15 Rosen, E. Ghost Airports: Unfinished, Abandoned or Forgotten, September 2015.
- 16 At Davis-Monthan Air Force Base in Arizona, over 4,400 scrapped fighter aircraft, bombers, transports, helicopters, and attack jets are kept.
- 17 The United States of America, Internal Revenue Service, IRS Pub 946.

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12

Engineering Cost Estimation

When someone comes to me with a cost saving idea, I don't immediately jump up and say yes. I ask: what's the effect on the customer?

Herb Kelleher, Southwest Airlines



This chapter presents a detailed description and evaluation of the cost structure of the aviation industry. In-depth understanding of the cost structure and accurate estimation is one of the most important functions of engineer economists. The accuracy of the cost estimation method plays a pivotal role in the financial sustainability of any company. Lockheed entered the commercial aircraft market in the early 1960s by producing their L-1011 TriStar. The new aircraft featured a twin-aisle interior with a maximum of 400 passengers and a three-engine layout. Lockheed faced strong competition, first from the McDonnell Douglas DC-10, and later from the Boeing 747 aircraft (Philip, 1998). After producing about 250 aircraft, Lockheed withdrew from the commercial aircraft business due to inadequate demand and the cost overrun in 1984.¹ The program did not earn back its development costs, and consequently Lockheed left the commercial airline industry after failing to properly estimate the production cost and the break-even number of aircraft sold.²

Cost estimation is fundamental in all aspects of a project management from the early stage of determining the initial investment size and incremental costs needed during the phase of production. Costs may be grouped and divided into many different categories according to the needs and the objectives of the project managers.

In professional engineering practices, the attention is more concentrated on cost estimation rather than revenue estimation; therefore the focus of this chapter is to enable students and practitioners to understand the fundamental cost concepts including total costs, fixed costs, variable costs, direct costs, indirect costs, opportunity

costs, and incremental costs. This chapter also aims to prepare analysts with cost estimation techniques to evaluate a project.

The following sections are addressed in this chapter:

- ♦ Definition of Cost
 - Fixed Costs (FC)
 - Variable Costs (VC)
 - Total Costs (TC)
 - Cubic Cost Function
 - Marginal Costs (MC)
 - Average Total Cost (ATC)
 - Average Fixed Cost (AFC)
 - Average Variable Cost (AVC)
 - Sunk Costs
 - Opportunity Costs
- ♦ Relationship Between Economic and Accounting Costs
 - Cost of Debt
 - Cost of Equity
 - Weighted Average Cost of Capital (WACC)
 - ROIC vs WACC
- ♦ Cost Divisions and Categories
 - Direct Costs
 - Indirect Costs
- ♦ Learning Curve
- ♦ Estimating Costs
- ♦ Cost Estimation Techniques
 - Unit Cost Method
 - Cost Index Method
 - Cost Capacity Equitation
 - Cost Factor Method
 - Indirect Cost Estimation and Allocation
- ♦ Airline Cost Structure
 - Operating Costs (OC)
 - Direct Operating Costs
 - Fixed Direct Operating Costs
 - Variable Direct Operating Costs
 - Indirect Operating Costs
 - Non-operating Costs (NOC)
- ♦ Summary
- ♦ Discussion Questions and Problems

Definition of Cost

After a steep rise in the price of oil in 2008, many airlines looked for different practices to reduce fuel cost and consumption. Airlines globally have been continually working toward restructuring their fleet and reducing their ownership as well as operating costs. Cost is defined as the value of the direct and indirect materials, human resources, know-how, technology, and equipment used to produce a certain amount of products. For example, the cost of producing an aircraft is the value of resources used to create the complex manufacturing plant, research and development, labor, material, and intellectual properties. Cost management should be closely aligned with the corporate culture, tailored productivity and efficiency targets, and carefully selected cost cutting measures. Southwest Airlines (LUV) by only flying Boeing 737s has been able to drastically lower the cost of training, maintenance, and inventory. Fleet homogeneity has enabled LUV to achieve significant cost reductions in labor costs and inventory spare holding costs. Generally, maintenance costs enjoy economies of scale which are only possible for airlines flying one type of aircraft.

A fundamental understanding of controlling costs is essential to ensure shareholders earn a required rate of return on the investment. In business, cost is usually a monetary valuation of

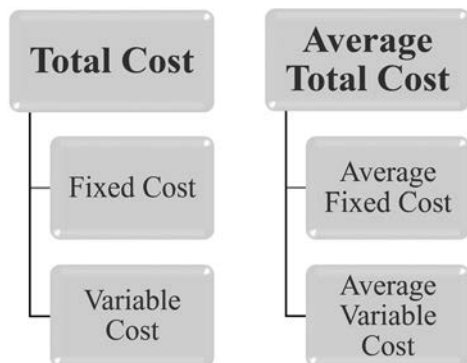
- ◆ Effort
- ◆ Material
- ◆ Opportunity forgone in production and delivery of a good or service
- ◆ Resources
- ◆ Risks incurred
- ◆ Time and utilities consumed

All expenses are costs, but not all costs (such as those incurred in acquisition of an income-generating asset) are expenses.

The following section takes a closer look at cost behavior and how to identify cost as fixed, variable, or total cost. The central idea for estimating future costs is to identify the cost behavior. The understanding of fixed, variable, and total cost provides engineer economists the vision needed to accurately forecast operations.

Fixed Costs (FC)

Fixed costs are costs that are independent of output, i.e., remain constant throughout the relevant range of production and may be considered sunk for the relevant range. Refer to Figure 12.1 to notice that the fixed cost remains constant throughout the range of production. Typical fixed costs differ widely among industries. Retail shops such as restaurants are an example of low fixed cost industries. Airlines, aircraft manufacturers, jet engine manufacturers, auto manufacturers, and drilling operations usually have very high fixed costs.



Aviation Snippets

Turkish Airlines boss quashes A380 rumor

"Also, if we buy it [the A380] we can't buy just one or two. We would need a dozen", the airline boss added, referring to the need for economies of scale when inducting a new aircraft type, "so it's not on the table."

Arabian Aerospace,
February 1, 2016

A major portion of cost production of a new aircraft is the fixed cost associated with development of prototype, research and development, and new physical plants and infrastructure. Airbus spent eight years and around \$14.7 billion developing the A350-XWB.³ The A350 is the first Airbus with both wing structure and fuselage made largely of carbon fiber reinforced polymer.

For airlines, an aircraft's fixed costs are the same no matter how much the airplane flies. Fixed costs can create economies of scale, which are reductions in per-unit costs through an increase in production volume. If a firm has a large

amount of fixed costs, profit margins could drop drastically when sales fall, which adds a certain level of risk to the stocks of these companies. Conversely, the same high-fixed-costs company will experience an abundance of profits when sales start to rise, because any revenue increases are applied across a constant cost level.

As we mentioned earlier, the airline industry is a highly capital-intensive business, requiring a massive amount of financial assets to operate effectively. The high fixed cost and the low variable cost have historically resulted in lower profitability and higher chance of financial insolvency.

Some examples of fixed costs include the following:

- ♦ Aircraft ownership
- ♦ Airport slot
- ♦ Airport terminals
- ♦ Crew member salaries (annual contracts)
- ♦ Insurance
- ♦ Jet engines
- ♦ Maintenance cost⁴
- ♦ Ownership cost of hangar and other physical assets
- ♦ Plant and equipment
- ♦ Regulatory fees and charges

Variable Costs (VC)

With fuel prices such a large part of their cost base, airlines are more than ever variable-cost companies whose expenses increase and decrease in tandem with rates of production. So growing doesn't yield the cost advantages it once did.

The Dallas Morning News, 2008

A variable cost is a cost that varies in relation to changes in the volume of production and activity. Variable cost is a constant amount per unit produced or used. Hence, the

total amount of the variable cost will change proportional to the volume of activity (Figure 12.1). There are many different forms and types of variable costs that an airline incurs that are based on the number of passengers, the number of departures, or the volume of cargo carried. Some of these costs may include aircraft fuel, some labor costs, supplies, and replacement parts. The total cost of any process is the sum of the fixed costs and variable costs as mentioned earlier.

In the airline industry, the maintenance cost could be both fixed and variable depending on the type and the classification of maintenance. Common examples of airline variable costs include:

- ◆ Catering
- ◆ Crew expenses (travel, hotel, and meals)
- ◆ Crew salaries (if paid per flight)
- ◆ Fuel
- ◆ Landing fees and air navigation charges
- ◆ Maintenance⁵
- ◆ Oil
- ◆ Other labor costs and salaries

Total Costs (TC)

There is one rule for the industrialist and that is: Make the best quality of goods possible at the lowest cost possible, paying the highest wages possible.

Henry Ford

Total cost is the sum of all the fixed and variable costs incurred during the completion of a project or business:

TC = Total Cost

TFC = Total Fixed Cost

TVC = Total Variable Cost

$TC = TFC + TVC$

With fixed cost and variable cost positive at all levels of output, total cost must be positive at all range of output. When estimating a cost function, cost behavior can be approximated either by a linear cost function within the relevant range, or by an exponential function. The general linear cost function is given by:

$$TC(Q) = FC + \beta \times Q$$

Here, $TC(Q)$ is the total cost, and Q is the number of items of output. In this context, the slope β is called the average variable cost per unit which at the same time is the marginal cost. With a linear total cost function, the total variable cost naturally becomes a linear function (constant variable cost). To better understand all the

Table 12.1 Short-run Cost Schedule

Quantity	Fixed Cost (FC)	Variable Cost (VC)	Total Cost (TC)	Average Variable Cost (AVC)	Average Fixed Cost (AFC)	Average Total Cost (ATC)
0	\$20,000,000	\$0	\$20,000,000			
50	\$20,000,000	\$2,500,000	\$22,500,000	\$50,000	\$400,000	\$450,000
100	\$20,000,000	\$4,500,000	\$24,500,000	\$45,000	\$200,000	\$245,000
150	\$20,000,000	\$6,000,000	\$26,000,000	\$40,000	\$133,333	\$173,333
200	\$20,000,000	\$7,000,000	\$27,000,000	\$35,000	\$100,000	\$135,000
250	\$20,000,000	\$10,000,000	\$30,000,000	\$40,000	\$80,000	\$120,000
300	\$20,000,000	\$13,500,000	\$33,500,000	\$45,000	\$66,667	\$111,667
350	\$20,000,000	\$17,500,000	\$37,500,000	\$50,000	\$57,143	\$107,143
400	\$20,000,000	\$22,000,000	\$42,000,000	\$55,000	\$50,000	\$105,000
450	\$20,000,000	\$27,000,000	\$47,000,000	\$60,000	\$44,444	\$104,444
500	\$20,000,000	\$32,500,000	\$52,500,000	\$65,000	\$40,000	\$105,000
550	\$20,000,000	\$38,500,000	\$58,500,000	\$70,000	\$36,364	\$106,364
600	\$20,000,000	\$45,000,000	\$65,000,000	\$75,000	\$33,333	\$108,333
650	\$20,000,000	\$52,000,000	\$72,000,000	\$80,000	\$30,769	\$110,769
700	\$20,000,000	\$59,500,000	\$79,500,000	\$85,000	\$28,571	\$113,571

elements of cost involved in any process of estimation, an illustrative example is constructed.

Example 12.1

A manufacturing company, JetParts, is seeking to develop a certain new aircraft. Assume the fixed cost for the project to be \$20 million. Some of the other elements have been considered to create Table 12.1 and the Figures following it, to demonstrate the general variations observed in the cost elements, when plotted against the quantity of a product produced.

Example 12.2

Assume the total cost of flying a passenger aircraft is a linear function, consisting of a fixed cost of operating a flight, and a variable cost based on the number of passengers onboard. Jet Aviation has fixed costs of \$10,000 per block hour and equipment and variable costs per passenger are \$30/hour. What is the total cost at varying levels of output?

$$TC = \$10,000 + \$30 \times Q$$

If the airline carries 100 passengers, the total cost per block-hour is:

$$TC = \$10,000 + \$30 \times 100$$

$$TC = \$13,000 \text{ per hour}$$

Figure 12.1 Short-run Cost Curves

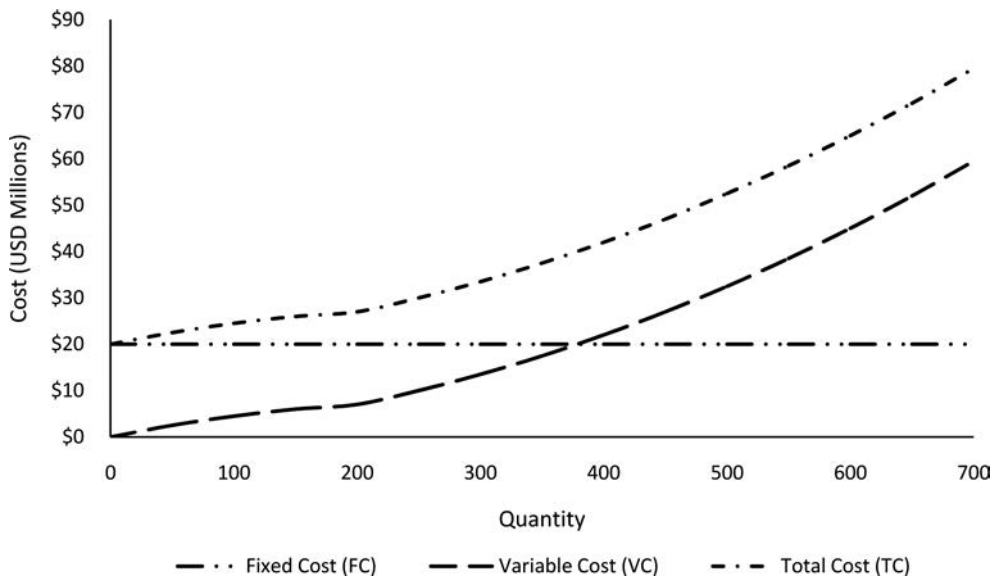
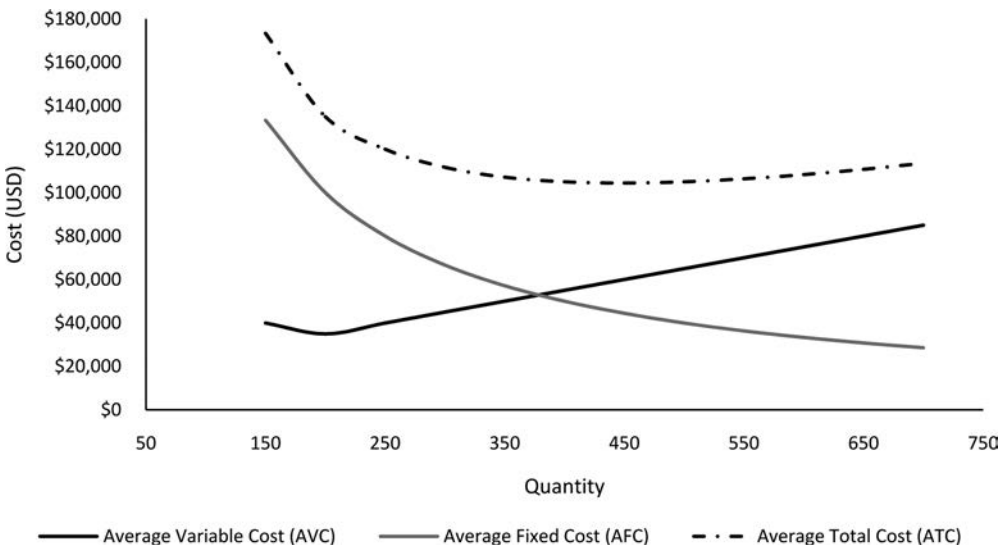


Figure 12.2 Average Fixed, Variable, Marginal, and Total Cost Curves



Cubic Cost Function

Quadratic or cubic cost specifications are frequently used to estimate the manufacturing cost function. A cubic specification for a total cost function is appropriate when the scatter diagram shows that the total cost initially increases at a decreasing rate, followed by increasing at an increasing rate throughout the range of output. Or, in economic terms, the cubic cost functions are used when the production represents increasing or decreasing return to scale. The general formula for a cubic cost function is presented by the following:

$$TC(Q) = FC + \beta Q + \gamma Q^2 + \delta Q^3$$

β , γ , and δ are the parameters of the function.

For example,

$$TC(Q) = 200 + 80 \times Q - 6 \times Q^2 + 0.25 \times Q^3$$

$$\text{At } Q = 0,$$

$$FC = TC = \$200$$

$$TVC = 80 \times Q - 6 \times Q^2 + 0.25 \times Q^3$$

Marginal Costs (MC)

Marginal cost is primarily defined as the change in the total cost of producing one additional unit of output. When plotted on a graph, it can be noted that the marginal cost curve may take a “U-shape.” The marginal cost decreases to a certain point, and then begins to rise. The marginal cost continues to decrease as long as the economies of scale are achieved. As soon as production enters diseconomies of scale, the marginal costs begin to rise again. In the airline industry, the marginal cost of providing an additional seat may be relatively insignificant. Marginal cost may include:

- ♦ Food and drink
- ♦ Airport luggage handling
- ♦ Additional fuel consumption

Marginal costs are used in production decisions. If the price charged for a product is greater than the marginal cost, then the revenue will be greater than the added cost and hence it is viable to continue production. However, if the price charged is less than the marginal cost, then it results in a loss of money and production should not expand.

Mathematically, marginal cost (MC) is the change in total cost resulting from a change in the quantity of output produced by a company in the short run. Marginal cost indicates how much total cost changes for a given change in the quantity of output. It is found by dividing the change in total cost by the change in quantity of output.

$$\text{Marginal Cost} = \frac{\Delta \text{Total Cost}}{\Delta \text{Quantity of Output}}$$

$$MC = \frac{\Delta TC}{\Delta Q}$$

To calculate marginal cost, we need to take the change in total cost divided by the change in total output, or take the first derivative of the total cost function.

Example 12.3

Assume the total cost function is given by the following equation. Determine the MC function, and identify the level of output that minimizes the marginal cost:

$$TC(Q) = 200 + 80 \times Q - 6 \times Q^2 + 0.25 \times Q^3$$

Marginal cost is the derivative of the total cost function, so take the derivative to find the MC:

$$MC(Q) = 80 - 12 \times Q + 0.75 \times Q^2$$

Given the marginal cost function, to determine the level of production that minimizes the marginal cost we just need to set the derivative of the marginal cost $\{MC(Q)\}$ to zero.

$$\frac{d MC(Q)}{d Q} = -12 + 1.5 \times Q$$

$$\frac{d MC(Q)}{d Q} = 0$$

$$-12 + 1.5 \times Q = 0$$

$$Q = 8$$

Hence a quantity of 8 units will minimize the marginal cost.

Average Total Cost (ATC)

ATC is primarily known as per unit total cost, or total cost divided by the quantity of output produced. Average total cost is also the sum of average fixed cost and average variable cost (Figure 12.2).

While average cost is a general term relating cost and the quantity of output, three specific average cost terms are average total cost (ATC), average variable cost (AVC), and average fixed cost (AFC). Average total cost is referring to the per unit cost that includes all fixed costs and all variable costs. Average total cost is a key factor in the short-run production decisions made by a firm when evaluated against the price, which is per unit revenue. A comparison between per unit revenue (price) and per unit cost (average cost) is a direct indication of whether a firm is making a profit, or incurring a loss. In many cases, a manager wants to know the cost per unit associated with producing a good, since the ATC information could be used to determine the price of the product.

To find the average cost, we just divide the total cost by the total number of units produced:

$$AC = \frac{TC}{Q}$$

$$\text{Average Total Cost} = \frac{\text{Total Cost}}{\text{Quantity of Output}}$$

$$ATC = \frac{TC}{Q}$$

From Example 12.3, the average total cost is found as,

$$ATC = \frac{200 + 80 \times Q - 6 \times Q^2 + 0.25 \times Q^3}{Q}$$

Example 12.4

Jet Services International (JSI) is one of the leading manufacturers of turbofan engines. JSI would like to analyze the variations in average total cost and marginal cost for a new engine manufacturing project. The estimated initial cost of the project is \$1,000,000,000. Table 12.2 illustrates the projected total costs, average total costs and marginal costs. (All costs are in USD millions.)

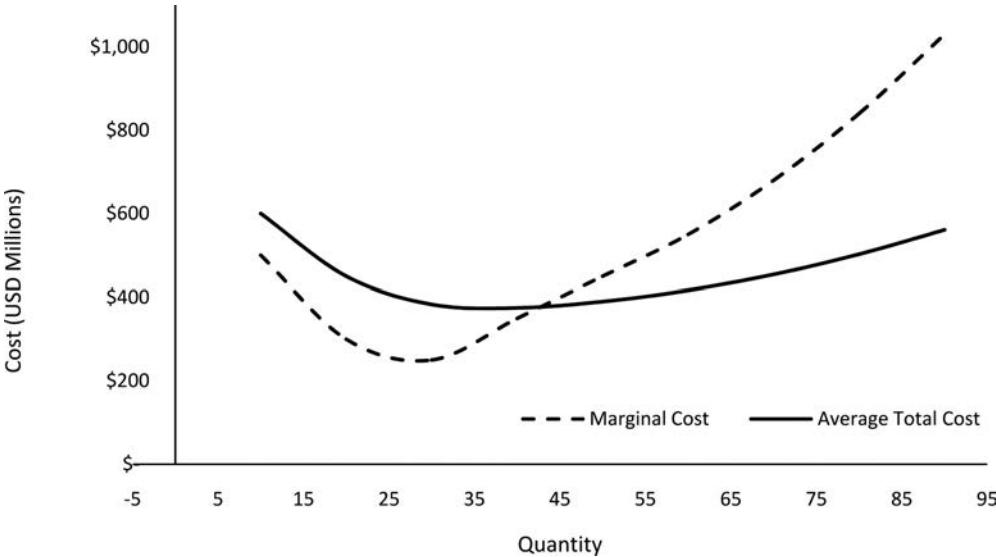
It can be inferred from Figure 12.3 that:

- ♦ When the marginal cost is less than the average total cost, average total cost declines.
- ♦ When the marginal cost is higher than the average total cost, average total cost increases.
- ♦ When the marginal cost is equal to the average total cost, there is a minimum or maximum at the average total cost.

Table 12.2 Jet Services International Project Costs

Quantity (Q)	Total Cost	Average Total Cost	Marginal Cost
0	\$1,000		
10	\$6,000	\$600	\$500
20	\$9,000	\$450	\$300
30	\$11,500	\$383	\$250
40	\$15,000	\$375	\$350
50	\$19,500	\$390	\$450
60	\$25,000	\$417	\$550
70	\$31,800	\$454	\$680
80	\$40,200	\$503	\$840
90	\$50,500	\$561	\$1,030
100	\$63,000	\$630	\$1,250
110	\$78,000	\$709	\$1,500
120	\$95,800	\$798	\$1,780

Figure 12.3 Marginal Cost and Average Total Cost



This concept could also be applied to a university setting. Imagine that you are currently on 3.5 in a class. If your next grade on an assignment is higher than your average (e.g., a 4.0), your average will increase. If your next grade is lower than your current average, then your new average will be lower than 3.5.

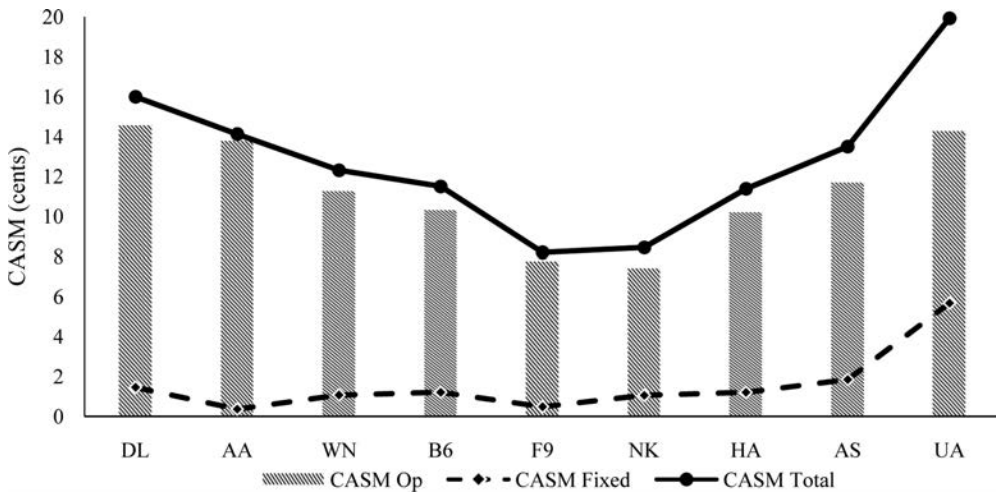
The calculation for marginal cost using the total cost value in Excel is carried out as shown in Figure 12.4.

In the airline industry, the cost per unit, or per flight, or even per seat mile, can be decreased with an increase of aircraft utilization. For example, DirectJet airline leases jet bridges for \$120,000 per year and operates the aircraft 3,000 times per year; the airline cost per aircraft operation for a jet bridge is \$40. If DirectJet leases the same jet

Figure 12.4 Calculating Marginal Cost Using Excel Spreadsheets

	A	B	C	D
1	Quantity	Total Cost	Average Total Cost	Marginal Cost
2	0	\$1,000		
3	10	\$6,000	\$600	$= (B3 - B2) / (A3 - A2)$
4	20	\$9,000	\$450	\$300
5	30	\$11,500	\$383	\$250
6	40	\$15,000	\$375	\$350
7	50	\$19,500	\$390	\$450

Figure 12.5 CASM Comparison for Q3, 2015



bridge, but operates 4,000 flights annually, DirectJet’s cost per aircraft operation decreases to \$30 per operation.

Short-run production analysis makes use of three average cost measures:

- ♦ Average Total Cost
- ♦ Average Fixed Cost
- ♦ Average Variable Cost

$$ATC = AFC + AVC$$

For airlines, the comparison of average fixed and average variable costs can be done using the cost per available seat mile (CASM), found for operating expenses (which can be considered as the average variable cost) and for the total expenses, from which the cost per available seat mile for average fixed costs can be found as the difference between the two. A depiction of the different CASMs for the major US carriers for the third quarter of 2015 is given in Figure 12.5.

Average Fixed Cost (AFC)

The costs for a plant or equipment needed for the production of goods and services (such as assembling cars or aircraft) are considered fixed costs. It does not matter if only one unit is made or 100, the cost of the facility and equipment will remain fixed. The average fixed cost, or fixed cost per unit is calculated as total fixed cost divided by the quantity of output produced. Since the fixed cost does not vary with output, AFC declines as the level of activity increases.

$$\text{AFC} = \text{TFC} \div \text{Q}$$

Example 12.5

From Example 12.3, determine the average fixed cost.

Given,

$$TC(Q) = 200 + 80 \times Q - 6 \times Q^2 + 0.25 \times Q^3$$

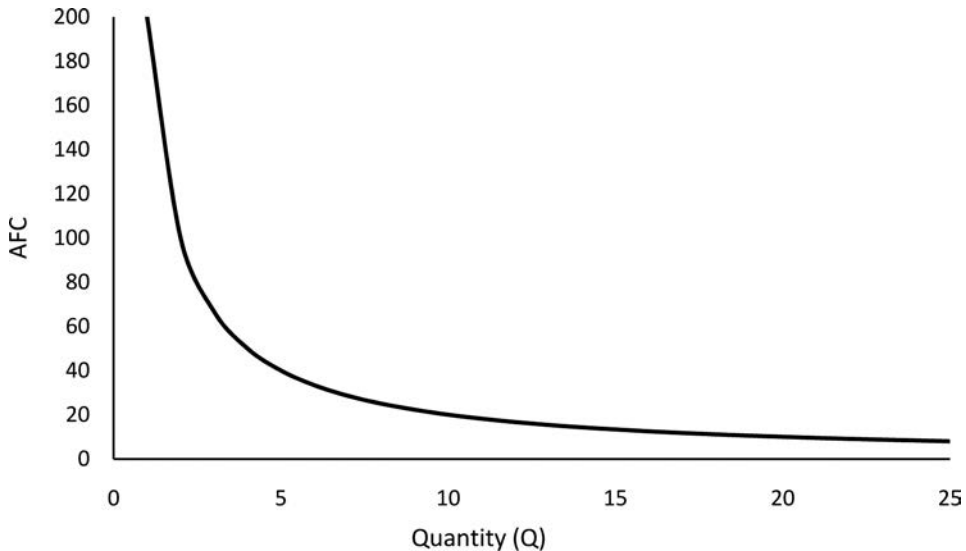
The fixed cost from the above equation is 200.

Hence AFC is,

$$AFC = \frac{200}{Q}$$

It can be noted from Figure 12.6 that, as stated previously, the AFC keeps decreasing since the fixed cost remains the same, while only the quantity keeps increasing.

Figure 12.6 Average Fixed Cost Curve

**Average Variable Cost (AVC)**

A good example of a variable cost is an airline fuel cost. If one mile of flight requires 10 gallons of fuel, then every mile requires 10 gallons, no matter how many miles the aircraft flies. Variable costs may vary in a linear fashion with the production level. In other words, the more the aircraft flies, the higher the total variable cost (fuel burn). Total variable costs vary in direct proportion to volume. Total variable cost is zero if the aircraft does not fly and just stays at the gate.

AVC is defined as the total variable cost divided by the quantity of output produced, and is generally referred to as per unit total variable cost.

$$AVC = \frac{TVC}{Q}$$

Example 12.6

Consider Example 12.3. Determine the level of production that minimizes the average variable cost. Given,

$$TC(Q) = 200 + 80 \times Q - 6 \times Q^2 + 0.25 \times Q^3$$

First, we need to find the average variable cost (AVC). The AVC is found by dividing the total variable cost by units produced (Q). The total variable cost is the same as the total cost minus the fixed cost of 200.

$$AVC(Q) = 80 - 6 \times Q + 0.25 \times Q^2$$

To determine the level that minimizes the average cost, the first derivative of AVC (Q) must be found and equated to zero.

$$\frac{d AVC(Q)}{d Q} = -6 + 0.5 \times Q$$

$$\frac{d AVC(Q)}{d Q} = 0$$

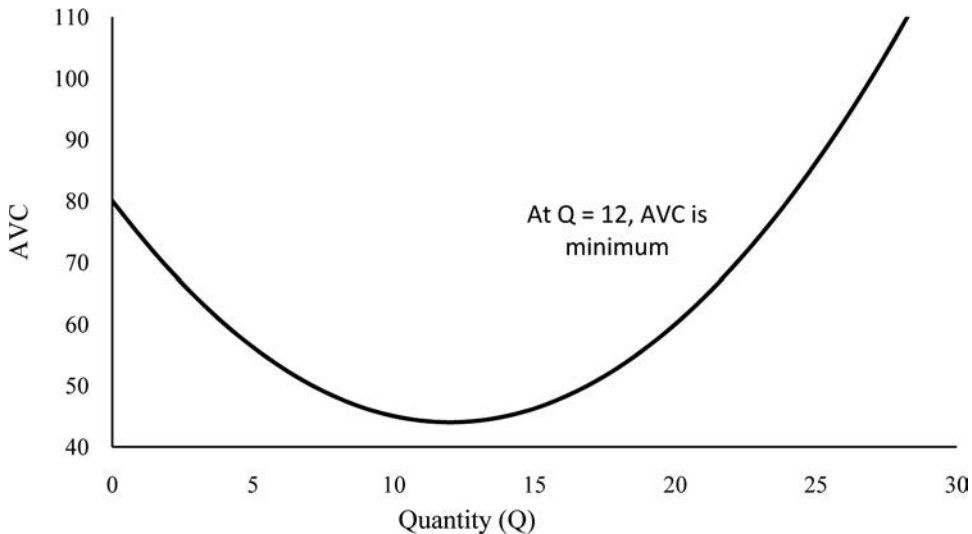
$$-6 + 0.5 \times Q = 0$$

$$Q = 12$$

Hence a quantity of 12 units will minimize the average variable cost. This can also be seen in Figure 12.7, which depicts the relationship between the quantity and the average variable cost for the equation stated above.

The three short-term alternatives faced by firms as a result of the analysis of price and average cost are as follows:

- ♦ Produce at a profit: If the price is equal to or greater than the average total cost, the firm generates a profit and can decide to continue operations.
- ♦ Produce at a loss: Some firms decide to continue production, even though the firm is at a loss in the short run. This alternative is chosen when the price is greater than the average variable cost but less than the average total cost. The loss from production is less than the fixed cost loss that would be incurred by shutting down production.
- ♦ Shut down operations: The last alternative is to shut down operations in the short run and incur the loss of fixed cost. A firm chooses this alternative if price is less than average variable cost. The loss from production is greater than the fixed cost loss incurred by shutting down production.

Figure 12.7 Minimum AVC with Change in Quantity

Sunk Costs

A sunk cost is defined as a cost that an entity has incurred and that cannot be recovered. Sunk costs are also called “stranded costs.” Sunk costs are independent of any event that may occur in the future. While making business decisions, typically a firm considers only future costs, ignoring the money already spent on the project, which now becomes a sunk cost. It is a common business tenet that sunk costs should never be considered as a relevant factor in decision making. When aircraft manufacturers spend millions of dollars on research and development that is not yet complete, any decision should not consider the millions already spent since they cannot get the money back.

Using sunk costs as one of the factors in a current decision is considered making a decision simply to justify past choices. There are several examples of sunk cost:

- ◆ Research and development
- ◆ Training
- ◆ Marketing study
- ◆ Pilot manuals in the cockpit

A **sunk cost** is a cost that has been incurred and cannot be recovered. When making a business decision on investing in a project or not, sunk costs should not be considered. For example, when an airline replaces £40 airline manuals with iPads, the purchase cost of the paper manuals is a sunk cost, since it occurred in the past and cannot be changed by any future course of action.

When facing a possible investment, a manager must only consider relevant costs and ignore all irrelevant costs. A sunk cost is a type of irrelevant cost.

In response to economic problems and reduction in traffic, many airlines reduce aircraft operations and park their aircraft. At Mojave Airport, more than 1,000 aircraft

have ended up in the California desert after their flying days.⁶ During an economic boom, airlines may bring them back.

Assume DirectJet is planning to purchase a 737-400 at a price of \$5 million. The management knows the resale market for this aircraft is not good. If DirectJet is able to resell the aircraft for \$2 million, then:

$$\text{Sunk Cost} = \$5,000,000 - \$2,000,000 = \$3,000,000$$

Sometimes, fixed costs are sunk, but not all fixed costs are sunk. In the B737 example above, if there was no alternative use or resale opportunity for the aircraft, the cost of the aircraft and other expenses rated to it would be irrelevant to the question of whether or not it is profitable to start another route.

The idea that a company or organization is more likely to continue with a project if they have already invested a lot of money, time, or effort in it, even when continuing is not the best thing to do, is popularly referred to as “Sunk cost fallacy” or “Concorde fallacy.”⁷ The greater the size of the sunk investment, the more people tend to invest further, even when the return on added investment appears not to be worthwhile. According to researchers Daniel Kahneman and Amos Tversky, a reason people fall prey to the sunk cost fallacy is loss aversion: people tend to have a much stronger preference for avoiding losses than for acquiring gains (Tversky and Kahneman, 1986).

A very apt example to describe the sunk cost fallacy is the construction of the Supersonic plane Concorde. Concorde development began in 1962 based on a treaty between Britain and France. The plane began commercial service in 1976 and flew for 27 years. The cost to develop the Concorde, which was funded by the UK and French governments, was £1.134 billion. It cost the company about £654 million (of which only £278 million were recovered through sales) to produce 16 Concorde planes during 1965–1979. This debt was also funded by the two governments.⁸ The governments kept “throwing more good money after bad” because they seemed prepared to pay the prestige premium no matter how high it rose. This was the reason the sunk cost fallacy also came to be known as the “Concorde fallacy.”

The Concorde used about three to four times as much fuel per passenger as any other aircraft. For example, Concorde managed about 17 passenger miles per gallon, which is worse than an A380 at about 78 passenger miles per gallon.

Opportunity Costs

The opportunity cost of an action or a decision is the value of the best alternative foregone. Everything has opportunity costs. One of the key ways that low-cost airlines keep costs down is by operating young fleets with the most fuel-efficient engines. That helps reduce fuel costs. The trade off is that they have to pay higher monthly

Aviation Snippets

Alaska is transitioning all 1,400-plus of its pilots to company-issued iPads. The plan aims to improve fuel economy by cutting back on the weight on board. Also, documents can be updated more frequently. Alaska is the first commercial airline to make such a move. Delta and American, among others, are considering similar steps.

USA Today, June 2011

aircraft costs, for the same reason that buying a new car is more expensive than buying a used car. An opportunity cost of an action or a decision is the potential gain given up when the choice of one action prevents the selection of a different action. When an airline is planning to bring an additional aircraft to the fleet, the airline may purchase the aircraft or lease it. In this case, the opportunity cost of leasing an aircraft may be the foregone benefits of depreciation tax saving.

Example 12.7

Consider the choice DirectJet faces when it decides to open a new route from two potential markets. Suppose that DirectJet has only one aircraft and one full crew. DirectJet is trying to decide whether to start serving airport (A) or airport (B). The accounting cost for one year is presented in Table 12.3:

Table 12.3 DirectJet Accounting Costs

Expenses	Airport (A)	Airport (B)
Landing fees	\$648,000	\$972,000
Gate fees	\$219,000	\$146,000
Airport personnel	\$150,000	\$150,000
Miscellaneous	\$320,000	\$300,000
Annual expenses	\$1,337,000	\$1,568,000

This tells us how much money DirectJet will need to spend if it decides to serve each of the airports. But in order to decide which airport to serve, DirectJet should figure out its opportunity cost. The first step is to determine the best alternative for each option (Table 12.4).

The opportunity cost of serving Airport A, then, is the value of what DirectJet would gain if it provided services instead to Airport B, or in this case $\$382,000 - \$323,000 = \$59,000$.

Table 12.4 DirectJet Net Gain

	Airport (A)	Airport (B)
Revenue	\$1,660,000	\$1,950,000
Expenses	\$1,337,000	\$1,568,000
Net gain	\$323,000	\$382,000

Relationship between Economic and Accounting Cost

Economists separate the world of cost into those costs that are explicit (that vary with the level of output) and those that are implicit. Explicit costs represent actual out of pocket cash outflows from a business that reduce its bottom-line profitability. Good examples of explicit costs would be items such as fuel expenses, gate fees and charges, wages and salaries, and the air navigation charges that go into the production of air transport services. In economic terms, an implicit cost, also called non-pecuniary cost or implied cost, is the opportunity cost of using your own resources (Chiang, 1984). This is the cost of lost income the airline has by not taking advantage of the opportunity of flying the aircraft and generating passenger revenue.



For example, according to the financial ratings agency Standard & Poor's the cost of the October 2013 Federal government shutdown was estimated at around \$24 billion.⁹ Hundreds of thousands of workers were furloughed during the shutdown, and it reduced fourth quarter Gross Domestic Product growth by 0.4 percentage points.¹⁰

Example 12.8

Assume after graduation with a master degree in Aerospace Engineering, you have been offered a job working for a company that pays you \$150,000 annually. In addition, you have the opportunity to start your own business by investing \$400,000, that provides \$180,000 year in revenue. You have always wanted to work in your own business. Should you invest in your own business, if you can invest your money earning 10 percent interest? Explain.

Solution:

The opportunity cost of your investment = $0.10 \times (400,000) = \$40,000/\text{year}$

The opportunity cost of your time = \$150,000

Total cost including the opportunity cost = \$190,000/year

You should invest in your business if the enjoyment from working for yourself is worth \$40,000 a year or more.

Cost of Debt

Debt typically has the following characteristics:

- ♦ Commitment to make fixed payments in the future
- ♦ The fixed payments are tax deductible
- ♦ Failure to make the payments can lead to either default or loss of control of the firm to the party to whom payments are due

The cost of debt is generally described as follows:

- ♦ The rate at which the company can borrow long-term today
- ♦ Composed of the risk-free rate and a default spread
- ♦ Corrected for the tax benefit it gets for interest payments

k_d = After-tax Cost of Debt

k_d = Long-term Borrowing Rate $\times$ (1 – Tax Rate)

Cost of Equity

Cost of equity is the return a firm theoretically pays to its equity investors, i.e., shareholders, to compensate for the risk they undertake by investing their capital. It is normally found using the Capital Asset Pricing Model (CAPM) or using the Dividend Discount Model (DDM). It is denoted by the symbol r_e . The dividend discount model is a technique for valuing the price of a stock by using the amount of dividends and discounting them back to the present value.

Weighted Average Cost of Capital (WACC)

The Weighted Average Cost of Capital (WACC) is a measure which combines the cost of equity and the after-tax cost of debt, based on the existing capital structure in the firm, i.e., the ratio of debt to value and equity to value. It is an average measure of the payout rate a company expects.

$$WACC = \left(\frac{D}{V} \right) \times k_d + \left(\frac{E}{V} \right) \times r_e$$

Where:

k_d = After-tax cost of debt

r_e = Cost of equity

V = Total value (debt + equity)

ROIC vs WACC

Return on Invested Capital (ROIC) is defined as the percentage return a company makes on its invested capital. Weighted Average Cost of Capital (WACC) is the rate at which a company is expected to pay on average all the security holders in order to finance its assets. As a company continues to increase the invested capital in more projects, it is simultaneously increasing its debt ratio. An increasing debt ratio raises the return on invested capital but, at the same time, decreases the weighted average

cost of capital. The optimal debt ratio for any company is at the point where the return on invested capital is the same as the weighted average cost of capital. This means that the company has utilized its profits as much as possible. Any investment beyond this point would result in the WACC being higher than the ROIC, making it a poor investing decision. This can be further explained with the help of an example.

Example 12.9

Jet-Services is an aircraft manufacturing firm headquartered in Brisbane, Australia. Jet-Services is trying to decide the level of investment that needs to be made in the production of different versions of a certain jet aircraft. Consider Table 12.5 which shows the estimates for ROIC and WACC for various levels of investment, and determine the optimal level of debt ratio that they have to invest in.

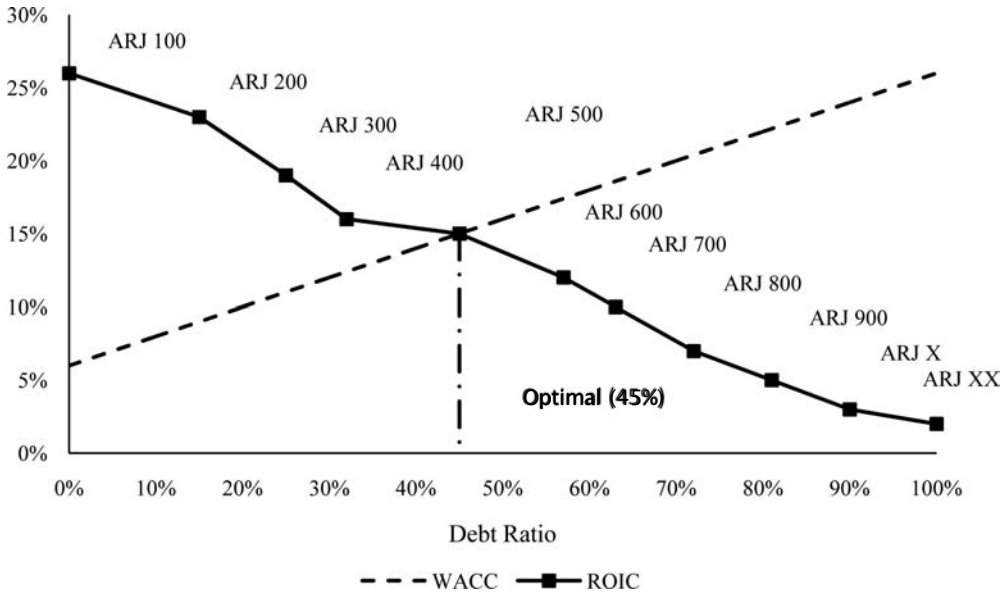
It can be clearly seen from Figure 12.8 that, at a debt ratio of 45 percent, the ROIC equals WACC and they converge at this point. Any investment beyond this point is a poor decision. Hence the company must decide to invest as long as ROIC is greater than or equal to WACC, i.e., until ARJ 500 (therefore including the investment into ARJ 100, 200, 300, and 400), which lowers the debt ratio to 45 percent.

Table 12.5 Jet-Services ROIC and WACC Estimates

Project	Debt Ratio for WACC	WACC	Debt Ratio for ROIC	ROIC
ARJ 100	0%	6%	0%	26%
ARJ 200	10%	8%	15%	23%
ARJ300	20%	10%	25%	19%
ARJ 400	30%	12%	32%	16%
ARJ 500	40%	14%	45%	15%
ARJ 600	50%	16%	57%	12%
ARJ 700	60%	18%	63%	10%
ARJ 800	70%	20%	72%	7%
ARJ 900	80%	22%	81%	5%
ARJ X	90%	24%	90%	3%
ARJ XX	100%	26%	100%	2%

Cost Divisions and Categories

All the costs associated with a business can be broadly classified as direct and indirect costs. The allocation and identification of direct and indirect costs contribute to more accurate profit calculations. Not only is the clarification of costs important to business owners, but also it may be of particular interest to some prime contractors, who may

Figure 12.8 ROIC, WACC, and Debt Ratio

actually impose a specific accounting methodology in order to meet their own policy requirements.

Direct Costs

An expense that can be traced directly to a specific cost center or activity is called a direct cost. In the airline industry, direct costs include labor, fuel, direct maintenance, and air navigation charges.

Direct costs are defined as costs which can be accurately traced to a particular cost object with little effort. Cost objects generally refer to a product, a department, a project, etc. Labor and direct materials, which are used in creating a specific product, constitute the majority of direct costs. While salaries tend to be a fixed cost, direct costs are frequently variable. Variable expenses increase as additional units of a product or service are created, whereas an employee's salary remains constant throughout the year. Determining whether a cost is direct or indirect must be done only by taking into consideration the cost object. A direct cost for one object might be an indirect cost for another. In the airline industry, direct costs may include the following:

- ◆ Airframe and power-plant maintenance and repair
- ◆ Flight and cabin crew expenses
- ◆ Fuel expense
- ◆ Station and ground handling service fees
- ◆ Landing fees
- ◆ In-flight meals and catering
- ◆ Airport load fees
- ◆ Navigation fees
- ◆ Commission on cargo

Indirect Costs

An indirect cost cannot directly be traced to an activity or a production. For example, at a local airport, airport security expenses such as the wages and fringe benefits of police officers are not easily traceable. Since indirect costs cannot be easily traced to a specific department, project or process, they are usually estimated or allocated using a standard rate for a particular industry. There are a number of techniques commonly used by professional engineers to estimate the cost of a project. These include the unit cost method, cost indexing method, power law method using a cost capacity equation, factor method, and a few more techniques for the allocation of indirect cost.

Indirect costs are defined as costs which cannot be accurately attributed to specific cost objects. Indirect costs incurred benefit multiple cost objects and cannot practically be assigned to a single object. They go beyond the costs associated with creating a particular product to include the price of maintaining an entire firm or project. Also referred to as overhead costs, indirect costs are the ones left over after direct costs have been computed. They provide an indication of the “real” costs of doing business. The materials and supplies for administrative purposes are examples of indirect costs. Similar to direct costs, indirect costs can be both fixed and variable. Fixed indirect costs include things like the rent paid for the building in which a company operates. Variable indirect costs include the ever-changing costs of electricity and gas. Some examples of indirect costs in the airline industry are:

- ♦ Advertising and publicity
- ♦ General and administrative expenses
- ♦ Passenger service
- ♦ Parking charges
- ♦ Crew and cabin salary

Example 12.10

An aircraft manufacturer produces different types of aircraft. There are many different types of costs incurred throughout the year that can be classified in terms of direct versus indirect and variable versus fixed:

	Direct	Indirect
Variable	♦ Materials and supplies for a specific aircraft	♦ Electricity for the corporate office
Fixed	♦ Rent for a building where only one specific aircraft is manufactured	♦ Salary of the CEO

Learning Curve

Change occurs rapidly and it is almost inevitable in the dynamic work environment existent today. The presence of change induces the power of learning. Improved instructions and repetition allow workers to learn to perform jobs more efficiently

and thereby reduce the number of direct labor hours per unit. This process is true not only for the workers, but also for the organization as a whole.

The learning effect can be represented by a line called a **learning curve**, which presents the relationship between the total direct labor per unit and the cumulative quantity of a product or service produced. The learning curve pertains to a repetitive job or task and shows the relationship between experience and productivity, which is that the time required to produce a unit decreases as the operator or firm produces more units (Krajewski et al., 2009).

The idea of learning curves was introduced into economics by Alchian in 1963. Analyzing the data of 22 different aircraft types produced by the American industry during World War II, Alchian (1963) found that the direct amount of labor required to produce a unit of a special aircraft type regularly declines when the total output of this type is expanded. This relationship can be graphically expressed by the so-called learning curve (Hartley, 1965).

Learning curves enable managers to project the manufacturing cost per unit for any cumulative production quantity. Firms that focus on low price as a competitive strategy rely on high volumes to maintain profit margins. The target for such firms is to move down the learning curve (lower labor hours per unit or lower costs per unit) by increasing volume. This tactic makes entry into a market by competitors difficult due to the large losses they would have to incur in order to lower prices to compete with the existing firm, while still experiencing large production costs in the initial phases.

The following assumptions are made when a learning curve is developed:

- ◆ The direct labor required to produce the $(n+1)$ th unit is always less than the direct labor required to produce the n th unit.
- ◆ Direct labor requirements will decrease at a declining rate as cumulative production increases.
- ◆ The reduction in time will follow an exponential curve.

In other words, the production time per unit is reduced by a fixed percentage each time production is doubled. We can use a logarithmic model to draw a learning curve. The direct labor required for the n th unit is k_n :

$$k_n = k_1 n^b$$

Aviation Snippets

Lockheed Martin has re-affirmed the full-rate production goal for the F-35 program remains 17 aircraft per month, despite a supplier's statement on 7 October that it is significantly less.

Full rate production volume is factored into the production learning curves that drive Lockheed's cost reduction targets for the F-35. In the early years of the program, Lockheed expected to build 20 F-35s per month by 2016, or nearly one per working day.

But program delays and spending deferrals trimmed Lockheed's full-rate production target to 17 aircraft monthly by the mid-2020s, compared to a current rate of about three per month.

Flightglobal, October 2015

Where:

k_1 = Direct labor hours required to produce first unit

n = Cumulative number of units produced

$$b = \frac{\log r}{\log 2}$$

r = Learning rate (as decimal)

Example 12.11

A component manufacturing company manufactures an electric starter which is used in jet engines manufactured by GE. The first time the component is produced, it takes 60 labor hours to complete manufacturing. Based on prior experience, assume the learning rate is 80 percent. The learning curve for the company can be developed as follows:

$$k_1 = 60 \text{ hours}$$

$$r = 80\% = 0.8$$

$$b = \frac{\log r}{\log 2} = \frac{\log 0.8}{\log 2}$$

$$b = -0.3219$$

$$k_n = 60 \times n^{-0.3219}$$

$$k_4 = 60 \times 4^{-0.3219} = 38.4 \text{ hours}$$

$$k_8 = 60 \times 8^{-0.3219} = 30.72 \text{ hours}$$

Figure 12.9 Learning Curve

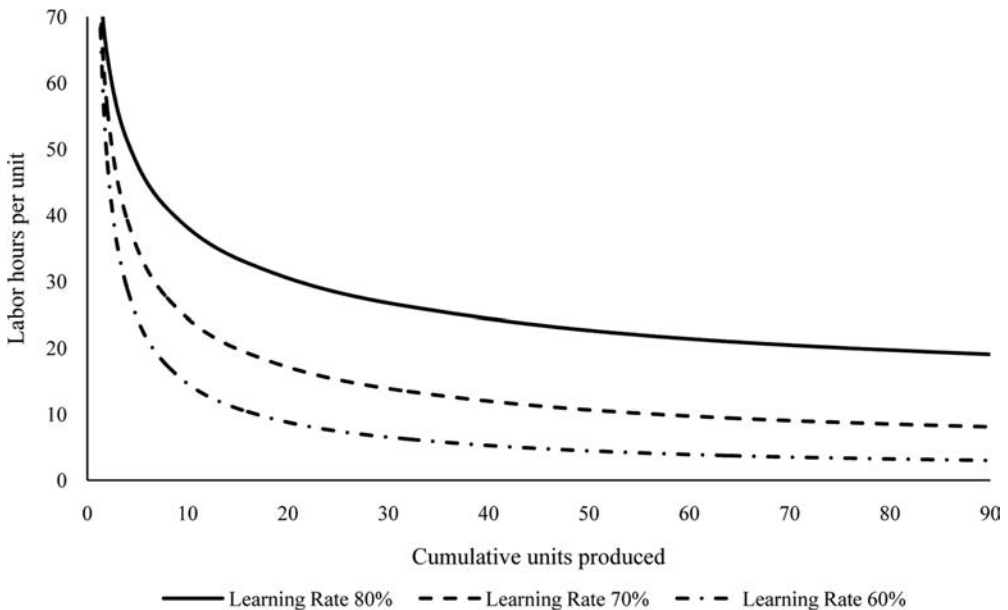
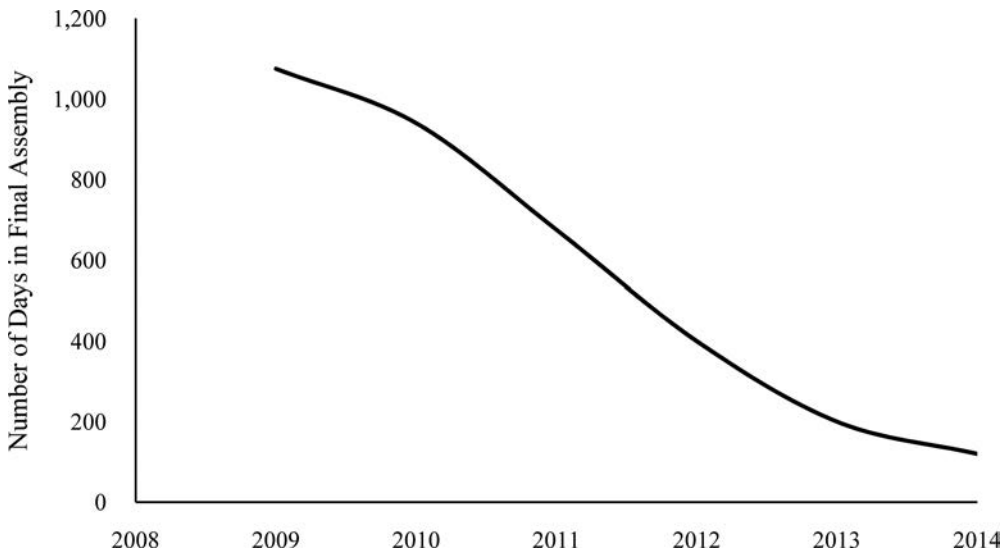


Figure 12.10 Sample Learning Curve for Boeing 787

Source: All things 787

Figure 12.9 shows a typical learning curve and is plotted by calculating the corresponding labor hours for various units based on the formula defined earlier. In this example, up to the manufacturing of 60 units could be denoted as the learning period, while beyond that the time period almost becomes standardized.

A learning curve can also be based on factors such as the number of days in the final assembly for an aircraft in manufacturing. For example, the Boeing 787 enjoys a learning curve as represented in Figure 12.10.¹¹

At the start of production, each aircraft required over 1,000 days in final assembly. Five years of continued production resulted in that time in the final assembly for a single aircraft dropping to as low as around 50 days.

Estimating Costs

Engineering economic analysis involves the consideration of present and future economic factors. Hence, it is critical to obtain reliable estimates of future costs, benefits, and other economic parameters. A characteristic of cost estimates is that errors in estimating are typically asymmetric as costs are more likely to be underestimated than overestimated.

The development of cost estimates involves certain difficulties or uncertainties arising from factors such as resource availability and estimator expertise. The quality of a cost estimate increases as the resources allocated to developing the estimate increase. However, the benefits expected from improving a cost estimate should outweigh the cost of devoting additional resources to the estimate improvement.

A good example of cost estimation is the use of the Cobb–Douglas production function to determine the total production cost for a particular project. The Cobb–Douglas production function relates the total cost to the labor cost and the cost of capital to the total cost of production with the use of a productivity factor (Houthakker, 1955).

$$TC = A L^{\alpha} K^{\beta}$$

Where:

TC = Total cost of production

A = Productivity factor

L = Labor input (number of hours)

K = Capital input

α, β = Output elasticities of labor and capital

For example, if $\alpha = 0.80$, a 1 percent increase in labor usage would lead to approximately a 0.80 percent increase in total cost. The data for the considered example is given in Table 12.6.

Table 12.6 Cobb–Douglas Data Assumptions

No of Units	L (Hours)	K	Total Cost
1	10	\$100	\$1,584.89
2	11	\$200	\$1,964.81
3	12	\$300	\$2,284.38
4	13	\$400	\$2,579.69
5	14	\$500	\$2,862.18
6	15	\$600	\$3,136.92
7	16	\$700	\$3,406.56
8	17	\$800	\$3,672.63
9	18	\$900	\$3,936.10
10	19	\$1,000	\$4,197.62

The total cost is found using the formula:

$$TC = 100 L^{0.8} K^{0.2}$$

Converting the above equation into logarithmic form makes it a linear equation, thereby allowing a regression analysis to be carried out on it. The linear equation is as follows:

$$\ln(TC) = \ln(100) + 0.8 \ln(L) + 0.2 \ln(K)$$

The results obtained from the regression analysis are as given in Table 12.7.

Table 12.7 Regression Analysis Results

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	4.6052	2.81E-15	1.60E+15	1.0E-104	4.605	4.605
X Variable 1	0.800	2.90E-15	2.76E+14	2.19E-99	0.800	0.800
X Variable 2	0.200	8.51E-16	2.35E+14	6.65E-99	0.200	0.200

The results are consistent with the linear equation obtained from the Cobb–Douglas production function. The intercept value (4.6052) obtained in the regression is the value of $\ln(100)$.

Cost Estimation Techniques

In the air transport industry, since the operation of an aircraft incurs a wide range of costs, cost minimization is very essential in order to improve the profitability of the aircraft and enhance the financial position of the company. But in order to minimize costs, we have to have a clear understanding of the cost components. The cost components for some projects such as building an aircraft hangar are much simpler and fewer than the components of manufacturing a commercial aircraft. For example, a Boeing 787 Dreamliner has huge cost components. While the aircraft is assembled in the US, most of the parts are coming from all over the world.

The Boeing Company, in order to reduce its development time and costs, the effective management of which are essential to the success of its new B787, decided to use a worldwide supply chain structure. This structure would allow Boeing to form partnerships with approximately 50 strategic partners worldwide.

These strategic partners would be responsible for the design, build, and assembly of entire sections of the plane and for delivering the complete sections of the aircraft to Boeing at its plant in Everett, Washington. This process would allow Boeing to assemble these complete sections within three to four days at its plant. While this example is used to magnify the size of the cost components, most manufacturers build components of their products in their factory and need to estimate the component costs as accurately as economically possible.

There are a number of techniques commonly used by professional engineers to estimate the cost of a project. These include unit cost method, cost indexing method, power law and sizing method using a cost capacity equation, factor method, and a few more techniques for allocation of indirect cost.

The following sections provide alternative approaches to estimate the direct and indirect costs.

Unit Cost Method

Unit cost method is a very preliminary but popular method to estimate the total cost of a project. This method usually provides a rough estimate of total costs based on the cost per unit and the total units needed to complete the project. The cost per unit is often available based on similar projects completed in the past, from sources within the company or within the industry. This method is commonly used for preliminary design stage estimates. It is also used by construction companies in building houses, commercial property, factories, parking garages, aircraft hangars, and school and university buildings. In order to get a good estimate of the total cost of any project using the unit cost method, the cost per unit has to be updated regularly to reflect the current cost. In unit cost method, the total cost is estimated as follows:

$$C_T = C_U \times N$$

Where:

C_T = Estimated total cost

C_U = Cost per unit

N = Number of units needed

There are two approaches used in estimating unit costs, namely, top-down approach and bottom-up approach.

Top-down approach: The top-down approach is based on a simple calculation involving the division of total expenditure (quantum of funding available) for a given project by total units of activity, to derive a unit cost. Typically, this approach uses aggregate budgetary data to estimate a unit cost. The main advantages of the top-down approach are the availability of the budgetary data, which also means that the cost and time required to estimate unit costs using this approach are minimal, and the simplicity of the calculation of unit cost. There are also a few limitations associated with a top-down approach. First, the approach does not identify the main cost drivers and results in often masking the underlying factors that determine the reason unit costs vary within a single yet heterogeneous group of service users. Second, top-down costing is not a reliable source to forecast the possible rise or fall of costs as a result of changes in the way that people use services or the possible changes in costs due to improvements in outcomes.

Bottom-up approach: The bottom-up approach provides a much more refined outlook than the top-down method. It involves identifying all of the resources that are used to provide a service and assigning a value to each of those resources. These values are summed and linked to a unit of activity to derive a total unit cost. A basis for assessment of which costs can be avoided as a result of reduced demand is obtained as a result of this approach. The advantages of this approach are as follows:

- ♦ It provides quality assurance because detailed cost data allows potential errors to be investigated and their impact to be tested.
- ♦ Variations in cost data are highlighted, and enable practitioners to explore the drivers of variation.

- ◆ It is highly versatile as it enables a practitioner to forecast change in costs as a result of a reduction in service usage or demand.

The main disadvantage associated with the bottom-up approach is that it is labor intensive, since the cost, time, and expertise required to apply it may be prohibitive for providers.

Example 12.12

Sea Breeze Regional Airport (SBR) is a regional airport serving the coastal communities in a 50-mile radius of the City of Sea Breeze, Australia. The airport is extremely convenient to residents of the area, and is very quick and efficient. A preliminary estimate for the cost of building an Executive hangar space at SBR is \$300 per square feet. Executive hangars are typically about 3,200 square feet. What is the estimated total cost of 20 Executive hangars at Sea Breeze Regional?

The estimated total cost per Executive hangar is

$$CT = \$300 \times 3,200 = \$960,000$$

Estimated total cost for 20 Executive hangars is

$$\$960,000 \times 20 = \$19,200,000$$

Unit cost method can be used to estimate the total cost of a multi-component system. To estimate the total cost of the system, we first calculate the total cost of each component using component unit cost and the quantity of components needed, then we add the total cost of all components.

Example 12.13

A multi-component system consists of paint materials, sheet metal, and labor for mechanical, electrical, casting, finishing, and painting jobs. The quantity of each component needed and its unit cost are provided in Table 12.8. What is the estimated total cost of the system?

Table 12.8 Components and Unit Cost Factor

Component	Quantity Needed	Unit Cost Factor
Paint material	500 gallons	\$95
Sheet metal	2,000 tons	\$62
Labor, mechanical/electrical	1,200 hours	\$85 per hour
Labor, casting	1,500 hours	\$60 per hour
Labor, finishing	1,000 hours	\$50 per hour
Labor, painting	200 hours	\$40 per hour

Table 12.9 Calculation of Estimated Total Cost

Component	Quantity Needed	Unit Cost Factor	$C_T = C_U \times N$
Paint material	500 gallons	\$95	\$47,500
Sheet metal	2,000 tons	\$62	\$124,000
Labor, mechanical/electrical	1,000 hours	\$85 per hour	\$85,000
Labor, casting	1,500 hours	\$60 per hour	\$90,000
Labor, finishing	1,000 hours	\$50 per hour	\$50,000
Labor, painting	500 hours	\$40 per hour	\$20,000
Estimated total cost of the system			\$416,500

Table 12.9 shows the computation of estimated total costs of each component and estimated total cost of the system.

Cost Index Method

Cost index can be defined as the ratio of the cost of a project today to the cost of a project in a particular time in the past, making the cost index an indicator of the changes in cost over time, so that calculating cost index accounts for inflation to a certain extent. An important quality of a cost index is that it is dimensionless.

$$C_t = C_0 \left(\frac{I_t}{I_0} \right)$$

Where:

C_t = Estimated cost at present time t

C_0 = Cost at previous time t_0

I_t = Index value at time t

I_0 = Index value at base time 0

The widely reported Consumer Price Index (CPI) is an example of an index which can denote changes over time. This index is computed and published by the US Department of Labor, Bureau of Labor Statistics. Table 12.10 presents the CPI for 15 years with 1999 as a base year.

Producer price index (PPI) measures the average change in the selling prices received by domestic producers for their output over time in a particular industry. Table 12.11, Table 12.12, and Table 12.13 present PPI for aircraft engine and engine parts, construction, and wholesales respectively.

There is another construction cost index for building, common labor, skilled labor, and materials produced by Engineering News-Record (ENR). Table 12.14 presents selected years' construction cost index with 1913 as the base year.

Table 12.10 CPI for All Urban Consumers as of December

Year	CPI	Year	CPI
1999	100	2008	121.6
2000	102.6	2009	124.5
2001	103.9	2010	126.1
2002	106.0	2011	129.8
2003	107.8	2012	131.8
2004	111.2	2013	133.5
2005	114.4	2014	134.2
2006	117.0	2015	134.6
2007	121.3		

Source: Bureau of Labor Statistics

Table 12.11 PPI for Aircraft Engine and Engine Parts as of January

Year	PPI	Year	PPI
2005	163.4	2011	202.3
2006	167.7	2012	207.7
2007	175.9	2013	213.6
2008	182.0	2014	217.0
2009	192.0	2015	217.8
2010	197.9	2016 (Preliminary)	220.6

Source: Bureau of Labor Statistics

Table 12.12 PPI for Construction as of December

Year	PPI	Year	PPI
2007	100	2011	106.3
2008	103.1	2012	109.9
2009	108.4	2013	111.8
2010	104.9	2014	115.7

Source: Bureau of Labor Statistics

Table 12.13 PPI for Wholesale Trade Industries as of December

Year	PPI	Year	PPI
2006	100	2011	116.9
2007	102.2	2012	121.5
2008	104.7	2013	125.9
2009	113.5	2014	127.8
2010	118.4	2015	131.1

Source: Bureau of Labor Statistics

Table 12.14 ENR Construction Cost Index

Year	Cons. Cost Index	Year	Cons. Cost Index
1913	100	2000	6221
1985	4195	2005	7446
1990	4732	2010	8802
1995	5471	2015 est.	10,405

Source: Engineering News-Record

Example 12.14

EZJET airline needs to compute the cost of labor today (in USD) in order to complete a certain project relative to the cost of completion of the same project in Europe five years ago. Labor index for 2010 was 790.60 and the labor index today is 1156.70. The labor cost of completing the project in Europe was €1.5 million. The conversion rate for today is 1 Euro equals 1.5 USD. The cost index method can be used to determine the labor cost of the project today.

C_t = The cost that needs to be determined in USD

C_0 = €1,500,000

I_t = 1156.70

I_0 = 790.60

$$C_t = C_0 \left(\frac{I_t}{I_0} \right)$$

$$C_t = €1,500,000 \left(\frac{1156.70}{790.60} \right)$$

$$C_t = €2,194.599.04$$

$$C_t = €2,194.599.04 \times 1.5 = \$3,291.898.56$$

Hence, C_t = \$3.29 million

Example 12.15

The average price of an engine in 2009 was about \$11.3 million. The aircraft engine and engine cost indexes are 192.0 and 218.9 for 2009 and 2015 respectively. What would be the estimated price of an engine in 2015?

$$C_t = \$11.3 \text{ m} \left(\frac{218.9}{192.0} \right)$$

$$C_t = \$12.88 \text{ m}$$

Cost Capacity Equation

The cost capacity equation accounts explicitly for economies of scale. This method, which is also called power law or sizing model, estimates the cost of an industrial plant, equipment, or system which has a different size or capacity as compared to a similar plant, equipment, or system that was completed earlier. This model accounts for economies of scale when we are estimating the cost of a larger size or capacity plant and it accounts for diseconomies of scale when we are scaling down the size of the plant. The cost capacity equation uses a correlating exponent (x) that reflects economies or diseconomies of scale in the capacity or size differentials. The cost capacity equation is presented below:

$$C_2 = C_1 \left(\frac{Q_2}{Q_1} \right)^x$$

Where:

C_1 = Cost at capacity Q_1

C_2 = Cost at capacity Q_2

X = Correlating exponent

In this model, the correlating exponent may have a value of equal to one, less than one, or greater than one. The impacts of the correlating exponent are presented in Table 12.15.

Since economies of scale generally exist when one purchases a plant or equipment with a larger size or capacity, it indicates that the value of the correlating exponent should usually be less than one.

Table 12.15 Possible Values for Correlating Exponent

$x = 1$	There is a linear relationship among two capacities and there are no economies or diseconomies of scale.
$x > 1$	Larger capacities will cost more than smaller capacities. This is due to diseconomies of scale.
$x < 1$	Larger capacities will cost less than smaller capacities. This is due to economies of scale.

Example 12.16

A turbojet engine with 20,000 pounds of thrust costs \$2,000,000. What is the estimated cost of a turbojet engine with 100,000 pounds of thrust if the correlating exponent is 0.60 for the pounds of thrust range of 10,000 to 200,000?

$C_1 = \$2,000,000$ for 20,000 pounds of thrust

$C_2 = \text{Estimated cost for 100,000 pounds of thrust } X = 0.6$

$$C_2 = \$2,000,000 \times \left(\frac{100,000}{20,000} \right)^{0.6}$$

$$C_2 = \$5,253,056$$

The capacity increased by five times from 20,000 to 100,000 pounds of thrust, but cost increased only by 2.63 times. This is due to economies of scale with a correlating exponent of 0.60 that is much less than one.

The cost capacity equation relates the cost of an industrial plant or equipment to its size or capacity. This equation assumes the costs of various sizes or capacities are current. The correlating exponents usually do not change over time but the cost of equipment or components changes over time. Therefore, one has to combine the cost capacity equation and cost indexing ratio (I_t/I_0) to adjust for time differential and to obtain the current cost of the equipment. The equation presented below provides the combined effects of the cost capacity equation and cost indexing ratio to estimate the cost at time t for capacity level 2.

$$C_2 = C_1 \left(\frac{I_t}{I_0} \right) \left(\frac{Q_2}{Q_1} \right)^X$$

Example 12.17

An aerobic digester with a flow rate of 1 million gallons per day (MGD) was installed in 2010 at a cost of \$800,000. Estimate the cost of an aerobic digester this year with a flow rate of 5 MGD, assuming the correlating exponent is 0.20 for range of 0.5 to 20 MGD and the cost index has increased from 152 in 2010 to 208 in the current year.

$$C_2 = \$800,000 \left(\frac{208}{152} \right) \left(\frac{5}{1} \right)^{0.2}$$

$$C_2 = \$800,000 (1.3684)(1.3797)$$

$$C_2 = \$800,000 (1.8880)$$

$$C_2 = \$1,510,385$$

The capacity of the aerobic digester increased by 400 percent, the cost index increased by about 37 percent, but the cost of the new aerobic digester increased only by about 89 percent. This is due to significant economies of scale or a very small correlating exponent.

Cost Factor Method

The cost factor method is especially useful in estimating total plant cost in processing industries. Both direct and indirect costs can be included. The cost factor is commonly the sum of all direct cost components and an indirect cost component, that is,

$$h = 1 + \sum f_i$$

For $i = 1, 2, 3 \dots n$ cost components, including direct and indirect costs.

Total plant cost estimate C_T is equal to overall cost factor (h) times cost of equipment or major component of a plant (C_E).

$$C_T = h \times C_E$$

Where,

C_T = Estimated total cost of a plant

C_E = Cost of equipment or major component of the plant (first cost)

h = Overall cost multiplier

Example 12.18

A manufacturing company is planning to shift a set of equipment to a new facility. The equipment costs \$20 million. A cost factor for all direct costs of 1.74 and an indirect cost factor of 0.35 will make the plant ready to operate. What is the total cost of the plant?

The total plant cost can be estimated as follows:

$$h = 1 + \sum f_i$$

$$h = 1 + 1.74 + 0.35 = 3.09$$

$$C_T = h \times C_E$$

$$C_T = 3.09 \times \$20 \text{ million}$$

$$C_T = \$61.8 \text{ million}$$

Hence the estimated total cost is \$61.8 million.

In some cases, where the indirect costs are charged separately against all direct costs, the indirect cost component is added separately. That is, the cost factor includes only the sum of all direct cost component factors and the indirect cost component factors are added separately, as follows:

$$h = 1 + \sum f_i \quad (\text{only direct cost components})$$

$$C_T = hC_E (1 + f_{\text{indirect}})$$

Example 12.19

Using relevant information from Example 12.13, consider the delivered-equipment cost to be \$20 million. Direct cost factor for installation is 1.20 and for training is 0.54;

and an indirect cost factor of 0.3 is applied to all direct costs. In this case, the total cost is estimated as follows:

$$h(\text{direct}) = 1 + 1.20 + 0.54 = 2.74$$

$$C_T = hC_E (1 + f_{\text{indirect}})$$

$$C_T = 2.74 \times \$20 \text{ million} (1 + 0.3) = \$71.24 \text{ million}$$

Hence the estimated total cost is \$71.24 million.

Indirect Cost Estimation and Allocation

Indirect costs are costs that cannot be directly traced to a production, process, or department. Examples of indirect costs include utilities, administrative costs, IT services, or accounting and marketing functions. Since indirect costs cannot be easily traced to a specific department, project, or a process, they are usually estimated and allocated using either a standard rate or an activity based method.

Standard rate estimation and allocation can be accomplished using an industry-specific rate. For a given industry, the standard rate can be computed either by expressing indirect cost as a factor of the first cost, major equipment, or main component cost, or by expressing indirect cost as a factor of total direct costs.

$$\text{Standard indirect cost rate} = \left(\frac{\text{Total expected indirect costs}}{\text{Estimated first cost}} \right),$$

$$\text{Standard indirect cost rate} = \left(\frac{\text{Total expected indirect costs}}{\text{Estimated total direct costs}} \right),$$

Once the standard rate for indirect cost is computed, one can use the factor method to estimate the total indirect cost, or estimate the total cost of the plant using both direct and indirect cost factors.

Example 12.20

Direct Jet's (DJ) total indirect costs or overhead costs were \$7,639 million and its total fixed assets were \$14,292 million in 2014. What was the indirect cost rate for DirectJet based on total fixed assets?

Utilizing the above equation, the indirect cost rate is computed below:

$$DJ \text{ indirect cost rate}_i = \left(\frac{\$7,639m}{\$14,292m} \right)$$

$$DJ \text{ indirect cost rate}_i = 0.53$$

In the activity based method, one would first identify the activities, cost driver, and total volume of cost drivers. Then establish the activity based allocation rate for the cost driver. Finally allocate the total indirect cost by multiplying the volume of each activity by the allocation rate.

The activity based allocation rate can be computed by dividing the total indirect cost of activities over the total volume of cost driver as shown below:

$$\text{Activity based allocation rate} = \left(\frac{\text{Expected total indirect costs of activities}}{\text{Total volume of cost driver}} \right)$$

Example 12.21

A startup airline company out of Phoenix, Arizona established four routes to Los Angeles, Denver, Chicago, and Dallas last year and incurred a total of \$2 million in overhead costs. The company wants to use the activity based method to allocate the indirect cost among the routes. The cost driver is identified as revenue passenger miles (RPM). Table 12.16 presents the available seat miles (ASM) and average load factor for each route. Allocate the overhead cost via the activity based method using number of revenue passenger miles.

To compute the total revenue passenger miles, multiply seat miles by the load factor for each route and add them together as shown in Table 12.17.

The indirect allocation rate per passenger revenue mile is computed as shown below:

$$\text{Allocation rate} = \left(\frac{\$2,000,000}{41,600,000} \right) = \$0.048$$

Given the allocation rate of \$0.048 per revenue passenger mile, the overhead costs can be allocated to four routes as shown in Table 12.18.

Table 12.16 Available Seat Miles and Load Factor for Each Route

Route	Total ASM	Average Load Factor
Chicago	12,000,000	75%
Dallas	15,000,000	78%
Denver	8,000,000	70%
Los Angeles	18,000,000	85%

Table 12.17 Computation of Revenue Passenger Miles

Route	Total ASM	Average Load Factor	Revenue Passenger Miles
Chicago	12,000,000	75%	9,000,000
Dallas	15,000,000	78%	11,700,000
Denver	8,000,000	70%	5,600,000
Los Angeles	18,000,000	85%	15,300,000
Total Revenue Passenger Miles			41,600,000

Table 12.18 Indirect Costs Allocation

Route	Revenue Passenger Miles	Allocation Rate	Indirect Cost Allocation
Chicago	9,000,000	\$0.04808	\$432,720
Dallas	11,700,000	\$0.04808	\$562,536
Denver	5,600,000	\$0.04808	\$269,248
Los Angeles	15,300,000	\$0.04808	\$735,624

Airline Cost Structure

It's like a wristwatch. You only see the face and hands, but all the parts inside are really necessary.

Gordon Bethune, Continental Airlines

The airline industry in the United States is an oligopoly dominated by the several major airlines, with the industry costs affected by a set of factors. The size and the type of aircraft, fuel efficiency and distance traveled are some examples of cost factors. The costs of airline resources can be divided into two major categories, namely, operating costs and non-operating costs. Operating costs can be attributed to the operation of an airline, make up the larger of the two categories, and can be further subdivided into two sub-categories: direct and indirect costs. Non-operating costs are costs not directly linked to the airline's operation, but required to conduct business. A description of all possible costs associated with an airline is presented below.

Operating Costs (OC)

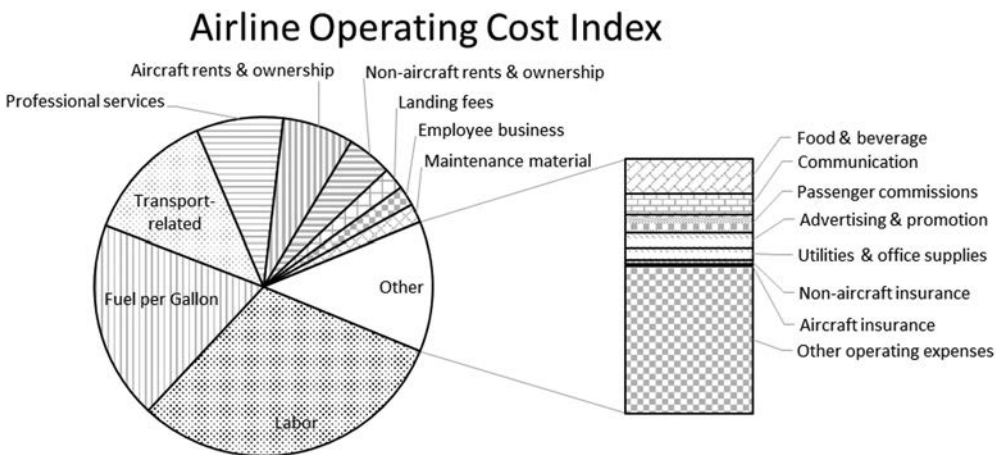
Airline operating costs are incurred in the process of providing air transport services to passengers or freight. Some of the operating costs include aircraft expenses, fuel expenses, maintenance expenses, employee expenses, landing fees, and other operating expenses. Operating costs could be both fixed and variable, although all the costs relate directly to the core operation of the business. The rise and fall of jet fuel prices, for example, influences airlines to improve their cost-efficiency techniques and levels of productivity. Examples of airline operating costs include:

- ♦ Aircraft rent and ownership
- ♦ Food and beverage
- ♦ Fuel
- ♦ Labor
- ♦ Landing fees
- ♦ Maintenance materials
- ♦ Non-aircraft rent and ownership
- ♦ Transport-related expenses

Fuel expense is one of the biggest operating costs encountered by airlines, followed by labor cost. In addition to fuel and labor, there is a variety of other operational costs such as the ownership cost of aircraft, landing fees, renting terminal facilities and gates.

Operating costs can be further broken down into direct and indirect operating costs. In the airline industry, direct operating costs constitute the greatest portion of operating costs, with labor wages and aircraft fuel comprising the bulk of airline operating costs. Airlines for America (A4A) provides a good overview on the cost index for all operating costs (Figure 12.11):

Figure 12.11 Airline Operating Cost Index



Source: Airlines for America

Table 12.19 is a portion of the actual income statement for Delta Air Lines and clearly states the different types of operating costs associated with an airline. Most airlines have this portion of the income statement similar to each other.

Direct Operating Costs (DOC)

Direct operating costs are those costs which can be directly traced to a specific level of production, which for the airline industry could be a specific airport or specific flight. From Table 12.19, it can be clearly noted that labor (salaries and related costs) and fuel expense form the biggest portion of the total operating expenses, making them the largest direct operating cost component. Labor cost exhibits tendencies of a mixed cost, with some wages, salaries, and benefits (WSB) components considered as fixed costs, while other components, such as overtime and profit sharing, are more likely to be variable costs.

Although proper understanding of direct operating costs is important from an external standpoint, internal analysis requires detailed understanding of the basic differences between variable direct operating costs and fixed direct operating costs. A clear understanding of this differentiation is crucial not only for devising

Table 12.19 Operating Expenses for Delta Air Lines (in \$ millions)

	Year Ended December 31			
	2015	2014	2013	2012
Aircraft fuel and related taxes	6,544	11,668	9,397	10,150
Salaries and related costs	8,776	8,120	7,720	7,266
Regional carrier expense	4,241	5,237	5,669	5,647
Aircraft maintenance materials and outside repairs	1,848	1,828	1,852	1,955
Depreciation and amortization	1,835	1,771	1,658	1,565
Contracted services	1,848	1,749	1,665	1,566
Passenger commissions and other selling expenses	1,672	1,700	1,603	1,590
Landing fees and other rents	1,493	1,442	1,410	1,336
Profit sharing	1,490	1,085	506	372
Passenger service	872	810	762	732
Aircraft rent	250	233	209	272
Restructuring and other items	35	716	402	452
Other	1,998	1,797	1,520	1,592
Total operating expense	32,902	38,156	34,373	34,495

Source: Compiled by the authors from financial statements

methods to reduce an airline's cost structure, but also in the shaping of business strategy.

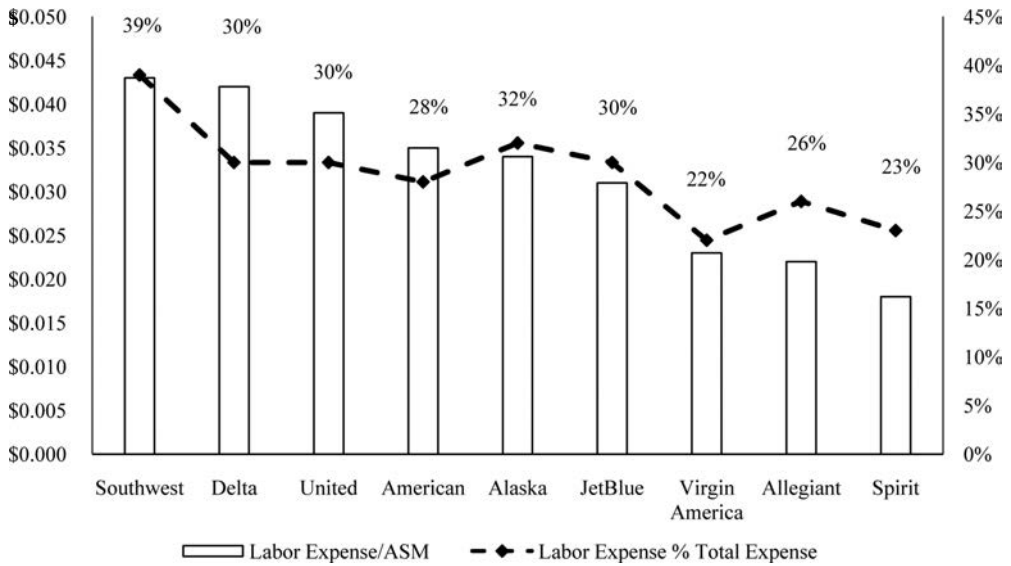
Figure 12.12 and Figure 12.13 represent the labor and fuel costs of major US carriers for the year 2015. It can be seen from the graphs that each of the two costs forms approximately 30 percent of the total expenses, indicating that they are the biggest contributors to the overall operating expenses.

Fixed Direct Operating Costs (FDOC)

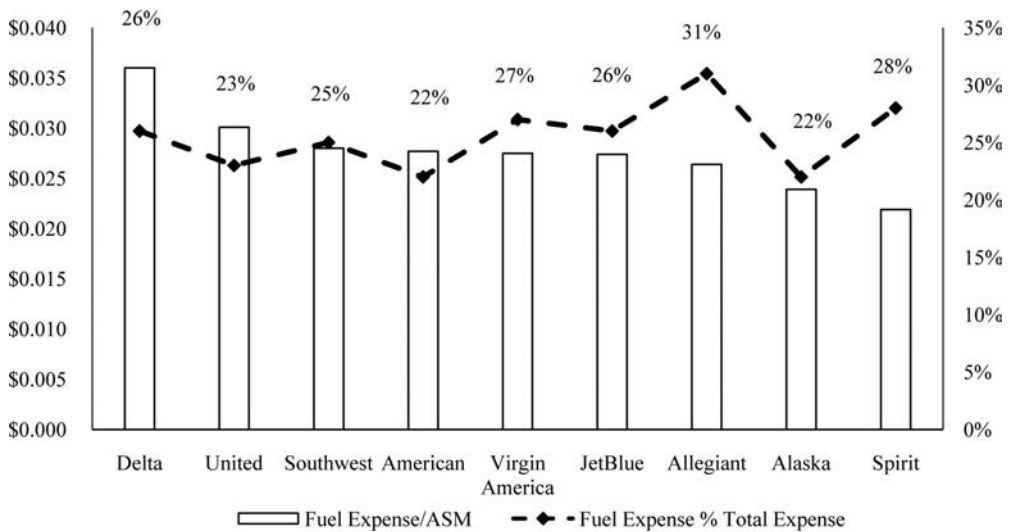
An airline's fixed operating costs are those costs of operation that occur whether or not specific flights are operated. These costs include aircraft leases, terminal leases, and crew salaries. The fixed operating costs are allocated to flights based on the percentage of operations that the flights make up. The fixed direct operating costs would still be incurred even if the flights were dropped.

Variable Direct Operating Costs (VDOC)

Variable direct operating costs are those costs that are incurred based on the level of activity of an airline (number of flights an airline operates). Fuel, landing fees, passenger services, and maintenance are all deemed to be variable operating costs since they vary depending on the number of flights taken over the course of a year.

Figure 12.12 Labor Costs of Selected US Airlines, 2015

Source: Compiled by the authors from financial statements

Figure 12.13 Fuel Costs of Selected US Airlines, 2015

Source: Compiled by the authors from financial statements

Some flight crew costs, such as overnight accommodations, could also be deemed variable operating costs.

An airline should examine the variable operating costs while attempting to determine the profitability of a certain flight since the fixed operating costs will be incurred whether or not the flight is carried out. Sometimes a flight is operated even if the

direct variable operating costs exceed the revenue. This can be attributed to the fact that the flight may be providing a passenger feed to a connecting flight that is profitable, but, without the connecting passengers, would turn unprofitable. Also, the direct costs associated with not operating the flight and having the aircraft grounded, such as routine maintenance and parking fees, could exceed the negative contribution. In such cases, the flight is mostly permitted to fly as well.

Indirect Operating Costs (IOC)

Indirect operating costs are costs that are not directly related to the core functioning of the airline, such as some selling and administration expenses, personnel training expenses, advertising and commissions. Indirect costs are necessary for an airline to operate, but cannot be directly linked to a specific flight since the expenses provide support to all operations associated with the airline. The direct impact of such costs on an airline's operation is negligible, allowing managers to focus on the reduction of indirect operating costs as the primary source of cutting down the total operating costs.

Non-operating Costs (NOC)

Non-operating costs are expenses arising from activities not associated with the rendering of air transport services. Ideally non-operating costs are related to the financial structuring of the company and are the result of a transcending financial strategy of an airline. The most common non-operating cost in the airline industry is the interest expense, accounting to the sizable debt loads airlines carry. The operation of the airline is in no way affected by the cost incurred due to the interest expense, making it a non-operating cost, although the debt itself is used to fund the operation of the airline. Other non-operating costs for an airline include any loss on the sale of aircraft and other assets and foreign currency exchange charges. Another major non-operating cost is depreciation. Depreciation of the assets owned by an airline has absolutely no impact on the operation of the airline, thereby making it a non-operating cost.

Summary

This chapter focuses on introducing readers to the different types of costs, such as fixed and variable, direct and indirect, total, marginal and average costs, and their varied applications. A brief discussion about the relationship between economic costs and accounting costs and the possible types of financing options is provided as well, incorporating the cost of capital, the return on investment and the optimal debt ratio. Understanding the learning curve is a crucial part of cost estimation in any business and a detailed description of it is provided in the chapter. Generally, costs tend to decrease as the amount of output increases, as a result of the learning effect occurring not only on the production floor but throughout the whole organization. Different methods of cost estimation are explained in order to facilitate accurate cost calculation in various scenarios: unit cost method, cost index method, cost capacity equation, cost factor method, and indirect cost estimation and allocation methods. The last section of the chapter focuses on the airline cost structure and how it matches up to the previously discussed types of costs. In addition to those, airline costs are subdivided into operating costs (costs directly related to the operation of the business such as aircraft rent and ownership costs, catering, fuel expenses,

labor costs, landing fees, maintenance costs, non-aircraft rent and ownership and transport-related expenses), and non-operating costs (costs that are not directly associated with the air transport services such as interest expense, loss on sale of aircraft and other assets, foreign currency exchange charges, and depreciation). In the airline industry, fuel and labor expenses constitute the major part of an airline's operating costs.

Discussion Questions and Problems

- 12.1 Distinguish between economic cost and accounting cost.
- 12.2 Describe the differences between variable costs and fixed costs.
- 12.3 Give an example of a fixed cost and a variable cost of an airline.
- 12.4 Define and give examples of opportunity costs and sunk costs.
- 12.5 The cost function for a Fixed Based Operator is given by $TC(Q) = 40,000 + 30Q^2$.
Find:
 - a. Marginal cost of producing 10 units
 - b. Average cost function
 - c. Average variable cost function
 - d. Average total cost function.
- 12.6 Suppose the cost function is:
 $TC(Q) = 50 + Q - 10Q^2 + 2Q^3$
 - a. What is the marginal cost of producing the 4th unit?
 - b. What is the marginal cost of producing the 10th unit?
- 12.7 John is thinking about opening a fixed base operation (FBO) at a local airport. John estimates that it would cost \$8,000,000 per year to rent a hangar and stock it with inventory of parts. In addition, John would have to give up \$800,000 per year as a manager for another FBO.
 - a. What are John's explicit costs of running the FBO for a year?
 - b. What are the implicit costs, if any, of running the FBO for a year?
 - c. If John's annual revenue (net other expenses) is \$8,450,000, what are his estimated accounting profits? His economic profits?
 - d. Would you advise John to open the FBO? Explain.
- 12.8 Aviation Supply is a small, North Korean-based company which provides parts for unmanned aircraft manufacturers. The company's total cost function is given by the following equation:
 $TC = 1,500,000 + 250 \times Q$
 - a. Graph the firm's TC , MC , and AC functions.
 - b. Find total fixed cost, total variable cost, and average cost for this function.
 - c. Break average cost into average fixed cost and average variable cost.
 - d. What happens to average variable cost and average fixed cost as output rises?
 - e. Will AC ever equal MC ?
- 12.9 Compute the time required to produce the 10th aircraft if the time to manufacture the first aircraft is three months and the learning rate of production is 0.75.

12.10 A multi-component system consists of paint materials, sheet metal, and labor for mechanical, electrical, casting, finishing, and painting jobs. The quantity of each component needed and its unit cost are provided in the following table. What is the estimated total cost of the system?

Component	Quantity Needed	Unit Cost Factor
Paint material	500 gallons	\$95
Sheet metal	2,000 tons	\$62
Labor, mechanical/electrical	1,200 hours	\$85 per hour
Labor, casting	1,500 hours	\$60 per hour
Labor, finishing	1,000 hours	\$50 per hour
Labor, painting	200 hours	\$40 per hour

12.11 The average price of a Rolls-Royce Trent 1000s engine was \$162.5 million in 2007. The producer price indexes (PPI) for an aircraft engine were 175.9 and 218.9 for 2007 and 2015 respectively. Assuming the price of Rolls-Royce engines increases according to PPI, what was the estimated price of a Rolls-Royce Trent 1000s engine in 2015?

12.12 The cost of production for 100 electric starter units is \$200,000. What would be the estimated cost of production of 500 electric starter units if the correlating exponent is 0.7 for a range of production from 100 to 1,000 units?

12.13 A compressor equipment with 2,000 horse power (hp) was installed at cost of \$200,000 in 2010. What is the cost of installing a compressor with 500 hp in this year assuming the correlating exponent is 0.62 for horse power range of 200 to 2,500 and the cost index has increased from 342 in 2010 to 512 in the current year?

12.14 A multi-component system is expected to have an equipment cost of \$400,000 including delivery and installation. The cost factors for the components are provided in the table below. If indirect costs are expressed as a factor of the cost of equipment, what is the estimated total cost of the system?

Components	Cost Factor
Concrete	0.85
Sheet metal	0.46
Construction	1.20
Indirect labor	0.55
Other indirect costs	0.42

- 12.15** In problem 12.4, re-compute the estimated total cost of the system if indirect costs are expressed as a factor of the total direct cost.
- 12.16** A startup airline company out of Denver, Colorado established five routes to Los Angeles, Los Vegas, Phoenix, Chicago, and Dallas last year and incurred a total of \$4.2 million in overhead costs. The company wants to use the activity based method to allocate the indirect cost among the routes. The cost driver is identified as available seat miles (ASM). The following table presents the revenue passenger miles (RPM) and average load factor for each route. Allocate the overhead cost by the activity based method using the number of available seat miles.

Route	Total RPM	Average Load Factor
Chicago	450,000	75%
Dallas	574,000	82%
Los Angeles	630,000	70%
Los Vegas	595,000	85%
Phoenix	480,000	80%

- 12.17** A compressor with 2,000 horse power (hp) was installed at a cost of \$200,000. What is the cost of installing a compressor with 500 hp assuming the correlating exponent is 0.32 for horse power range of 200 to 2,500?
- 12.18** The price of a jet engine for the B737-MAX by CFM, a joint venture between GE and SAFRAN, is about \$11 million in 2016. What was the price of the same engine in 2010 if the aircraft engine and engine cost indexes are 197.9 and 217.0 for 2010 and 2016 respectively?

Notes

- 1 Saddened Lockheed Workers Still View L-1011 with Pride, *Los Angeles Times*, December 8, 1981.
- 2 Greenwald, J., J. Hannifin, and J. Kane. Catch a Falling TriStar, *Time*, December 21, 1981.
- 3 The Airbus A350, Up and Away, *The Economist*, June 14, 2013.
- 4 Aircraft maintenance is typically scheduled to be completed whether the airplane flies or not, as well as scheduled overhauls and avionics updates. Hence, these costs can be considered fixed costs since they do not necessarily change with aircraft usage.
- 5 Delta Air Lines is retiring its 747s, older 757s, and domestic 767s in order to lower its maintenance costs.
- 6 BBC, September 18, 2014.
- 7 Definition retrieved from Cambridge Business Dictionary.
- 8 Concorde, F.A.Q.

- 9 Shutdown Took \$24 Billion Bite Out of Economy, CNN, October 17, 2013.
- 10 Impacts and Costs of the October 2013 Federal Government Shutdown, Executive Office of the President of the United States, November 2013.
- 11 All things 787.

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13

Analysis of Financial Statements

The immediate effect of the deficit is to make you feel good, like when you go on a trip and pay later. You feel good, and then you get a hangover. The deficit makes you feel good—until you pay later.

Franco Modigliani



Financial statements present a collection of reports about a company's operation by providing information on financial activities over the course of a given period. Financial statements are the most important sources of information about companies for a wide variety of stakeholders such as creditors, employees, investors, and tax authorities. Corporations use four types of financial statements to report the financial results of their operations including the balance sheet, the cash flow statement, the income statement, and the statement of retained earnings. The income statement represents the company's revenues and expenses over a given time period, whereas, the balance sheet is a snapshot of the organization's financial position on a particular day. Another important financial statement is the statement of cash flows which shows the changes in cash and cash equivalents over the period. In this chapter, we will learn how to read aviation industry financial statements and do basic financial analysis. We will explore the income statement, the balance sheet, the statement of retained earnings, and the cash flow statement.

- ◆ The Income Statement: Measuring a Firm's Profits
 - Revenue, Sales, or Income
 - Cost of Goods Sold
 - Gross Profit (Operating Profit)
 - Operating Expenses
 - Depreciation

- Operating Profit
- Financial Costs
- Net Income
- ♦ The Balance Sheet: Measuring a Firm's Book Value
 - Current Assets
 - Cash and Cash Equivalents
 - Marketable Securities
 - Accounts Receivable
 - Inventories
 - Prepaid Expenses
 - Fixed Assets
 - Property, Plant, and Equipment
 - Other Long-term Assets
 - Total Liabilities
 - Current Liabilities
 - Accounts Payable
 - Accrued Expenses
 - Current Portions of Long-term Debt
 - Non-current Liabilities
 - Long-term Debt
 - Other Long-term Liabilities
 - Stockholders' Equity
 - Preferred Stock
 - Common Stock and Additional Paid-in Capital
 - Retained Earnings
- ♦ Statement of Cash Flow
 - Cash Flow from Operating Activities
 - Cash Flow from Investing Activities
 - Cash Flow from Financing Activities
- ♦ Measuring Free Cash Flow
 - Common Size Financial Statements
 - Common Size Income Statement
 - Common Size Balance Sheet
- ♦ Summary
- ♦ Discussion Questions and Problems

The Income Statement: Measuring a Firm's Profits

The income statement, also called an earnings statement or a profit and loss statement, is where you will find the profits and losses of a company. The income

statement is an accounting statement that matches a company's revenues with its expenses over a period of time, usually a quarter or a year. Income statements begin by listing the company's income from the sale of products or services, less total cost of the operations.

The principle of the income statement involves a company's recognition of earnings and expenses related to the production of goods and services during a given period of time. The difference between revenues and expenses results in a profit or loss.



The income statement is a cumulative account of items through a specific accounting period. Thus, the expenses on a first quarter income statement equal the cumulative amount of all expenses during the first three months of the firm's fiscal year. The expenses on the second quarter income statement equal the cumulative amount of all expenses during the second three months of the firm's fiscal year. The same applies to all other values included in the income statement. The various components of the income statement are explained in the following sections. For the purpose of illustration, we have selected the Boeing Company to discuss income statements. Table 13.1 presents the Boeing Company's income statement for the year ended December 31, 2015. As is shown in Table 13.1, Boeing's total revenue is \$96,114 million for the year ended December 31, 2015. Boeing's cost of goods and services sold is \$82,088 million resulting in a gross profit of \$14,026 million. Its operating expenses for the year were \$6,583 million resulting in an operating profit of \$7,443 million. Subtracting financial cost, interest and debt expenses, and taxes resulted in a net income for the year of \$5,176 million. The components of income statements for aviation and aerospace companies are discussed in detail in the following section.

Revenue, Sales, or Income

Revenue is the amount of money a company earns over a given period of time by selling products and services. Airbus Group's global revenue amounted to around 64 billion euros, by delivering a record-setting 635 aircraft to airlines in 2015.¹ For the Boeing company, the major sources of income come from:

- ◆ Commercial airplanes
- ◆ Defense, space, and security
- ◆ Boeing capital
- ◆ Other income

For Boeing, most of the revenue comes from sales of airplanes. Other types of revenue include defense, space, and security revenue and Boeing capital revenue. Defense, space, and security revenue is an important revenue source for Boeing. Over the next 20 years, Boeing forecasts that long-haul international traffic will grow at about

Table 13.1 Boeing Income Statement, 2015

Consolidated Income statement	
The Boeing Company	
For the year ended December 31, 2015	
(in millions, except per share data)	
Operating Revenue:	
Commercial airplanes	66,048
Defense, space, & security	30,388
Boeing capital	413
Unallocated items	(735)
Total revenues	\$96,114
Cost of Sales:	
Cost of products	(73,446)
Cost of services	(8,578)
Others	(64)
Total cost of goods and services sold	(\$82,088)
Gross Profit	\$14,026
Operating Expenses:	
General and administrative expenses	(3,525)
Research and development expenses	(3,331)
Others	273
Total operating expenses	(\$6,583)
Operating Income	\$7,443
Other Expense:	
Interest and debt expenses	(275)
Other losses	(13)
Total other expenses	(\$288)
Earnings before Income Taxes	\$7,155
Income before Income Taxes	\$7,157
Income tax expense	(\$1,979))
Net Income	\$5,176
Basic Earnings Per Share	\$7.52
Diluted Earnings Per Share	\$7.44

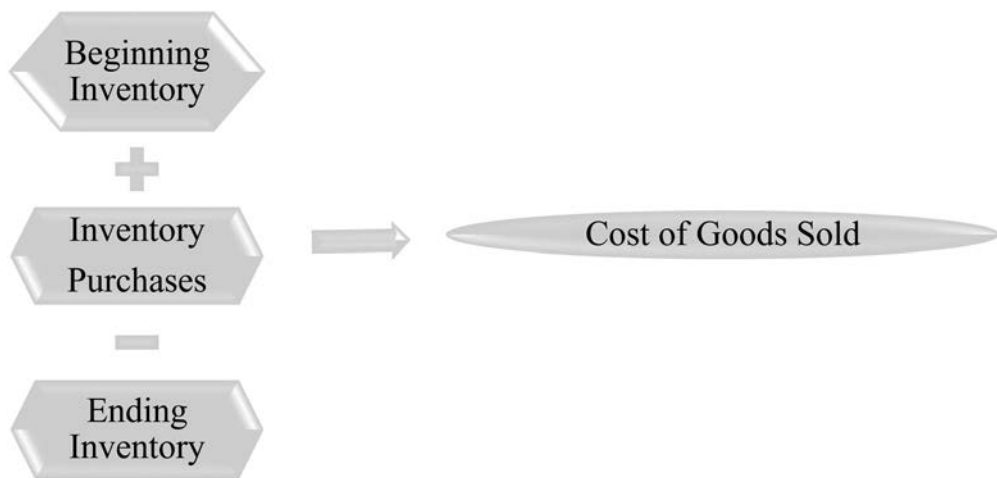
Source: Compiled by the authors from financial statements

5 percent per year.² This growth drives the need for about 8,830 new airplanes, valued at approximately \$2.7 trillion. This is a major source of revenue for both the Boeing and Airbus companies.

Cost of Goods Sold (COGS)

Whenever a product is manufactured or service is provided, certain direct costs are incurred. These costs are labeled on the income statement as the cost of goods sold. For the airline industry, the cost of goods sold includes direct materials such as fuel, direct labor, and indirect costs. For an aircraft manufacturer, it would include the material costs for the parts and the labor costs that go into production of the aircraft. Generally, it includes direct costs and the overhead associated with production of the product or delivery of a service. COGS excludes indirect expenses such as distribution costs and sales force costs.

COGS are the direct costs associated with the production of the goods and services provided by a company.



One of the major players in the aviation manufacturing industry is Boeing. Because it is a manufacturing company, the cost of goods sold is included in the income statement and can be seen in Table 13.2.

Table 13.2 COGS Section of Boeing Income Statement, 2015

Cost of products	(\$73,446)
Cost of services	(8,578)
Boeing capital interest expense	(64)
Total costs and expenses	(\$82,088)

Cost of goods sold is a key figure on the income statement. To compute cost of goods sold, start with the cost of beginning inventory, add inventory purchased, and then subtract the cost of ending inventory of finished goods.

Example 13.1

Alpine Aviation is a business selling landing gears for general aviation aircraft at a local airport. An inventory count at the beginning of the year showed that the company held \$800,000 worth of inventory on hand. Over the year, Alpine purchased another \$2,400,000 worth of parts. At the end of the year, Alpine had \$600,000 worth of inventory:

$$\text{Cost of Goods Sold} = \$800,000 + \$2,400,000 - \$600,000$$

$$\text{Cost of Goods Sold} = \$2,600,000$$

Example 13.2

Aspen Engineering Company (AEC) operates a small machine shop. AEC manufactures hydraulic parts used primarily in production in the helicopter industry. AEC has three main products used for light, medium, and heavy lift copters. The company had a beginning inventory of \$700,000 last month and purchased another \$1,500,000 in inventory. Last month was a great month, and Aspen's remaining inventory at the end of the month was \$200,000. We can calculate the COGS as follows:

$$\text{COGS} = \$700,000 + 1,500,000 - 200,000$$

$$\text{COGS} = \$2,000,000$$

Gross Profit (Operating Profit)

A recent Deutsche Bank report projects a total airline industry first-quarter pretax profit of \$3.5 billion, up from \$700 million this time last year—a 400-percent improvement. The bank estimates that nearly 110 percent of the earnings gain will derive from lower fuel prices. The industry over the past several years has demonstrated its ability to successfully offset most, if not all, of the rise in fuel expense via a combination of cost savings and various revenue initiatives.

Deutsche Bank, April 26, 2015

Gross profit is calculated as the difference between total revenue and cost of goods sold. This basically denotes the gross profit of a company taking into account purely the revenue generated and the costs incurred due to the sale of a particular product.



A firm's gross profit is analyzed to determine its efficiency and leanness of operation, though creditors, labor groups, investors, and company management tend to pay more attention to the net income, because the net income is a good barometer of the company's financial strength and its ability to meet future financial obligations. Table 13.1 shows that the Boeing Company had net sales of \$96,114 million and a total cost of goods and services sold of \$82,088 million, resulting in a gross profit of \$14,026 million for the year 2015.

Example 13.3

Sanford Regional is a small airport that handles only commuter traffic. During the most recent fiscal year, Sanford generated \$40,000,000 in revenue and \$10,000,000 in cost of goods sold. Sanford's gross profit is:

$$\text{Gross Profit} = \$40,000,000 - \$10,000,000 = \$30,000,000$$

Operating Expenses

Operating expenses are expenses other than the cost of goods sold that a company sustains in the normal course of business. For aerospace manufacturing, they may include items such as management salaries, advertising expenditures, research and development expenditures, general and administrative expenses, and depreciation of assets. Aircraft and jet engine manufacturers need a research team to work on designing next year's cool products, so that they don't fall behind.

For airlines, general management, flight personnel, aircraft maintenance, and traffic handling personnel are all included in the calculation of operating expenses.

Depreciation

Depreciation expense is included as part of the operating expenses, because it is considered tax deductible. The various methods of depreciation are:

- ◆ Straight Line Depreciation Method
- ◆ Declining Balance Depreciation
- ◆ Sum-of-the-Years Digits
- ◆ Modified Accelerated Cost Recovery System (MACRS)

In the United States, the IRS follows the Modified Accelerated Cost Recovery System in order to determine the depreciation of various assets to ensure that the process of tax deduction is uniform for all organizations.

Two of the most basic but important accounting estimates airline managements make are the useful lives and the residual values of aircraft. These estimates determine effective depreciation rates. Useful lives and residual values of existing aircraft fleets are being increasingly impacted by "new generation" aircraft. These aircraft have reduced the operating costs and are adversely impacting the values of older aircraft in the secondary market. When decisions are made to retire aircraft earlier than anticipated, accelerated depreciation may need to be applied prospectively to reduce the carrying value of aircraft.³ Different airlines have different depreciation policies based on the type of aircraft and business objectives.

Carriers such as Qatar Airways and Emirates Airlines, which are based in the Middle East, are required to depreciate their airplanes using only a Straight Line

depreciation method for a specified duration. This provides them with a disadvantage compared to their US-based counterparts, such as United Airlines and American Airlines, since the US-based carriers are allowed to depreciate their assets in an accelerated manner using the MACRS depreciation methodology.

Depreciation has been explained in more detail in Chapter 11.

Example 13.4

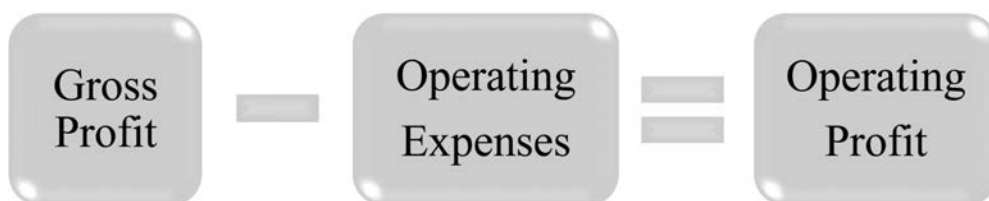
Jet Propulsion Company purchased several simulators at a total cost of \$6,000,000. The installation cost for this equipment was \$500,000. The company plans to depreciate the equipment using the MACRS five-year normal recovery period. Prepare a depreciation schedule showing the depreciation expense for each year.

Table 13.3 Jet Propulsion Depreciation Schedule

Year	Depreciation Base	Depreciation Rate	Depreciation Expense
0	6,500,000		
1	6,500,000	0.20	\$1,300,000
2	6,500,000	0.32	\$2,080,000
3	6,500,000	0.19	\$1,235,000
4	6,500,000	0.12	\$780,000
5	6,500,000	0.12	\$780,000
6	6,500,000	0.05	\$325,000

Operating Profit

Referring to the operating expenses section in Table 13.1 for the Boeing Company for the year ending December 31, 2015, Boeing had a total operating expense of \$6,583 million. We need to take the operating expenses out of the gross profit of \$14,026 million to get the operating profit of \$7,443 million. Operating profit presents a more accurate picture of how much money the company made from its operation.



Financial Costs

Interest expense refers to the cost faced by a firm as a result of borrowing money. It depends on the overall level of firm indebtedness and the interest rate associated with this debt. Companies need capital to maintain business operations and to finance their growth. Start-up airlines borrow money to acquire aircraft, flight equipment,

and to meet other related expenses. Interest is the cost of borrowing money. If a debtor is not able to pay the interest expenses as they come due but has assets out of which creditors might get paid, an involuntary bankruptcy may be filed by creditors to get paid when they otherwise would not collect. States West Airlines announced that it had a critical shortage of cash and would seek bankruptcy protection if it could not get short-term financing soon.⁴ Interest expense is generally a small fraction of total firm expenses. Undoubtedly, the higher the interest expenses, the higher the risk of financial insolvency.

Income taxes are a necessary component imposed by the government on any company carrying out operations under its jurisdiction. Income obtained after deducting the interest expense can be taxed at the federal, state, and/or local levels and the provision for income taxes can be calculated using published tax tables from the respective government agencies (refer to the appendix to this chapter for detailed tax rates). From the company's point of view, there are tax strategies that can be implemented to reduce or defer their tax liabilities. If a company incurs a significant operating loss in one year, the loss may carry back up to two years and/or carry forward up to 20 years from the year the loss was realized.

An operating loss can be carried back to the preceding two years and applied against the taxable income in those years to receive an immediate tax rebate. IRS stipulates that a loss carryback can be applied only to the two years preceding the loss. For instance, if a company were to record a net operating loss in year 7, the company could apply a loss carryback to years 6 and 5.⁵

A company may waive carrying loss back and proceed with loss carryforward up to 20 years. A loss carryforward occurs when a business carries loss forward to future years and applies it against the net income in those years, which reduces the taxable income; they consequently pay less tax in future years.

Example 13.5

DirectJet is a regional carrier with the following pattern of net income/losses over the years 2014 to 2019. They made a large Net Operating Loss (NOL) in year 2016 and they can offset this loss by means of a loss carryback and a loss carryforward. The schedule of actual earnings for 2014 and 2015 and the expected earnings from 2017 onwards is presented below. Provide a tax carryback and carryforward schedule for DirectJet to help them compensate for the NOL in 2016. Assume they are in the 35 percent tax bracket.

After loss carryback of \$15,000,000 to years 2014 and 2015, the rest of the loss is carried forward and applied to net income in years 2017 through 2019, which reduces taxable income to zero in those years and therefore no taxes are paid in years 2017 through 2019. The sole purpose of a carryback and carryforward is to realize the maximum possible tax refunds and to eliminate or reduce future tax liabilities to assist in covering a large loss incurred during a particular year.

The depreciation expense is another way of reducing the effective taxable income. Even though it is not an actual cash expense, Generally Accepted Accounting Principles (GAAP) allows companies to deduct depreciation from the revenue. Depreciation refers to accounting for the hypothetical loss in value of an asset upon usage. Depreciation and the types of depreciation are discussed in detail in Chapter 11.

Boeing Company earnings before interest and taxes (EBIT) for the year ending in December 2015 were \$7,802 million and earnings before taxes (EBT) were

Table 13.4 DirectJet Schedule of Earnings and Taxes Paid

Year	Net Operating Profit/Loss	Actual Taxes Paid
2014	\$5,000,000	(\$1,750,000)
2015	\$10,000,000	(\$3,500,000)
2016	(\$30,000,000)	
2017	\$3,000,000	
2018	\$2,000,000	
2019	\$10,000,000	(\$3,500,000)

Table 13.5 DirectJet Schedule for Loss Carryforward and Loss Carryback

Year	Net Operating Profit/Loss	Carryback / Carryforward	Adjusted Taxable Income	Tax Refund
2014	\$5,000,000	(\$5,000,000)	\$0	\$1,750,000
2015	\$10,000,000	(\$10,000,000)	\$0	\$3,500,000
2016	(\$30,000,000)			
2017	\$3,000,000	(\$3,000,000)	\$0	\$0
2018	\$2,000,000	(\$2,000,000)	\$0	\$0
2019	\$10,000,000	(\$10,000,000)	\$0	\$0

\$7,157 million. Boeing Company paid \$2,631 million in taxes for the year ended December 2015.

Net Income

Net income (also called net profit or earnings) is the last part of the income statement. It represents the base profit earned by a firm in a given accounting period. Boeing Company's operating profit or earnings before interest and taxes (EBIT)

for the year ending in December 2015 were \$7,443 million and earnings before taxes (EBT) were \$7,155 million. Boeing paid \$1,979 million in taxes resulting in a net income of \$5,176 million for the year ended December 2015.

Net income divided by the number of outstanding common shares yields the earnings per share (EPS). This value represents the profit earned by an investor for each share of stock owned.

Table 13.1 shows that for 2015, Boeing Company portrayed positive earnings of \$5,176 million with basic earnings per share of \$7.52. This means that if Boeing

Net income is what remains of an airline's revenue after subtracting all expenses.

distributed every dollar of income to its shareholders (EPS), each share would receive \$5.68 dollars.

$$EPS = \frac{\text{Net Income}}{\text{Number of Shares Outstanding}}$$

Corporations may pay part of their earnings as dividends to shareholders (dividends per share, DPS) as a return on their investment.

$$DPS = \frac{\text{Total dividends paid}}{\text{Number of Shares Outstanding}}$$

Boeing paid \$1.09 dividend per share in the third quarter of 2016.^{6a} A few airlines such as Southwest Airlines and SkyWest Airlines pay dividends regularly,⁷ while other airlines, such as the Boeing Company, pay intermediately. In the year 2015, Boeing Company paid \$0.91 per share dividend to its shareholders.^{6b}

The Delta Air Lines income statement for the year ended December 31, 2015 is presented in Table 13.6. For a service industry such as airlines the cost of services sold is included in the total operating expenses as shown in Table 13.6.

Table 13.6 Delta Air Lines Income Statement, 2015

Consolidated Statements of Operations	
Delta Air Lines, Inc.	
For the year ended December 31, 2015	
(in millions, except per share data)	
Operating Revenue:	
Passenger:	
Mainline	\$28,898
Regional carriers	5,884
Total passenger revenue	34,782
Cargo	813
Other	5,109
Total operating revenue	\$40,704
Operating Expense:	
Aircraft fuel and related taxes	(6,544)
Salaries and related costs	(8,776)

(continued)

Table 13.6 (Continued)

Consolidated Statements of Operations	
Delta Air Lines, Inc.	
For the year ended December 31, 2015	
Regional carrier expense	(4,241)
Aircraft maintenance materials and outside repairs	(1,848)
Depreciation and amortization	(1,835)
Contracted services	(1,848)
Passenger commissions and other selling expenses	(1,672)
Landing fees and other rents	(1,493)
Profit sharing	(1,490)
Passenger service	(872)
Aircraft rent	(250)
Restructuring and other items	(35)
Other	(1,998)
Total operating expense	(\$32,902)
Operating Income	\$7,802
Other Expense:	
Interest expense, net	(481)
Loss on extinguishment of debt	
Miscellaneous, net	(164)
Total other expense, net	(\$645)
Income before Income Taxes	\$7,157
Income tax (provision) benefit	(2,631)
Net Income	\$4,526
Earnings Per Share	\$5.68
Diluted Earnings Per Share	\$5.63
Cash Dividends Declared Per Share	\$0.45

Source: Compiled by the authors from financial statements

For the airline industry, the major sources of income are:

- ◆ Passenger revenue
- ◆ Cargo revenue
- ◆ Ancillary revenue
- ◆ Other income

In the airline industry most of the revenue comes from ticket sales. Other types of revenue include cargo revenue and ancillary revenue. Air cargo is an important revenue source for airlines. Over the next 20 years, world air cargo traffic will grow 4.7 percent per year.⁸ Most of the cargo carried is on large wide-body aircraft such as the B767, B777, B747, A330, and A380. Narrow-body aircraft have significant limitations on the volume and the type of cargo they can carry. Recently, airlines have increasingly depended on revenue from other sources rather than ticket prices. Another source of revenue, termed ancillary revenue, is the revenue airlines make from the sales of in-flight services.⁹

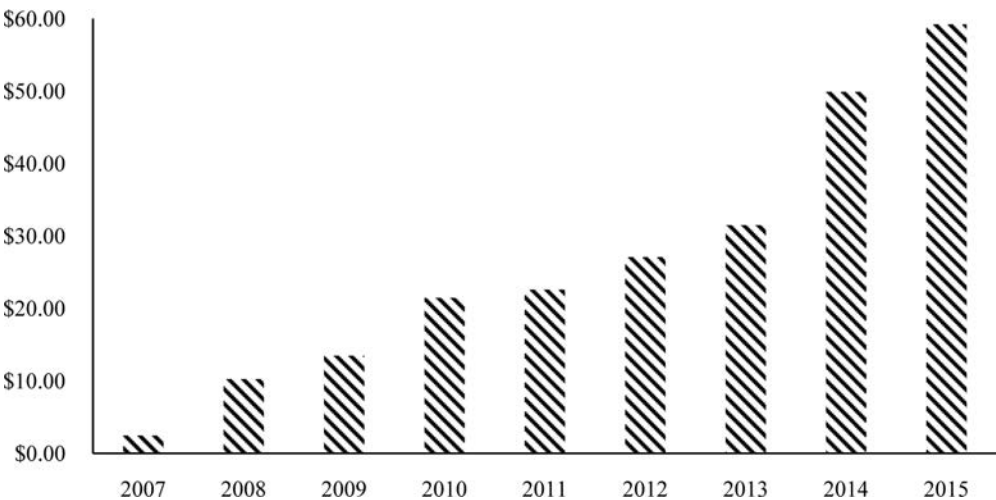
Ancillary revenues are increasingly becoming a major source of income for airlines around the world. The growth in the amount earned as ancillary revenues is staggering over the past seven to eight years.

The consistent growth in this category of revenue suggests that numerous airlines are slowly realizing the potential of this source of revenue and beginning to utilize it. A significant change has been seen in pricing of passenger tickets which used to be mostly bundled a few decades ago, meaning the purchase of a ticket included many services added into it such as luggage charges and meals. Now, airlines are focused on unbundling these charges when offering a ticket. There

Table 13.7 Annual Financial Disclosures of Ancillary Revenue¹⁰

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015
USD Billions	\$2.45	\$10.25	\$13.47	\$21.46	\$22.60	\$27.10	\$31.50	\$49.90	\$52.9

Figure 13.1 Ancillary Revenue for Global Airlines (in USD billions)



have also been some innovative methods developed to generate ancillary revenue, such as the “seat-to-seat” service offered by Virgin America Airlines, which enables a passenger to buy a drink for a co-passenger in-flight by looking at a digital seat map. This trend toward increasing revenue from ancillary services is expected to increase in the coming years.

Example 13.6

MyJet Airlines has \$5,000,000 of earnings before interest and taxes at the year end. Interest expenses for the year were \$100,000. The airline expects to distribute \$1,000,000 in dividends. Calculate the earnings after taxes for the firm assuming a 30 percent tax on ordinary income. The solution is shown in Table 13.8.

Table 13.8 MyJet Airlines Financial Data

Earnings before interest and taxes	\$1,000,000
Less: Interest	\$100,000
Earnings before taxes	\$900,000
Less: Taxes (30%)	\$270,000
Earnings after taxes	\$630,000

Example 13.7

General Engine Manufacturing Company (GEM) has announced that the company made \$50 million profit. The company has paid \$30 million in operating expenses and \$5,000,000 in interest expenses. The company is subject to a 30 percent tax rate and has declared \$5,000,000 total preferred stock dividends.

- How much are the earnings available for common stockholders?
- Compute the increased retained earnings if the company was to declare a \$5.00 dividend per share of common stock. The company has 1,000,000 shares of common stock outstanding.

Part a

Table 13.9 General Engine Manufacturing Company Financial Data (a)

Gross profits	\$50,000,000
Less: operating expenses	(\$30,000,000)
Operating profits	\$20,000,000
Less: interest	(\$5,000,000)
Net profits before taxes	\$15,000,000
Less: taxes (30%)	(\$4,500,000)
Net profits after taxes	\$10,500,000
Less: preferred stock dividend	\$5,000,000
Earnings available for common stock	\$5,500,000

Part b

Table 13.10 General Engine Manufacturing Company Financial Data (b)

Earnings available for common stock	\$5,500,000
Dividend (\$5.00), 1,000,000 shares	\$5,000,000
Retained earnings	\$500,000

The Balance Sheet: Measuring a Firm's Book Value

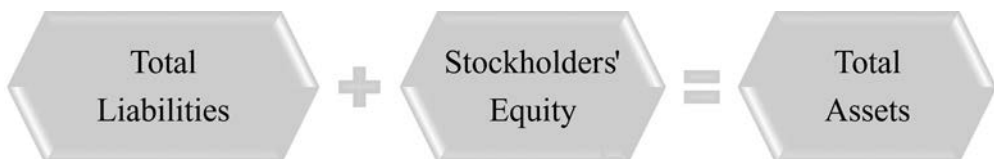
The balance sheet is a typical snapshot of a company's finances at a particular point in time. It represents the balance between what the company owns and what the company owes. In addition, looking at a balance sheet, three of the major components are total assets, total liability, and stockholders' equity. The left side of the balance sheet lists everything the company owns (total assets). Total assets include current assets and non-current (fixed) assets. Non-current assets consist of plants, equipment, machinery, and non-tangible assets, where non-tangible assets are those that can't be touched but nevertheless exist and have value, such as trademarks and patents.¹¹ In 1993, Pan Am's name was sold for \$1.3 million after the airline filed for bankruptcy.¹² The right side of the balance sheet lists the claims against the company, called total liabilities and shareholders' equity.

A balance sheet is a stock measure statement, implying that each value on a balance sheet is the value of that account on the specific date associated with the balance sheet. The value of the account (particularly liquid assets and liability accounts) at a later date may differ substantially from what has been reported on the balance sheet.

The asset component of the balance sheet for Delta Air Lines is shown in Table 13.11a.

The liability and stockholders' equity components of the balance sheet for Delta Air Lines is shown in Table 13.11b.

The airline industry is widely known for its poor profitability. A subsequent impact of this lack of profitability can be seen on the balance sheet in some cases.



Current Assets

Braniff Inc. announced Monday it was ceasing all scheduled passenger service immediately because a severe cash shortage doomed the airline's bankruptcy reorganization plan.

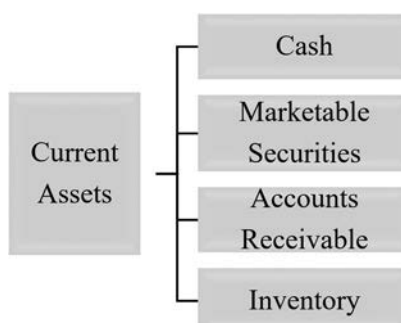
Associated Press, November 7, 1989

Table 13.11a Balance Sheet for Delta Air Lines

ASSETS			
Current Assets:		December 31, 2015	December 31, 2014
	Cash and cash equivalents	\$1,972	\$2,088
	Short-term investments	1,465	1,217
	Accounts receivable, net	2,020	2,297
	Hedge margin receivable	119	925
	Fuel inventory	379	534
	Expendable parts and supplies inventories, net	318	318
	Hedge derivatives asset	1,987	1,078
	Prepaid expenses and other	796	701
	Total current assets	9,056	9,158
Property and Equipment and Others:			
	Property and equipment, net	23,039	21,929
	Goodwill	9,794	9,794
	Identifiable intangibles, net	4,861	4,603
	Deferred income taxes, net	4,956	7,595
	Other non-current assets	1,428	926
	Total other assets	21,039	22,918
Total Assets	Total assets	\$53,134	\$54,005

Source: Compiled by the authors from the company's financial statements

The first subsection in the balance sheet is the assets section. This can be split into current and non-current assets. Current assets are those which are considered to be more liquid and can be converted into cash with substantial ease. The order in which the various entities are presented is based upon the liquidity of the assets. The most liquid asset being first and the non-current assets, i.e., fixed assets, being last.



Cash and Cash Equivalents

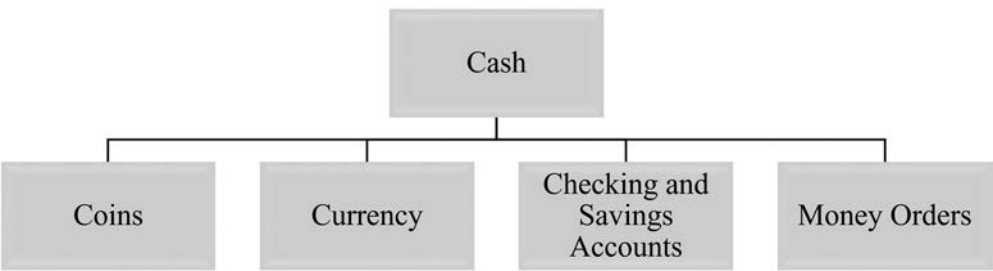
Items on the balance sheet are listed in order of liquidity and the most liquid asset is cash and cash equivalents. Companies hold cash for various reasons and in various forms, some of which are restricted for special purposes. In the United States, if a company has foreign bank accounts, these accounts must be translated into dollars

Table 13.11b Balance Sheet for Delta Air Lines

Liabilities and Stockholders' Equity			
Current Liabilities:		December 31, 2015	December 31, 2014
	Current maturities of long-term debt and capital leases	\$1,563	\$1,184
	Air traffic liability	4,503	4,296
	Accounts payable	2,743	2,622
	Accrued salaries and related benefits	3,195	2,266
	Hedge derivatives liability	2,581	2,772
	Frequent flyer deferred revenue	1,635	1,580
	Other accrued liabilities	1,306	2,127
	Total current liabilities	17,526	16,847
Non-current Liabilities:			
	Long-term debt and capital leases	6,766	8,477
	Pension, postretirement and related benefits	13,855	15,138
	Frequent flyer deferred revenue	2,246	2,602
	Other non-current liabilities	1,891	2,128
	Total non-current liabilities	24,758	28,345
Commitments and Contingencies			
Stockholders' Equity:			
	Common stock at \$0.0001 par value; 1,500,000,000 shares authorized, 799,850,675 and 845,048,310 shares issued at December 31, 2015 and 2014, respectively		
	Additional paid-in capital	10,875	12,981
	Retained earnings	7,623	3,456
	Accumulated other comprehensive loss	-7,275	-7,311
	Treasury stock	-373	-313
	Total stockholders' equity	10,850	8,813
	Total liabilities and stockholders' equity	\$53,134	\$54,005

Source: Compiled by the authors from the company's financial statements

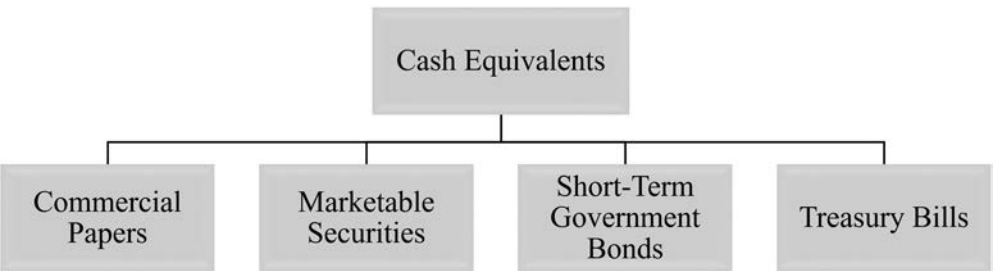
at the prevailing exchange rate on the balance sheet statement date. Shortage of available cash and cash equivalents and short-term investments to meet liquidity could force a company into a complex bankruptcy. Eastern Air Lines, Continental Airlines, Mesaba, and many other airlines succumbed to cash shortage and were forced into bankruptcy.¹³ Boeing Company, on December 31, 2015, had cash and cash equivalents of \$1,972 million.



Marketable Securities (Short-term Investments)

Some companies temporarily invest excess cash in marketable securities such as certificates of deposits (CDs), Treasury bills, notes, and bonds, other US government securities, high-grade corporate commercial paper, or bankers' acceptances. These short-term investments can earn interest income until the cash is needed in the business. Appropriate marketable securities are highly liquid and maintain a stable market value. It is critical that companies choose the right type of security to make an investment, so that it can ensure a good return.

During the September 2014 quarter 3, Boeing Company modified their approach to managing short-term investments by investing \$1.5 billion of cash reserves in externally managed investment accounts. Rather than let the cash remain idle in its bank accounts, Boeing Company invested in marketable securities. These new investments were made up of US government and agency securities, asset- and mortgage-backed securities, corporate obligations, and other fixed-term securities.¹⁴ Table 13.11a shows that Delta Air Lines had short-term investments worth \$1,465 million as of December 31, 2015.



Accounts Receivable

The sale of a product or service on credit is shown on the balance sheet as accounts receivable. Credit sales often stipulate payment terms that allow the purchaser to pay

within a specified time and may offer a discount as an incentive for early payment. Receivables are defined as the amounts to be collected from travel agencies, corporate agencies, accounts, or governmental units. Accounts receivable remain on the balance sheet until they are collected.

Some amount of accounts receivable will never be collected and therefore, constitute a bad debt, or loss. A company calculates the percentage of possible bad debt (from prior actual experience) and creates a corresponding reserve that is deducted from its gross accounts receivable. This reserve is often referred to as an allowance for doubtful accounts. Thus,

$$\text{Net Accounts Receivable} = \text{Gross Accounts Receivable} - \text{Allowance for Doubtful Accounts}$$

For an airline, passenger tickets are primarily sold on credit; hence, accounts receivable is an important asset, as seen in the balance sheet of Delta Air Lines for 2015, where the accounts receivable are \$2,020 million (Table 13.11a).

Inventories

As Boeing continues to transform itself into an integrator of large parts and systems and implements lean manufacturing principles, it is relying more heavily on its supply base to achieve the results customers demand. This reliance is causing a fundamental shift in the role of the supplier to Boeing. And Boeing suppliers say they are ready for the challenge.

The Boeing Company, 2005

Inventories correspond to products that need to be stored to facilitate future operation when necessary. For some industries, inventory can be among a company's most expensive assets. For an airline, the main types of inventory constitute fuel inventory and expendable parts and supplies inventory. For Delta Air Lines, the net inventory amounted to \$697 million in the year 2015, as can be seen from Table 13.11a. On the aircraft manufacturing side, inventory comprises the total number of aircraft in the process of production and their spare parts. Inventory may include:

- ◆ Raw material
- ◆ Parts
- ◆ Work-in-progress (WIP)
- ◆ Finished goods

For example, Boeing had a net inventory of \$47,257 million for the year 2015. This shows the importance of inventory in the manufacturing industry.

Prepaid Expenses

The demand faced by aircraft and jet engine manufacturers for new orders is derived from the perceived future demand for commercial air activities. Generally, airlines are required to make prepayments to sign aircraft purchase agreements, then there

are structured payments of about 25 percent of the purchase price, in stages over an 18-month period with the remainder due at delivery. The actual price an airline pays for a given aircraft is not publicly announced, and most of the payments are proprietary information. Prepaid expenses refer to advance payments made in cash and recorded as assets until they are used or consumed. Prepaid expenses are costs that expire with the passage of time (i.e., rent and insurance) or through use (i.e., supplies). Prepaid expenses increase during a period because cash paid for expenses is higher than expenses reported on the accrual basis. Examples of prepaid expenses may include:

- ◆ Advance payments
- ◆ Fuel
- ◆ Insurance
- ◆ Legal fees
- ◆ Rent
- ◆ Supplies

Fixed Assets (Non-current Assets)

This is the second subsection under assets, consisting of equipment, buildings, vehicles, tools, computers, office equipment, leasehold improvements, furniture—in general any items of a fairly permanent nature that are required for the normal operation of a business. A major portion of an airline’s fixed assets consists of aircraft and other flight equipment. An airline puts a fixed asset on its balance sheet at historical cost, meaning that its stated value is whatever it costs the company to obtain the asset. The company then depreciates the asset, that is, reduces its reported value over the life of the asset. Accounting standards indicate that balance sheets report the value of fixed assets at book value. Book value is defined as the original historical cost (the purchase price paid by the company) minus allowable depreciation to date. Thus appearing as “Net” fixed assets on the balance sheet. Boeing Company’s fixed assets include:

- ◆ Long-term Investments
- ◆ Property Plant and Equipment
- ◆ Goodwill
- ◆ Intangible Assets
- ◆ Accumulated Amortization
- ◆ Other Assets
- ◆ Deferred Long-term Asset Charges

Property, Plant, and Equipment (PP&E)

For an airline, PP&E refers to the amount of long-lived non-current assets owned by the company and used in principal business operations. Most of the assets of airlines or of aircraft and jet engine manufacturers will fall into this category. These companies must continue to buy aircraft to continue their operations. As mentioned earlier, PP&E is shown net of depreciation when displayed on the balance sheet. It can be seen from Table 13.11a that Delta Air Lines had a net PP&E of \$23,039 million in the year 2015.

Other Long-term Assets

This mainly refers to investments in other privately or publicly held companies, as these are not easily liquefiable and have a life of certainly more than one year. Another type of long-term asset is the deferred tax which is the difference between actual taxes paid and those calculated prior to payment and recorded in the books. The value of a company's brand name, solid customer base, good customer relations, good employee relations and any patents or proprietary technology represent goodwill. Goodwill is considered an intangible asset because it is not a physical asset like buildings or equipment. Boeing Company had goodwill of \$9,794 million recorded for the year 2015 as seen in Table 13.11a.

Total Liabilities

Liabilities are the financial obligations of one business entity to other individuals or businesses. Liabilities have two main categories:

- ◆ Current Liabilities
- ◆ Long-term Liabilities

Current Liabilities

Liability represents the amount a company owes to all its creditors, investors, and shareholders. Current liabilities denote payments which are due within the next year, though the category can be divided into a few subsections. Delta Air Lines had total current liabilities worth \$17,526 million as of December 31, 2015 (Table 13.11b). There are three main categories of current liabilities:

- ◆ Accounts Payable
- ◆ Accrued Expenses
- ◆ Current Portions of Long-term Debt

Accounts Payable

One of the most important parts of current liabilities is accounts payable. Accounts payable represent purchases made on credit by the company from suppliers. They are included under current liabilities because they are expected to be settled by the end of the current year. Those dealing with accounts payable are responsible for examining all accounts, claims, and demands against the company and for making payments of all the legally incurred obligations. Accounts payable form the largest portion of current liabilities for an airline and can be seen from Table 13.11b, where Delta has accounts payable amounting to \$2,743 million in the year 2015.

Accrued Expenses

Accrued expenses represent specific direct and operating costs that a company has expensed on its income statement which are yet to be paid at the close of the reporting period. These expenses are usually paid at regular intervals and include such items as utilities, rent, wages and salaries, and taxes. For example, if wages are paid every three weeks and the firm's balance sheet is prepared in the middle of the pay period, wages owed as of that date would be reported as an accrued expense.

Current Portions of Long-term Debt

At times companies borrow long-term money on an installment basis, implying that the company makes periodic payments over the life of the loan that include principal reduction as well as interest. The current maturities of long-term debt represent the principal portion of these installment payments that is due over the next 12 months. The sum of the current portion of long-term debt and the rest of the long-term debt is the total long-term debt that a company has outstanding at any point in time. Entitling some debt as current is merely a transfer process. Delta Air Lines had current maturities of long-term debt at the end of 2015, worth \$1,563 million (Table 13.11b).

Non-current Liabilities

Non-current liabilities refer to payments which are not due within the next year or next reporting period and therefore, can be classified as non-current. In 2015, the total non-current liabilities for Boeing Company summed up to \$24,758 million.

Long-term Debt

Long-term debt represents liabilities with maturities in excess of one year. It is usually used to finance long-term assets such as land, buildings, and equipment. Occasionally, long-term debt is used to finance “permanent” current assets. Long-term debt includes capital lease obligations, corporate bonds, and other long-term leases. Excessive long-term debt and low liquidity are critical factors leading to an airline’s bankruptcy. A fairly large contributor to non-current liabilities is long-term debt which was \$6,766 million for Delta Air Lines for the year 2015 (Table 13.11b).

Other Long-term Liabilities

Some of the other liabilities not due over the immediate year include pension and post-retirement-related benefits for employees, and also a separate type of liability specific to airlines called frequent flyer deferred revenue, which represents the mileage credit an airline provides to passengers or sells to credit card companies and other third parties. Pension and post-retirement benefits are the biggest contributors to other long-term liabilities. For Boeing Company, for the year 2015, pension and post-retirement benefits amounted to \$13,855 million, which was more than 50 percent of the total non-current liabilities.

Stockholders’ Equity

Stockholders’ equity is one of the three main components of a corporation’s balance sheet. It is the difference between the total assets and the total liabilities and ideally represents what the owners of the company contribute to it.



Preferred Stock

Preferred stock or “preferred securities” share many characteristics of both a stock and a bond. Like bonds, they have a relatively high fixed-rate payment. Like common stock, they are generally listed on a stock exchange. A preferred stock is an equity that may or may not have maturity. Preferred stock can pay either a dividend or interest, usually quarterly or semi-annually, and represents a non-voting ownership in a company. These securities are called “preferred” because preferred shareholders have superior rights to assets and cash flows of a company versus common shareholders in the event of a bankruptcy or liquidation; however, bondholders’ claims are superior to those of preferred shareholders. Preferred stock is usually cumulative, meaning that skipped payments accrue and must be paid when earnings allow.

Common Stock and Additional Paid-in Capital

Common stock refers to the shares a company offers on its ownership. Common stock is divided into two components: common stock at par value and additional paid-in capital (or, capital surplus). The first component can be used to determine the number of shares currently outstanding. The second component represents the additional money (over and above par value) generated when the company actually sold the stock.

Delta Air Lines had 799,850,675 shares of common stock authorized on December 31, 2015 at a par value of \$0.0001 and made an additional paid-in capital of \$10,875 million (Table 13.11b).

Retained Earnings

Retained earnings denote the cumulative total of all net income that has been reinvested into the company. Many companies retain some of their annual profit to fund the expansion (replacement) of assets to reduce their reliance on outside capital markets.

Hence it can be represented as:



Retained earnings on the balance sheet refer to the cumulative retained earnings of the previous years along with what has been retained in the current reporting period.

Delta Air Lines reported retained earnings of \$7,623 million as of December 31, 2015 (Table 13.11b).

Example 13.8

Metro Jet’s retained earnings account had a beginning balance of \$25,000,000 at the end of 2016. During 2017, Metro Jet earned \$75,000,000 of net income and distributed \$15,000,000 of dividends among common shareholders. Metro Jet’s retained earnings account at the end of 2017 is:

Table 13.12 Metro Jet Statement of Retained Earnings

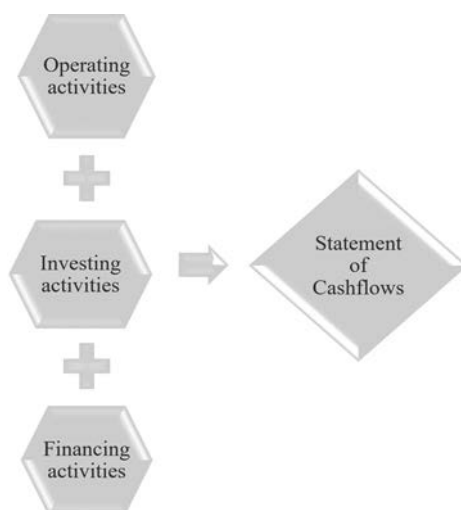
Metro Jet Statement of Retained Earnings For the year ended December 31, 2017	
Beginning balance	\$25,000,000
+ Net income	\$75,000,000
– Dividends	\$15,000,000
Ending balance	\$85,000,000

Statement of Cash Flow

The statement of cash flows reports the *cash* generated and used during the time interval specified. The period of time that the statement covers is chosen by the company. For example, the heading may state “The Fiscal Year Ending December 31, 2015” or “The three months ending September 30, 2015.”

Because the income statement is prepared under the accrual basis of accounting, the revenues stated may not have been collected. Similarly, the expenses reported on the income statement might not have been paid. The cash flow statement primarily integrates all that information in order to depict the operation of a firm purely based on the inflow and outflow of cash. As a result, investors frequently utilize this important financial statement.

It is mainly divided into three sections as explained below.



Cash Flow from Operating Activities

This section consists of the revenue-generating activities of a business. Examples of operating activities are cash received and disbursed for product sales, royalties, commissions, fines, lawsuits, supplier and lender invoices, and payroll.

There are two ways to present the cash flow from operating activities, which are the direct method and the indirect method. The direct method requires you to present cash flow information that is directly associated with the items triggering cash flows. But most companies do not maintain a record with information in that format. Hence, they use the indirect method in which the statement begins with the

net income or loss reported on the company's income statement, and then makes a series of adjustments or alterations to this figure to arrive at the amount of net cash provided by operating activities. Some of these adjustments include elimination of non-cash expenses such as depreciation and amortization. After all the necessary changes are made, the net cash provided by operating activities is found. For the Boeing Company, the net cash flow from operating activities for the year 2015 was found to be \$7,927 million.

Cash Flow from Investing Activities

Cash flow from investing activities includes the changes caused in cash flow due to the purchase and sale of long-term assets (fixed assets). It also includes cash inflow from income in investments and cash inflow from disposal of investments. Table 13.13 shows that the cash flow used in investing activities for Delta Air Lines for the year 2015 was \$3,955 million. Because it is a cash outflow, it is represented as a negative value.

Cash Flow from Financing Activities

This section constitutes activities that will alter the equity or borrowings of a business. The entries included are the sale of company shares, issuance of debentures and bank loans, the repurchase of shares, and dividend payments. In 2015, Boeing Company used up \$4,088 million in financing activities and, like investing activities, because it is a cash outflow, it is represented as a negative value.

The sum of the three aforementioned cash flows results in the net increase/decrease in cash and cash equivalents for the corresponding year. For 2015, Delta Air Lines had a net cash decrease of \$166 million. This reduced the beginning cash balance of \$2,088 million to \$1,972 million (Table 13.13).

Southwest Airlines during the years 2008, 2009, and 2010 had an interesting look to their statement of cash flows. Southwest Airlines was one of the very few airlines in the US that was able to sustain its operations during the global economic crisis post 2008. A list of the statement of cash flows (in \$millions) during that period is given in Table 13.14.

According to the Southwest Airline report, the company received \$3,238,000,000 cash from operating activities but spent \$1,913,000,000 to purchase new property and equipment. Southwest supplemented the cash from operating activities by issuing long-term debt and additional shares of common stock (financing activities). It used excess cash to repurchase stock, pay down debt, and pay out dividends. In total, it generated more cash from operating activities than it paid for investing and financing activities resulting in a net increase in cash for 2015.

The variations in the net change in cash and cash equivalents over the three years show the progress of Southwest Airlines and the stability they were able to maintain during the period of the global economic crisis. An important part of that stability is attributed to the fuel hedging activities which Southwest was very actively part of, enabling them to make considerable profits from their derivatives. In 2008, Southwest Airlines also issued \$1,000 million worth of long-term debt which enabled a large positive cash flow from the financing activities, reducing the overall impact of net change in cash and cash equivalents to a smaller negative value.

Table 13.13 Delta Air Lines, 2015 Consolidated Statements of Cash Flows

	Year Ended December 31,	
(in millions)	2015	2014
Cash Flows From Operating Activities:		
Net income	\$ 4,526	\$ 659
Adjustments to reconcile net income to net cash provided by operating activities:		
Depreciation and amortization	1,835	1,771
Hedge derivative contracts	(1,366)	2,186
Deferred income taxes	2,581	414
Pension, postretirement and postemployment payments greater than expense	(1,013)	(723)
Restructuring and other	35	758
Extinguishment of debt	22	268
Equity investment (earnings) loss	(35)	106
SkyMiles used pursuant to advance purchase under American Express Agreements	–	–
Changes in certain assets and liabilities:		
Receivables	(56)	(302)
Restricted cash and cash equivalents	7	62
Fuel inventory	155	172
Hedge margin	806	(922)
Prepaid expenses and other current assets	(102)	58
Air traffic liability	207	174
Frequent flyer deferred revenue	(301)	(238)
Profit sharing	734	264
Accounts payable and accrued liabilities	(201)	(36)
Other, net	93	276
Net cash provided by operating activities	\$7,927	\$4,947
Cash Flows From Investing Activities:		
Property and equipment additions:		
Flight equipment, including advance payments	(2,223)	(1,662)
Ground property and equipment, including technology	(722)	(587)
Purchase of equity investments	(500)	–

Source: Compiled by the authors from the company's financial statements

Purchase of short-term investments	(998)	(1,795)
Redemption of short-term investments	739	1,533
Acquisition of London-Heathrow slots	(276)	–
Other, net	25	48
Net cash used in investing activities	(\$3,955)	(\$2,463)
Cash Flows From Financing Activities:		
Payments on long-term debt and capital lease obligations	(2,558)	(2,928)
Repurchase of common stock	(2,200)	(1,100)
Cash dividends	(359)	(251)
Fuel card obligation	(340)	(41)
Payments on hedge derivative contracts	(71)	–
Proceeds from hedge derivative contracts	429	–
Proceeds from long-term obligations	1,038	1,020
Other, net	(27)	60
Net cash used in financing activities	(\$4,088)	(\$3,240)
Net (decrease) increase in cash and cash equivalents	(116)	(756)
Cash and cash equivalents at beginning of period	2,088	2,844
Cash and cash equivalents at end of period	\$1,972	\$2,088

Table 13.14 Statements of Cash Flow Highlights, Southwest Airlines

Year	2015	2014	2013	2012
Cash flow from operating activities	\$3,238,000	\$2,902,000	2,477,000	\$2,064,000
Cash flow from investing activities	–\$1,913,000	–\$1,727,000	–\$1,384,000	–\$833,000
Cash flow from financing activities	–\$1,248,000	–\$1,248,000	–\$851,000	–\$947,000
Net change in cash and cash equivalents	\$301,000	–\$73,000	\$242,000	\$284,000
Year	2011	2010	2009	2008
Cash flow from operating activities	\$1,356,000	\$1,561,000	\$985,000	–\$1,521,000
Cash flow from investing activities	\$1,022,000	–\$1,265,000	–\$1,569,000	–\$978,000
Cash flow from financing activities	–\$766,000	–\$149,000	\$330,000	\$1,645,000
Net change in cash and cash equivalents	–\$432,000	\$147,000	–\$254,000	–\$845,000

All numbers in thousands

Source: Southwest Airlines, Annual Reports

Measuring Free Cash Flow

Free cash flow (FCF) is cash flow available for distribution among all the security holders of an organization, thus, measuring the ease with which businesses can grow. They include debt holders, equity holders, preferred stock holders, and so on. There are three different methods for calculating free cash flows.

- ♦ $FCF = EBIT \times (1 - \text{Tax rate}) + \text{Depreciation \& Amortization} - \text{Changes in Working Capital} - \text{Capital Expenditure}$

- ♦ $FCF = \text{Net Profit} + \text{Interest Expense} - \text{Net Capital Expenditure (CAPEX)} - \text{Net Change in Working Capital} - \text{Tax Shield on Interest Expense}$

Where:

$$\text{Net Capital Expenditure (CAPEX)} = \text{CAPEX} - \text{Depreciation \& Amortization},$$

$$\text{Tax Shield} = \text{Net Interest Expense} \times \text{Effective Tax Rate}$$

- ♦ When Profit after Tax and Debt/Equity ratio (d) is available:

$$FCF = \text{Profit after Tax} - \text{Changes in Capital Expenditure} \times (1-d) + \text{Depreciation \& Amortization} \times (1-d) - \text{Changes in Working Capital} \times (1-d)$$

Example 13.9

DirectJet has an Earnings Before Interest and Taxes (EBIT) of \$2,500 million and a depreciation and amortization expense of \$1,600 million for the year ending December 31, 2016. If DirectJet purchased property and equipment worth \$500 million during the same period and it falls under the 35 percent tax bracket, determine the Free Cash Flow (FCF) for DirectJet for the year 2016. Assume the net working capital for the considered period to be \$300 million.

In order to determine the Free Cash Flow, we use the formula:

$$FCF = EBIT \times (1 - \text{Tax rate}) + \text{Depreciation \& Amortization} - \text{Changes in Working Capital} - \text{Capital Expenditure}$$

$$FCF = \$2,500 \times (1 - 0.35) + \$1,600 - \$300 - \$500$$

$$FCF = \$2,425 \text{ million}$$

Hence, DirectJet had a Free Cash Flow of \$2,425 million for the year ending December 31, 2016.

Common Size Financial Statements

A common size financial statement is the one that displays all items in the financial statement as percentages of a common base figure. This type of financial statement facilitates easy analysis between companies or between time periods of a company. The bias existing when comparing companies of different sizes for a given time period is largely eliminated when they are compared as percentages of a common base figure.

Common Size Income Statement

Generally, the base figure utilized for an income statement is the total revenue or total sales. All the items in the income statement are presented as a percentage of total revenue. Table 13.15 provides an example of a common size income statement for

Table 13.15 Delta Air Lines Common Size Income Statement, 2015

Consolidated Statements of Operations		
Delta Air Lines, Inc.		
For the year ended December 31, 2015		
(in millions of US \$, except per share data)		
Operating Revenue:		
Passenger:		
Mainline	\$28,898	71.00%
Regional carriers	5,884	14.46%
Total passenger revenue	34,782	85.45%
Cargo	813	2.00%
Other	5,109	12.55%
Total operating revenue	\$40,704	100.00%
Operating Expense:		
Aircraft fuel and related taxes	6,544	16.08%
Salaries and related costs	8,776	21.56%
Regional carrier expense	4,241	10.42%
Aircraft maintenance materials and outside repairs	1,848	4.54%
Depreciation and amortization	1,835	4.51%
Contracted services	1,848	4.54%
Passenger commissions and other selling expenses	1,672	4.11%
Landing fees and other rents	1,493	3.67%
Profit sharing	1,490	3.66%
Passenger service	872	2.14%
Aircraft rent	250	0.61%
Restructuring and other items	35	0.09%
Other	1,998	4.91%
Total operating expense	\$32,902	80.83%

(continued)

Table 13.15 (continued)

Consolidated Statements of Operations		
Delta Air Lines, Inc.		
For the year ended December 31, 2015		
Operating Income	\$7,802	19.17%
Other Expense:		
Interest expense, net	(481)	−1.18%
Miscellaneous, net	(164)	−0.40%
Total other expense, net	(645)	−1.58%
Income before income taxes	7,157	17.58%
Income tax (provision) benefit	(2,631)	−6.46%
Net Income	\$4,526	11.12%

Source: Compiled by the authors from the company's financial statements

Boeing Company, for the year 2015. Forming a common size income statement also inadvertently calculates the various margins associated with the firm. For example, the net profit margin is simply the net income divided by sales. For Boeing Company as seen in this table, the net profit margin is 11.12 percent.

Example 13.10

Phoenix Aviation is a small manufacturer of landing gear for general aviation aircraft. Phoenix believes a market exists for both general aviation and commercial aircraft. The company has hired a consultant to prepare a common size income statement. Table 13.16 shows the company's income statement.

The right side of the income statement, which shows each expense as a percentage of total revenue, is a common size income statement. Phoenix Aviation has a total revenue of \$250,000, and shows operating expenses of \$75,000, then operating expenses are $\$75,000 / \$250,000 = 30$ percent of total revenue, and the operating profit is 50 percent of the revenue.

Common Size Balance Sheet

The base figure utilized for the balance sheet is the total assets which will also equal the total liabilities and stockholders' equity. Table 13.17 gives an account of the common size balance sheet for Boeing

A common size balance sheet expresses each item on the balance sheet as a percentage of total assets.

Table 13.16 Phoenix Aviation Income Statements

Phoenix Aviation				
	Income Statement		Common Size Income Statement	
Total revenue		\$250,000		100%
Cost of goods sold		\$50,000		
Gross profit		\$200,000		80%
Operating expenses				
Salaries	\$25,000		10%	
Rent	\$25,000		10%	
Utilities	\$12,500		5%	
Depreciation	<u>\$12,500</u>		<u>5%</u>	
Total operating expenses		\$75,000		30%
Operating profit (EBIT)		\$125,000		50%
Interest expense		\$25,000		10%
Earnings before tax (EBT)		\$100,000		40%
Taxes		\$25,000		10%
Net income		\$75,000		30%

Table 13.17 Delta Air Lines Common Size Balance Sheet, 2015

Consolidated Balance Sheet		
Delta Air Lines, Inc.		
For year ended December 31, 2015		
(in millions of US \$)		
ASSETS		
Current Assets:		
Cash and cash equivalents	\$1,972	3.71%
Short-term investments	1465	2.76%
Accounts receivable, net	2020	3.80%
Hedge margin receivable	119	0.22%
Fuel inventory	379	0.71%
Expendable parts and supplies inventories, net	318	0.60%

(continued)

Table 13.17 (Continued)

Consolidated Balance Sheet		
Delta Air Lines, Inc.		
For year ended December 31, 2015		
Hedge derivatives asset	1987	3.74%
Prepaid expenses and other	796	1.50%
Total current assets	\$9,056	17.04%
Property and Equipment and Others:		
Property and equipment, net	23039	43.36%
Goodwill	9794	18.43%
Identifiable intangibles, net	4861	9.15%
Deferred income taxes, net	4956	9.33%
Other non-current assets	1428	2.69%
Total other assets	\$21,039	39.60%
Total assets	\$53,134	100%
LIABILITIES AND STOCKHOLDERS' EQUITY		
Current Liabilities:		
Current maturities of long-term debt and capital leases	1563	2.94%
Air traffic liability	4503	8.47%
Accounts payable	2743	5.16%
Accrued salaries and related benefits	3195	6.01%
Hedge derivatives liability	2581	4.86%
Frequent flyer deferred revenue	1635	3.08%
Other accrued liabilities	1306	2.46%
Total current liabilities	\$17,526	32.98%
Non-current Liabilities:		
Long-term debt and capital leases	6766	12.73%
Pension, post-retirement and related benefits	13855	26.08%

Frequent flyer deferred revenue	2246	4.23%
Other non-current liabilities	1891	3.56%
Total non-current liabilities	\$24,758	46.60%
STOCKHOLDERS' EQUITY:		
Common stock at \$0.0001 par value; 1,500,000,000 shares authorized, 799,850,675 and 845,048,310 shares issued at December 31, 2015 and 2014, respectively		
Additional paid-in capital	10875	20.47%
Retained earnings	7623	14.35%
Accumulated other comprehensive loss	-7275	-13.69%
Treasury stock	-373	-0.70%
Total stockholders' equity	\$10,850	20.42%
Total liabilities and stockholders' equity	\$53,134	100.00%

Source: Compiled by the authors from the company's financial statements

Company, for the year 2015. The common size analysis applied to a balance sheet lends insight into a firm's capital structure and how it compares to rivals. An investor can also look to determine an optimal capital structure for an industry and compare it to the firm being analyzed. It provides a perspective on whether debt is too high, excess cash is being retained on the balance sheet, or inventories are growing too high. The goodwill level on a balance sheet also helps indicate the extent to which a company has relied on acquisitions for growth. The current liabilities for Boeing Company are 32.98 percent of the Total Assets and the goodwill is at 18.43 percent.

Summary

Financial statements provide business owners with the necessary information for determining how their company performs during a given period of time. Financial statements are comprised of four main components: the balance sheet, the income statement, the cash flow statement, and the statement of retained earnings. Prospective investors, financial managers, vendors, creditors and other stakeholders use financial statements to measure the financial performance of a company. Public companies are required by law to publish the primary financial statements in an Annual Report to Shareholders. When you consider comparing the results of different companies, their financial statements may not provide accurate information, because the entities may be of different sizes, operate under different jurisdictions, and have different business plans. For example, regional airliners typically seat fewer than 100 passengers and

operate a short haul market, naturally a higher unit cost than network airlines. To compare the performance of distinct companies such as major airlines to regional airlines or to low-cost carriers, one can use the common size financial statements. The common size financial statement is the one that displays all items in the financial statement as percentages of a common base figure. This type of financial statement facilitates easy analysis between companies or between time periods of a company. The bias existing when comparing companies of different sizes for a given time period is largely eliminated when they are compared as percentages of a common base figure.

Discussion Questions and Problems

- 13.1 Get Airbus and Boeing companies' latest financial statements. Which of the current assets is likely to be considered the most liquid?
- 13.2 Analyze the latest financial statements of Airbus Company and decide whether to buy its capital stock or, possibly, loan money to the company. If you have been offered a job at the company, would you accept the job or take a job offer from another organization? Explain your answers.
- 13.3 What types of financial data will be available on a typical income statement of a business like Pratt and Whitney, Rolls-Royce, or GE Capital?
- 13.4 DirectJet is a firm with \$800 million total revenues, \$700 million cost of goods sold, \$10,000,000 depreciation expense, and \$2,400,000 interest expense. Find the company's operating profit and net income.
- 13.5 Determine the method or methods of depreciation that Ryanair Airlines uses to depreciate its property, plant, and equipment.
- 13.6 Analyze the information disclosed in the most recent Air France Balance Sheet on property, plant, and equipment. Recommend additional data that Air France could include that would be useful to potential investors and creditors.
- 13.7 Access the most recent financial statements and use the formulas provided in your textbook to calculate the following ratios for Ryanair. (You may have to search around the Profit & Loss Statement and Balance Sheet to find the appropriate values to plug in!)
 - a. Gross profit margin
 - b. Operating profit margin
 - c. Current ratio
 - d. Debt to assets ratio
 - e. Total assets turnover
 - f. Average collection period
- 13.8 Under MACRS, an aircraft, which originally cost \$20,000,000, incurred \$5,000,000 repair and overhaul expenses, and has an estimated salvage value of \$5,000,000, is being depreciated using a 10-year normal recovery period. What is the annual depreciation amount?

- 13.9** Fun Air had a cash flow from operating activities of \$20,000,000, cash flow from investment activities of $-\$8,000,000$, and cash flow from financing activities of \$18,000,000 as of December 31, 2016. What is the impact of these activities on the cash flows?
- 13.10** Fun Air has just ended the calendar year with a total revenue \$30,000,000 of merchandise purchased during the year at a total cost of \$12,500,000. Although the firm paid in full for the merchandise during the year, it has yet to collect \$2,000,000 at year end from the customer. Depreciation and amortization for the year is \$3,500,000. The net profit and the net cash provided by operating activities for the year are_____.

Appendix—US Tax Rates for 2016

Single:

Taxable Income	Tax Rate
\$0—\$9,275	10%
\$9,276—\$37,650	\$927.50 plus 15% of the amount over \$9,275
\$37,651—\$91,150	\$5,183.75 plus 25% of the amount over \$37,650
\$91,151—\$190,150	\$18,558.75 plus 28% of the amount over \$91,150
\$190,151—\$413,350	\$46,278.75 plus 33% of the amount over \$190,150
\$413,351—\$415,050	\$119,934.75 plus 35% of the amount over \$413,350
\$415,051 or more	\$120,529.75 plus 39.6% of the amount over \$415,050

Married Filing Jointly or Qualifying Widow(er):

Taxable Income	Tax Rate
\$0—\$18,550	10%
\$18,551—\$75,300	\$1,855 plus 15% of the amount over \$18,550
\$75,301—\$151,900	\$10,367.50 plus 25% of the amount over \$75,300
\$151,901—\$231,450	\$29,517.50 plus 28% of the amount over \$151,900
\$231,451—\$413,350	\$51,791.50 plus 33% of the amount over \$231,450
\$413,351—\$466,950	\$111,818.50 plus 35% of the amount over \$413,350
\$466,951 or more	\$130,578.50 plus 39.6% of the amount over \$466,950

Married Filing Separately:

Taxable Income	Tax Rate
\$0—\$9,275	10%
\$9,276—\$37,650	\$927.50 plus 15% of the amount over \$9,275
\$37,651—\$75,950	\$5,183.75 plus 25% of the amount over \$37,650
\$75,951—\$115,725	\$14,758.75 plus 28% of the amount over \$75,950
\$115,726—\$206,675	\$25,895.75 plus 33% of the amount over \$115,725
\$206,676—\$233,475	\$55,909.25 plus 35% of the amount over \$206,675
\$233,476 or more	\$65,289.25 plus 39.6% of the amount over \$233,475

Head of Household:

Taxable Income	Tax Rate
\$0—\$13,250	10%
\$13,251—\$50,400	\$1,325 plus 15% of the amount over \$13,250
\$50,401—\$130,150	\$6,897.50 plus 25% of the amount over \$50,400
\$130,151—\$210,800	\$26,835 plus 28% of the amount over \$130,150
\$210,801—\$413,350	\$49,417 plus 33% of the amount over \$210,800
\$413,351—\$441,000	\$116,258.50 plus 35% of the amount over \$413,350
\$441,001 or more	\$125,936 plus 39.6% of the amount over \$441,000

For Corporations:

If Taxable Income is			
Over ---	But Not over --	Tax is:	Of the amount over
\$0	\$50,000	15%	\$50,000
\$50,000	\$75,000	\$7500 + 25%	\$25,000
\$75,000	\$100,000	\$13,750 + 34%	\$25,000
\$100,000	\$335,000	\$22,250 + 39%	\$235,000
\$335,000	\$10,000,000	\$113,900 + 34%	\$9,665,000
\$10,000,000	\$15,000,000	\$3,400,000 + 35%	\$5,000,000
\$15,000,000	\$18,333,333	\$5,150,000 + 38%	\$3,333,333
\$18,333,333	-----	35%	

Source: IRS Official Website

Notes

- 1 The Airbus Group was known as EADS (European Aeronautic Defense and Space Company) prior to the group's restructuring in January 2014.
- 2 The Boeing Company (2014). Long-term Market. Current Market Outlook 2015–2034.
- 3 KPMG (2007). Components of Aircraft Acquisition Cost, Associated Depreciation and Impairment Testing in the Global Airline Industry, p.10.
- 4 *The New York Times*, December 15, 1992.
- 5 IRS Publication on Net Operating Loss (NOL), <https://www.irs.gov/publications/p536/ar02.html>. Accessed March 23, 2016.
- 6a Boeing Quarterly Report, 2016
- 6b Boeing Quarterly Report, 2015.
- 7 Sky West, Inc. Announces Authorization for Stock Repurchases and Declares 80th Consecutive Quarterly Dividend. PR Newswire, May 7, 2015.
- 8 Boeing World Air Cargo Forecast, 2014–2015.
- 9 Airfares Slip 5.6% in July, *Airways News*, August 24, 2015.
- 10 Ancillary revenue statistics by IdeaWorks Company, 2015.
- 11 So are investments a company makes.
- 12 *The New York Times*, December 3, 1993.
- 13 *The New York Times*, December 9, 1988
- 14 Delta Air Lines Inc. Form 10-K, SEC Filings.

14

Analysis of Financial Ratios

From an aerospace and defense industry perspective, falling oil prices and the increasingly likely 'lower for longer' outlook elevate the risk of aircraft order deferral and/or cancellation as the value proposition for airline carriers in terms of reduced cost of ownership for new/next-generation equipment relative to existing/current-generation equipment is markedly reduced.

Moody's R. Solomon, February 20, 2015



Financial analysis of a company is mainly used to relate to how well a company can utilize its assets and other resources to maximize its efficiency and productivity. The application of financial ratios is a useful management tool that helps the understanding of financial results and trends over time. These ratios are key indicators of organizational performance, and allow us to compare our performance directly against other businesses regardless of size or market share. Managers must apply financial ratios to determine how efficiently a company's assets are being used. Ratio analysis can be used to pinpoint strengths and weaknesses to form strategies and is also useful in benchmarking performance to industry standards. One important ratio for airlines is break-even load factor. Airlines use income statements to measure how cost of goods sold and operating expenses affect profits. If the costs and expenses of providing a flight were exactly equal to the revenue generated from that flight, there would be neither a profit nor a loss and net income would be zero.

An important application of financial ratios is in the process of credit rating a particular organization. Credit ratings are vital in an industry like aviation which is heavily dependent on leasing and external sources of financing. Ratio analysis can be applied to an aircraft, a runway, an employee, a passenger, a market, a company, or even an entire industry. The ratios can be broadly classified under seven different types which will be discussed in detail in this chapter.

- ◆ Liquidity Ratios
 - Current Ratio (CR)
 - Quick Ratio (QR)
- ◆ Asset Management Ratios
 - Accounts Receivable Turnover Ratio (ART)
 - Average Collection Period (ACP)
 - Inventory Turnover (ITO)
 - Fixed Asset Turnover Ratio (FAT)
 - Total Asset Turnover Ratio (TAT)
- ◆ Financial Management Ratios
 - Debt Ratio (D/A)
 - Debt to Equity Ratio (D/E)
 - Times Interest Earned Ratio (TIE)
- ◆ Profitability Ratios
 - Gross Profit Margin (GPM)
 - Operating Profit Margin (OPM)
 - Net Profit Margin (NPM)
 - Return on Total Assets (ROA)
 - Return on Equity (ROE)
- ◆ Dividend Ratios
 - Dividend Payout Ratio (DPR)
 - Dividend Yield
- ◆ Market Value Ratios
 - Market to Book Ratio
 - Price–Earnings Ratio (P/E Ratio)
- ◆ DuPont System Ratios
 - Return on Assets (ROA)
 - Return on Equity (ROE)
- ◆ Window Dressing
- ◆ Altman Insolvency Index
- ◆ Key Financial Ratios for the Airline Industry
 - Available Seat Mile (ASM)
 - Revenue Passenger Mile (RPM)
 - Cost per Available Seat Mile (CASM)
 - Revenue per Available Seat Mile (RASM)
 - Revenue per Revenue Passenger Mile (RRPM) or Passenger Yield
 - Average Stage Length (ASL)
 - CASM (Stage Length Adjusted)
 - Average Daily Block Hour Utilization
 - Load Factor (LF)
 - Break-even Load Factor (LF_{b-e})

- ♦ Summary
- ♦ Discussion Questions and Problems

Liquidity Ratios

Liquidity ratios are used to determine a firm's ability to meet its short-term (usually, within the next reporting period) financial obligations. Investors often consider liquidity ratios when performing a fundamental analysis of a company. Because a company that is consistently facing difficulty in meeting its short-term debt is at a higher risk of bankruptcy, liquidity ratios are a decent measure of whether a company will be able to securely continue as a going concern.

Liquidity is not only a measure of the amount of cash possessed by a company. It is also a measure of how easy it will be for the company to raise enough cash or convert assets classified as current into cash. Assets like accounts receivable, marketable securities, and inventory are relatively simple for many companies to convert into cash in the short term. Thus, all of these assets go into the liquidity calculation of a firm.

Aviation Snippets

To help solve immediate cash flow problems, Continental announced that it had agreed to sell its route authority from Seattle to Tokyo to American Airlines for \$150 million. Continental will continue to operate the route through most of next year, and American will soon make an advance payment of \$140 million.

The Los Angeles Times,
December 1990

Current Ratio (CR)

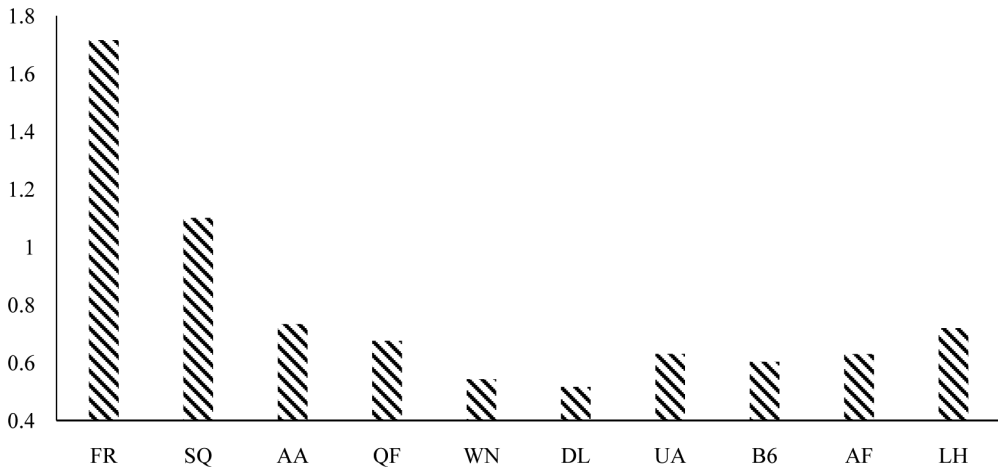
Current ratio is the most direct measure described as a liquidity ratio. It measures the ratio of all current assets (CA) to all current liabilities (CL).



$$CR = \frac{\text{Current Assets}}{\text{Current Liabilities}}$$

$$CR = \frac{CA}{CL}$$

Ideally, this measure should be greater than 1, ensuring that a firm is liquid enough to continue operation without resulting in bankruptcy.

Figure 14.1 Current Ratio for Various Airlines (2015)

As of December 31, 2015, Delta Air Lines¹ had current assets worth \$9,056 million and current liabilities worth \$17,526 million, which resulted in a current ratio of 0.52.

$$CR = \frac{\$9,056}{\$17,526}$$

$$CR = 0.5167$$

When the current ratio is low, especially less than one, and the current liabilities exceed current assets then the company may have problems meeting its short-term obligations paying its employees' salaries, interest expenses, rent expenses, etc.

Figure 14.1² illustrates a comparison of different global airlines with respect to their current ratio. International carriers Ryanair (FR) [1.716] and Singapore Airlines (SQ) [1.1] have the highest current ratio. Among the US carriers, American Airlines (AA) [0.7339] has a better current ratio than its peers. Delta Air Lines (LH) [0.5167] has the smallest current ratio. For the selected airlines the current ratio on average is 0.787.

Example 14.1

DirectJet Airlines has current assets worth \$13,500 million and current liabilities worth \$15,000 million. Determine the current ratio for DirectJet Airlines.

$$CR = \frac{\$13,500}{\$15,000}$$

$$CR = 0.90$$

In this scenario, DirectJet has \$0.90 in current assets for every dollar of current liabilities. The company appears to be unable to easily service its short-term debt obligations.

Quick Ratio (QR)

Quick ratio (also known as the acid-test ratio) is a measure of a company's ability to meet its short-term obligations using its most liquid assets (near cash or quick assets). Current assets that presumably can be quickly converted into cash at close to their book values can be considered quick assets. Quick ratio signifies a company's financial strength or weakness; it gives insight into a company's short-term liquidity. The ratio tells creditors what portion of a company's short-term debt can be settled by the sale of all the company's liquid assets at very short notice.



$$QR = \frac{\text{Cash} + \text{Short-term Investment} + \text{Accounts Receivable}}{\text{Current Liabilities}}$$

It can also be given by the formula:

$$\text{Quick Ratio} = \frac{\text{Total Current Assets} - \text{Inventory} - \text{Prepaid Expenses}}{\text{Current Liabilities}}$$

As of December 31, 2015, with the amounts expressed in millions, Delta Air Lines' quick assets amounted to \$7,563 million, while current liabilities amounted to \$17,526 million. By dividing, the equation gives us a quick ratio of 0.43.

$$QR = \frac{(\$9,056 - \$697 - \$796)}{\$17,526}$$

$$QR = \frac{\$7,563}{\$17,526}$$

$$QR = 0.4315$$

Because we subtracted current inventory, it means that for every dollar of current liabilities there are only \$0.43 dollars of cash, marketable securities and accounts receivable. Both of these ratios indicate that Delta Air Lines has a significant shortage of liquidity.

Example 14.2

MyJet Airlines has the following assets and liabilities in the year ending December 31, 2016:

Table 14.1 MyJet Airlines Financial Data

Cash	\$34,000,000
Marketable securities	\$12,000,000
Accounts receivable	\$56,200,000
Prepaid insurance	\$9,000,000
Total current assets	\$111,590,000
Total current liabilities	\$73,780,000

Determine the quick ratio for MyJet Airlines.

$$QR = \frac{\$34,000,000 + 12,000,000 + 56,200,000}{\$73,780,000}$$

$$QR = 1.38$$

Thus, a quick ratio of 1.38 means that MyJet has \$1.38 of liquid assets (cash, marketable securities, and accounts receivable) available to cover each \$1 of current liabilities.

Asset Management Ratios

Asset management ratios attempt to measure a company's success in managing its assets to generate sales. Asset management ratios are also called efficiency ratios, activity ratios, or turnover ratios. For example, these ratios can help comprehend the success of the firm's credit policy and inventory management.

A company desires high asset turnover ratios because it implies that the assets are efficiently utilized to produce sales. The higher the asset turnover ratios, the more sales the company is generating from its assets. However, this measure is highly industry specific as a low asset turnover ratio in one industry might be high in another. Hence, it is important to restrict comparisons to within a particular industry.

Accounts Receivable Turnover Ratio (ART)

The accounts receivable turnover ratio (Receivables Turnover) indicates the speed with which a company collects its debt, i.e., the number of times average receivables are turned over during a year. This ratio determines how fast a company collects outstanding cash balances from its customers during an accounting period. It is an important measure of a company's financial and operational performance and can be used to identify if a company is having difficulties collecting sales made on credit.

$$\text{Accounts Receivable Turnover Ratio} = \frac{\text{Annual Credit Sales}}{\text{Accounts Receivable}}$$

$$ART = \frac{ACS}{AR}$$

Net credit sales of Delta Air Lines during the year ending December 31, 2015 were \$40,704 million. Its accounts receivable were \$2,139 million and hence the accounts receivable turnover becomes 19.03.

$$\text{Accounts Receivable Turnover Ratio} = \frac{\$40,704}{\$2,139}$$

$$\text{Accounts Receivable Turnover Ratio} = 19.03$$

Average Collection Period (ACP)

The receivables turnover is often reported in number of days, which denotes the number of days the credit sales remain in accounts receivable before they are collected. This is known as average collection period. It can be denoted as follows:

$$ACP = \frac{\text{Accounts Receivable}}{\text{Annual Credit Sales}/365}$$

$$ACP = \frac{AR}{ACS/365}$$

It is also given by the formula:

$$ACP = \frac{365}{ART}$$

Consider the example of Delta Air Lines for the year ending December 31, 2015.

$$ACP = \frac{365}{19.03}$$

$$ACP = 19.18 \text{ days}$$

On average, Delta Air Lines has to wait for 19.18 days before the receivables are collected. Whether the accounts receivable turnover ratio of 19.18 is good or bad depends on Delta's past ratios, the average for other airlines, and the specific credit terms given to this company's creditors and investors.

Example 14.3

MyJet Airlines has an accounts receivable turnover ratio of 24.6. Determine the average collection period for MyJet Airlines.

$$ACP = \frac{365}{ART}$$

$$ACP = \frac{365}{24.6}$$

$$ACP = 14.84 \text{ days}$$

Hence the Average Collection Period for MyJet Airlines is 14.84 days.

Inventory Turnover (ITO)

Inventory turnover is a measure of the number of times inventory is sold or used in a given time period such as one year. It is a good indicator of inventory quality (whether the inventory is obsolete or not), efficient buying practices, and inventory management. This ratio is important because gross profit is earned each time inventory is turned over. The Inventory Turnover Ratio is calculated by dividing Cost of Goods Sold by Inventory.



$$ITO = \frac{\text{Cost of Goods Sold}}{\text{Inventory}}$$

The ride-sharing services Uber and Lyft charge higher rates Saturday nights and at other peak times of demand. Sellers on Amazon.com and other e-commerce sites are using dynamic pricing more and more to match their inventories with demand. Prices of toll lanes on Southern California freeways move up and down in tandem with traffic.³

What makes service industries such as airlines so distinct from manufacturing ones is the high level of inventory turnover due to the low levels of inventory associated with the service industry.

An airline, normally being a service-oriented company, does not have the cost of goods sold, although some of the operating expenses such as fuel expense and selling and administrative expenses can be considered as cost of goods sold.

To illustrate how the ratio is useful, look at the 2015 financial statements for Delta Air Lines. Delta's average inventory was equal to \$774.5 million and the airline reported its cost of goods sold as \$17,096 million.

$$ITO = \frac{\$17,096}{\$774.5}$$

$$ITO = 22.07$$

Therefore, the resulting inventory turnover ratio was equal to a little more than 22.

Example 14.4

AvAir, a general aviation aircraft manufacturing company, has an inventory worth \$45 billion during the year 2015 and the cost of revenue for the chosen period is \$76.8 billion. Determine the inventory turnover ratio.

$$ITO = \frac{\$76.8}{\$45}$$

$$ITO = 1.71$$

Hence, AvAir has an Inventory Turnover Ratio (ITO) of 1.71, which is typical for a manufacturing firm as opposed to the higher values obtained for a service-oriented industry.

Fixed Asset Turnover Ratio (FAT)

The fixed assets turnover ratio measures how productively the firm manages its fixed assets to generate sales (revenue). If a company has a high fixed asset turnover ratio, it shows that the company manages its fixed assets very efficiently. Fixed assets are important because they usually comprise the largest component of total assets. This ratio is calculated by dividing Total Revenue by Net Fixed Assets.

$$\text{Fixed Asset Turnover Ratio} = \frac{\text{Total Revenue}}{\text{Net Fixed Assets}}$$

$$FAT = \frac{TR}{NFA}$$

As of December 31, 2015, with amounts expressed in millions, Delta Air Lines had net sales, or revenue, of \$40,704 million and net fixed assets, or PP&E, of \$23,039 million. By dividing, the equation gives us a fixed asset turnover of 1.767.

$$FAT = \frac{\$40,704}{\$23,039} = 1.767$$

$$FAT = 1.767$$

1.767 is a fairly high number indicating that Delta Air Lines quite efficiently manages its fixed assets in order to generate revenue.

Example 14.5

EZjET Airlines nurtured a revenue of \$35,000 million during the year 2014 and at the end of the year owned property, plant, and equipment worth \$24,350 million. Determine the fixed asset turnover ratio for EZjET.

$$FAT = \frac{\$35,000}{\$24,350}$$

$$FAT = 1.44$$

Thus, EZjET had a reasonable Fixed Asset Turnover ratio of 1.44 for the year 2014.

Total Asset Turnover Ratio (TAT)

The total asset turnover ratio measures how effectively the firm manages all of its assets to generate revenue. This ratio is calculated by dividing Total Revenue by Total Assets.

$$\text{Total Asset Turnover Ratio} = \frac{\text{Total Revenue}}{\text{Average Total Assets}}$$

$$TAT = \frac{TR}{TA}$$

Table 14.2 Total Asset Turnover for Various Airlines

Airline Name	Total Revenue (TR)	Total Assets (TA)	TAT (TR/TA)
Lufthansa (LH)	15669	22249	70%
United Airlines (UA)	37,864	40,861	93%
Southwest Airlines (WN)	19,820	21,312	93%
Qantas Airways (QF)	15,816	17,530	90%
American Airlines (AA)	40,990	48,415	85%
Emirates Airlines (EK)	86,728	111,362	78%
Delta Air Lines (DL)	40,704	53,134	77%
JetBlue (B6)	6,416	8,660	74%
Air New Zealand (NZ)	4,925	6,775	73%
Ryanair (FR)	5654	12,185	46%

Source: Compiled from respective annual reports. Reporting periods may slightly vary.

Note: Numbers are in millions of respective local currencies.

Consider the example of Delta Air Lines for the year ending December 31, 2015. Delta Air Lines had net sales, or revenue, of \$40,704 million and average total assets of \$53,570 million. By dividing, the equation gives us a total asset turnover of 0.76.

$$TAT = \frac{\$40,704}{\$53,570}$$

$$TAT = 0.76$$

Because fixed assets are fewer in the airline industry, although a high value for fixed asset turnover ratio was obtained, the total asset turnover is comparatively less. The higher the total asset turnover ratio, the better and more efficiently the airline uses the company's assets to generate sales.

It can be seen from Table 14.2 and from Figure 14.2, that both United Airlines and Southwest Airlines have the highest total asset turnover, while a low-cost carrier such as Ryanair has the lowest total asset turnover ratio.

Example 14.6

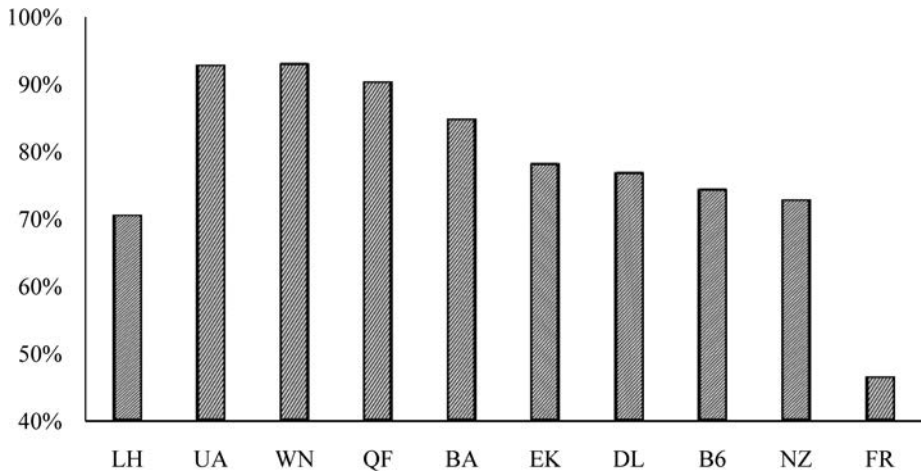
DirectJet Airlines has current assets worth \$23 million and non-current assets worth \$19 million for the year 2015. DirectJet manages overall sales of \$37 million during the same year. Determine the Total Asset Turnover for DirectJet Airlines.

$$TAT = \frac{\$37,000,000}{\$23,000,000 + \$19,000,000}$$

$$TAT = 0.881$$

Hence, DirectJet has a Total Asset Turnover ratio of 0.881.

Figure 14.2 Total Asset Turnover for Various Airlines (2015)



Financial Management Ratios

Liquidity ratios are concerned only with short-term assets and liabilities, while financial management ratios measure the extent to which the firms use long-term debt to finance capital projects. There are many reasons for an airline to file for bankruptcy or be forced to go out of business. The high volume of debt makes it more difficult for any company to remain profitable and to pay interest on the debt. By the end of 1988, Trans World Airlines (TWA) was facing nearly \$4 billion in debt which forced the airline to file for Chapter 11 bankruptcy in 1992. Delta Air Lines was heading into bankruptcy with a remarkably high \$28 billion of debt on its balance sheet. Since the airline did not have enough cash to meet its liquidity and financial obligations, it was forced to file for Chapter 11 protection in 2015.

Debt Ratio (D/A)

Debt ratio is a measure of a firm’s total liabilities to its total assets. A figure of 0.5

Aviation Snippets

Continental Airlines filed for bankruptcy protection in 1983, citing declining passenger traffic and high labor costs. Monday’s Chapter 11 filing, according to the airline, was brought about by high fuel costs and a high debt load, which has caused a cash shortage. Listed below is a comparison of figures from Continental’s annual reports for the respective years that preceded the two bankruptcy moves.

	1982	1989
Revenue (in billions of dollars):	\$1.40	\$4.80
Revenue Passenger Miles (in billions of miles):		
Domestic Traffic:	8.26	28.43
International Traffic:	1.62	10.35
Employees:	14,500	33,600
Fleet (number of planes):	105	325
ESG Aviation Services, 1990		

or less is ideal for this particular ratio. In other words, no more than half of the firm's assets should be financed by debt.

$$D/A = \frac{\text{Total Liabilities}}{\text{Total Assets}}$$

Delta Air Lines had total liabilities worth \$42,284 million and total assets worth \$53,134 million, as of December 31, 2015. By dividing, the equation provides a debt ratio of 0.796.

$$D/A = \frac{\$42,284}{\$53,134}$$

$$D/A = 0.796$$

Although it is higher than the ideal value of 0.5 (or less), a debt ratio of 0.796 implies that total assets are greater than total liabilities, and for an airline this is a fair number because most airline operations are financed by debt.

Figure 14.3⁴ is a depiction of debt ratio for various global carriers. Lufthansa Airlines (LH) is the only carrier with an ideal debt ratio of 0.487, which is less than 0.5, and Singapore Airlines (SQ) is quite close [0.504] as well. American Airlines (AA) [0.884] is better than its peers in the US. Overall, American Airlines (AA) [0.884], Qantas Airways (QF) [0.803], Delta Air Lines (DL) [0.796], and United Airlines (UA) [0.781] form the higher end of the spectrum.

Example 14.7

Assume MyJet has the following values (in millions of \$) corresponding to its balance sheet for the year 2015.

Figure 14.3 Debt Ratio for Various Airlines (2015)

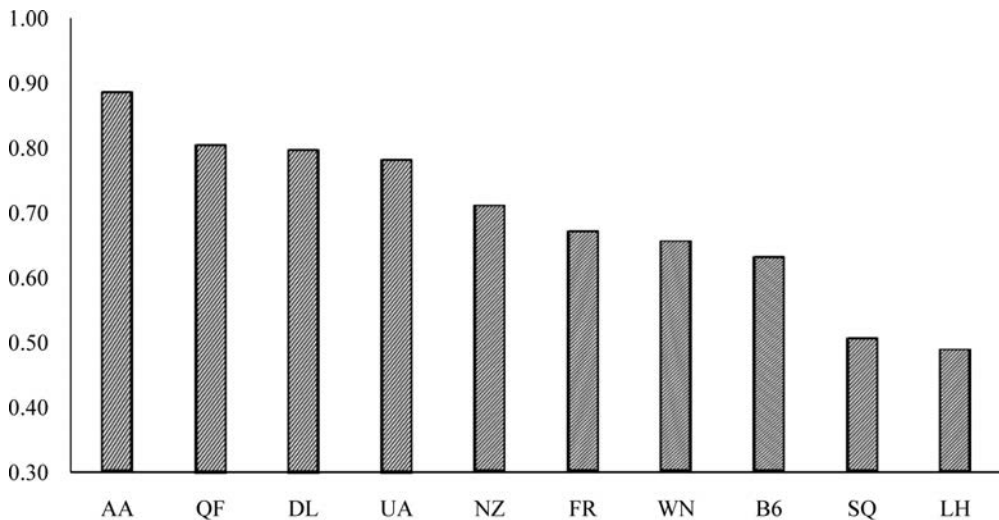


Table 14.3 MyJet Balance Sheet Data

Current assets	\$13,500
Current liabilities	\$16,000
Non-current assets	\$42,000
Non-current liabilities	\$27,500

Determine the Debt Ratio for MyJet for the year 2015.

Total Assets = \$13,500 + \$42,000 = \$55,500 million

Total Liabilities = \$16,000 + \$27,500 = \$43,500 million

$$D/A = \frac{\$43,500}{\$55,500}$$

$$D/A = 0.784$$

The above example shows 78.40 percent of MyJet assets have been financed by debt, therefore only 21.60 percent were financed by equity capital. A higher percentage indicates greater leverage and risk.

Debt to Equity Ratio (D/E)

The debt to equity ratio compares a firm's debt to its stockholders' equity. The ideal value for this ratio would be 1.0, implying that a company can cover all its debt using its equity.

$$D/E = \frac{\text{Total Liabilities}}{\text{Stockholder's Equity}}$$

Consider the example of Delta Air Lines for the year ending December 31, 2015.

$$D/E = \frac{\$42,284}{\$10,850} = 3.894$$

It can be seen that the value for this ratio is fairly high for Delta Air Lines. This means that for every dollar of Delta owned by the shareholders, Delta owes almost \$4 to creditors. In the same period, for some of the other airlines, such as American Airlines, this same measure was \$7.60. Historically, airlines have had a very high debt to equity ratio and this is due to the nature of the industry where returns are typically very poor but the functioning of the industry cannot be avoided. A number of airlines have been pushed to bankruptcy due to very high levels of debt and have either ceased operations, or reorganized under bankruptcy

D/E Anomaly in Airline Industry

D/E Delta Air Lines = 3.894

D/E American Airlines = 3.65 First Quarter 2016

Airline industry in general has very high debt due to poor returns.

protection. Airlines such as Braniff,⁵ Eastern Airlines, Kiwi International, PanAm, and Southern Air all filed for bankruptcy and ceased operation.

Times Interest Earned Ratio (TIE)

The times interest earned ratio determines how efficiently a firm can cover the interest payments on its debts. It is the ratio of earnings before interest and tax (EBIT) to the interest expense. Creditors would favor a company with a much higher times interest earned ratio because it shows the company can afford to pay its interest payments when they come due, hence the ratio should ideally be greater than 1.0.

$$TIE = \frac{EBIT}{Interest\ Expense}$$

To illustrate the times interest earned ratio, look at Delta Air Lines' financial statement of 2015 (in thousands). For Delta Air Lines in 2015 the times interest earned ratio is:

$$TIE = \frac{\$7,638,000}{\$481,000} = 15.88$$

In other words, a ratio of 15.88 means that Delta Air Lines makes enough income to pay for its total interest expense 15.88 times over.

Example 14.8

Let's assume that a corporation's net income after tax was \$1,000,000; its interest expense was \$4,000,000 and its income tax expense was \$6,000,000. Calculate the times interest earned ratio for the corporation.

$$EBIT = \$1,000,000 \text{ (Net Income)} + \$4,000,000 \text{ (Interest Expense)} + 6,000,000 \text{ (Income Tax Expense)}$$

$$EBIT = \$11,000,000$$

$$TIE = \frac{\$11,000,000}{\$4,000,000} = 2.75$$

Hence, the corporation had a times interest earned ratio of 2.75.

Profitability Ratios

Apple, a single company, earned \$13.6 billion in the second quarter of this year. That's just under half the expected full-year profit of the entire airline industry.

IATA Industry Outlook, 2015

The airline industry historically has achieved one of the lowest levels of average rate of return on capital (ROIC) of any industry and has been one of the few that consistently fails to meet its WACC.⁶ For the first time, the industry level average ROIC

will be in excess of its cost of capital. Profitability ratios associate various income statement accounts and categories to show a company's ability to generate profits from its operations. Profitability ratios focus on a company's return on investment in its varied assets. These ratios essentially depict how well companies can achieve profits from their operations. In this regard, profitability ratios relate to efficiency ratios. Profitability is also vital to the concept of solvency and going concern.

$$ROIC = \frac{EBIT (1 - \text{Tax rate})}{\text{Book Value of Equity} + \text{Book Value of Debt} - \text{Cash}}$$

If the airline leases aircraft (operating lease) rather than owning them, the formula will be as follows:

$$\begin{aligned} &\text{Return on Capital} \\ &= \frac{(\text{EBIT} + \text{Operating Lease Expense} - \text{Depreciation of OL}) (1 - \text{Tax rate})}{\text{Book Value of Equity} + \text{Book Value of Debt} - \text{Cash} + \text{Present Value of OL}} \end{aligned}$$

Gross Profit Margin (GPM)

Gross profit margin (gross margin) is the ratio of gross profit (gross sales less cost of sales) to sales revenue. It calculates the percentage by which gross profits exceed production costs. The gross profit margin is essentially the percentage markup on merchandise from its cost. This is the pure profit from the sale of inventory that can go to paying operating expenses. The levels of gross margin can vary drastically from one industry to another depending on the type of business.

$$GPM = \frac{\text{Sales} - \text{Cost of Goods Sold}}{\text{Net Sales}}$$

To clarify, let's look at Delta Air Lines. As of December 31, 2015, a gross profit of \$23,608 million, divided by the net sales of \$40,704 million, provides a gross profit margin (GPM) of 0.5799 or 57.99 percent.

$$GPM = \frac{\$40,704 - \$17,096}{\$40,704}$$

$$GPM = 0.5799$$

$$GPM = 57.99\%$$

Hence, for every dollar of sales made, a gross profit of \$0.5799 is obtained, which can be used to cover the operating expenses. Gross profit margin is a key measure of profitability by which investors and analysts compare similar businesses to the overall industry.

Example 14.9

Let's assume that EZjet Airline has net sales of \$160,000,000 and its cost of goods sold is \$120,000,000. This means its gross profit is \$40,000,000 (net sales of \$160,000,000 minus its cost of goods sold of \$120,000,000) and its gross margin ratio is 25 percent.

$$GPM = \frac{\$160,000,000 - \$120,000,000}{\$160,000,000} = 0.25 = 25\%$$

Gross margin ratios vary significantly between industries, so it is advisable to compare a company's gross margin ratio to companies within the same industry.

Operating Profit Margin (OPM)

Operating profit margin measures the operating income as a ratio of the net sales. The primary difference is that the operating income considers other expenses as well and not just cost of goods sold.

$$OPM = \frac{\text{Operating Income}}{\text{Net Revenue}}$$

Consider the example of Delta Air Lines for the year ending December 31, 2015, the ratio between an operating income of \$7,802 million and net sales of \$40,704 million results in an operating profit margin (OPM) of 0.1917 or 19.17 percent.

$$OPM = \frac{\$7,802}{\$40,704}$$

$$OPM = 0.1917$$

$$OPM = 19.17\%$$

Even after all the operating expenses are considered, for every dollar of sales made, an operating profit of 19.17 cents is obtained.

Net Profit Margin (NPM)

Net profit margin (profit margin or net margin) is a ratio of profitability calculated as after-tax net income (net profits) divided by sales (revenue). It shows the amount of each sales dollar left over after all expenses have been paid.

$$NPM = \frac{\text{Net Income}}{\text{Net Revenue}}$$

Let's look at the Delta Air Lines income statement as of December 31, 2015. By dividing net profit by total revenue, we can see what percentage of revenue made it all the way to the bottom line, which is good for investors. The ratio between the net income of \$4,526 million and the net sales of \$40,704 million results in a net profit margin (NPM) of 0.1112 or 11.12 percent.

$$NPM = \frac{\$4,526}{\$40,704}$$

$$NPM = 0.1112$$

$$NPM = 11.12\%$$

After the completion of tax payment, a net profit of 11.12 cents was made to every dollar of sales made. Compared with different airlines, the NPM tells investors which airlines are relatively more profitable than others.

Example 14.10

Assume we have collected the following information about DirectJet. Determine the GPM, OPM, and NPM for DirectJet.

Gross profit margin is calculated as:

$$GPM = \frac{\text{Sales} - \text{Cost of Goods Sold}}{\text{Net Sales}}$$

$$GPM = \frac{\$50,000,000}{\$52,500,000}$$

$$GPM = 0.9524$$

$$GPM = 95.24\%$$

Operating profit margin is calculated as:

$$OPM = \frac{\text{Operating Income}}{\text{Net Revenue}}$$

$$OPM = \frac{\$50,000,000 - 3,750,000 - 3,750,000}{\$52,500,000}$$

$$OPM = 0.8095$$

$$OPM = 80.95\%$$

Net profit margin is calculated as:

$$NPM = \frac{\text{Net Income}}{\text{Net Revenue}}$$

$$NPM = \frac{\$50,000,000 - 3,750,000 - 3,750,000 - 2,500,000 - 1,250,000}{\$52,500,000}$$

$$NPM = 0.7381$$

$$NPM = 73.81\%$$

A high ratio indicates the efficient management of business affairs.

Table 14.4 DirectJet Financial Data

Sales	52,500,000
Cost of goods sold	2,500,000
Gross profit	50,000,000
Administrative expenses	3,750,000
Selling expenses	3,750,000
Interest on investment	2,500,000
Loss on investment	1,500,000
Income tax	1,250,000

Return on Total Assets (ROA)

The return on assets ratio, often called the return on total assets, is a profitability ratio that measures the net income produced by total assets during a period against the average total assets. This ratio helps both management and investors to see how well the company can convert its investments in assets into profits.

$$ROA = \frac{\text{Net Income}}{\text{Average Total Assets}}$$

Return on total assets (ROA) of 0.0845 or 8.45 percent, was calculated for Delta Air Lines by dividing the net income of \$4,526 million by the average total assets of \$53,569 million, as of December 31, 2015.

$$ROA = \frac{\$4,526}{\$53,569}$$

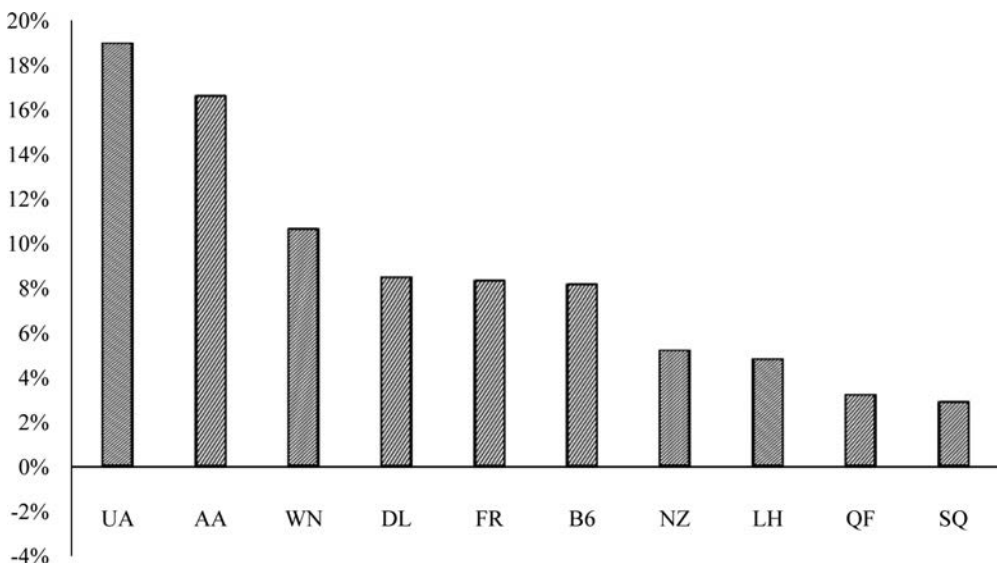
$$ROA = 0.0845$$

$$ROA = 8.45\%$$

This ratio illustrates how and how efficiently the airlines utilized their assets, and reflects the return which is generated by 1 dollar of asset.

Figure 14.4⁷ is an illustration of the return on total assets achieved by various airlines in the year 2015. United Airlines (UA) [18.95 percent], American Airlines (AA) [16.61 percent], and Southwest Airlines (WN) [10.63 percent] are doing well in this respect. It can be seen that Lufthansa Airlines (LH) [4.81 percent], Qantas Airways (QF) [3.21 percent], and Singapore Airlines (SQ) [2.85 percent] have the lowest ROA

Figure 14.4 Return on Total Assets for Various Airlines (2015)



in the comparison group. Overall, these results demonstrate significant progress for both leading and lagging airlines in the sample in contrast with the year 2014.

Return on Equity (ROE)

The return on equity ratio or ROE is a profitability ratio that denotes the ability of a firm to generate profits from its shareholders' investments in the company. In other words, the return on equity ratio signifies how much profit each dollar of common stockholders' equity generates. A rising ROE suggests that a company is increasing its ability to generate profit without needing as much capital. This is a crucial quantity for potential investors because they want to see how efficiently a company will use their money to generate net income.

$$ROE = \frac{\text{Net Income}}{\text{Average Shareholders' Equity}}$$

Consider the example of Delta Air Lines for the year ending December 31, 2015. Return On Equity (ROE) of 0.4171 or 41.71 percent was calculated by dividing the net income of \$4,526 million by the average shareholders' equity of \$10,850 million.

$$ROE = \frac{\$4,526}{\$10,850}$$

$$ROE = 0.4171$$

$$ROE = 41.71\%$$

In the year 2015, 41.71 percent of shareholder's equity was converted into net income by Delta Air Lines, i.e., for every dollar of shareholder's equity there was a contribution of 41.71 cents into the net income.

Dividend Ratios

Dividend ratios provide insight into the dividend policy of a firm and also a possible prospect for future growth. The value of any stock primarily depends on future dividend payments and so it is vital to maintain good dividend ratios to portray a profitable picture to potential investors and to existing shareholders. Boeing paid its shareholders a dividend of \$0.91 per share in December 2015. In contrast to a manufacturer like Boeing, American Airlines paid its shareholders a dividend of \$0.10, indicating the bleak nature of returns in the airline industry.

Dividend Payout Ratio (DPR)

The dividend payout ratio is the percentage of earnings paid out to shareholders as dividends. Airlines are not generally known for their regular dividend payments, and many airlines do not pay any dividends at all. The ratio also reveals whether a firm can maintain its current level of dividend payouts. If the ratio is greater than 100 percent, then the company is utilizing its cash reserves to pay dividends. This circumstance is not sustainable, and may result in the eventual termination of all dividends or the financial decline of the company.

$$DPR = \frac{\text{Dividends per Share}}{\text{Earnings per Share}}$$

The dividend payout ratio for Delta Air Lines for the year ended December 31, 2015 was found by dividing the dividends per share, which were \$0.135 and the earnings per share, which were \$1.26.

$$DPR = \frac{\$0.135}{\$1.26}$$

$$DPR = 0.1071$$

$$DPR = 10.71\%$$

A dividend payout ratio of 10.71 percent is a healthy quantity, as it implies that a considerable amount of the earnings are utilized for the firm, because only 10.71 percent of the earnings are paid as dividends. It has to be noted that the Delta Air Lines dividends per share and earning per share used in the above calculation are provided by NASDAQ as of December 31, 2015.

Dividend Yield

Dividend yield indicates how much a company pays out in dividends each year relative to its share price. Dividend yield is an important part of the rate of return on an investment. If the price of a stock goes up by 8 percent in a year while paying a 3 percent dividend, then it results in a net 11 percent earning on the investment.

$$\text{Dividend Yield} = \frac{\text{Dividends per Share}}{\text{Current Share Price}}$$

Consider the example of Delta Air Lines for the year ending December 31, 2015.

$$\text{Dividend Yield} = \frac{\$0.135}{\$50.69} = 0.0027 = 0.27\%$$

The ratio between the dividends per share of \$0.135 and the current share price of \$50.69, as of December 31, 2015, results in a dividend yield of just 0.27 percent. An extremely low dividend yield is proof that the airline industry does not possess a high rate of return in comparison to many of the other existing industries.

Example 14.11

MyJet Airlines has a dividend policy of \$0.22 per share and has 790 million shares outstanding. If MyJet Airlines earned a net income of \$580 million in the year 2014, determine the dividend payout ratio and the dividend yield. Assume the stock price on December 31, 2014 was \$32.71.

$$\text{Earnings per Share (EPS)} = \frac{\$580 \text{ million}}{790 \text{ million}}$$

$$EPS = \$0.73$$

$$DPR = \frac{\text{Dividends per Share}}{\text{Earnings per Share}}$$

$$DPR = \frac{\$0.22}{\$0.73}$$

$$DPR = 0.3014$$

$$DPR = 30.14\%$$

$$\text{Dividend Yield} = \frac{\text{Dividends per Share}}{\text{Current Share Price}}$$

$$\text{Dividend Yield} = \frac{\$0.22}{\$32.71}$$

$$\text{Dividend Yield} = 0.0067$$

$$\text{Dividend Yield} = 0.67\%$$

Market Value Ratios

Market value ratios are used to relate an observable market value (i.e., stock price) to values available on the financial statements of a company (i.e., book values). A few of these ratios are discussed below.

Market to Book Ratio

The market to book ratio relates a company's market value per share to its book value per share. Because the book value reflects historical cost accounting, this ratio signifies management's success in creating value for its shareholders. This ratio is used by "value-based investors" to identify undervalued stocks.

$$\text{Market to Book Ratio} = \frac{\text{Price per Share}}{\text{Book Value per Share}}$$

Where:

$$\text{Book Value per Share} = \frac{\text{Total Owner's Equity}}{\text{Number of Shares Outstanding}}$$

Delta Air Lines has a Market to Book ratio of 4.716, which is obtained by dividing the market price per share of \$50.69 to the book value per share which is \$13.93, as of December 31, 2015.

$$\text{Market to Book Ratio} = \frac{\$50.69}{\$13.93}$$

$$\text{Market to Book Ratio} = 3.64$$

It can be noted that a ratio of 3.64 indicates that Delta Air Lines stock is undervalued, and thus is creating a certain value for the shareholders.

Price–Earnings Ratio (P/E Ratio)

The price–earnings ratio is calculated by dividing the current market price per share of a stock by earnings per share (EPS). The P/E ratio indicates how much investors are willing to pay per dollar of current earnings. Generally, high P/E ratios are associated with stocks with good growth potential.

$$P/E = \frac{\text{Price per Share}}{\text{Earnings per Share}}$$

Consider the example of Delta Air Lines for the year ending December 31, 2015. The market price per share of \$50.69 is divided by the earnings per share of \$1.26, in order to obtain a P/E ratio of 40.23.

$$PE = \frac{\$50.69}{\$1.26}$$

$$PE = 40.23$$

Figure 14.5 Price–Earnings Ratio for Various Airlines in USD (2015)

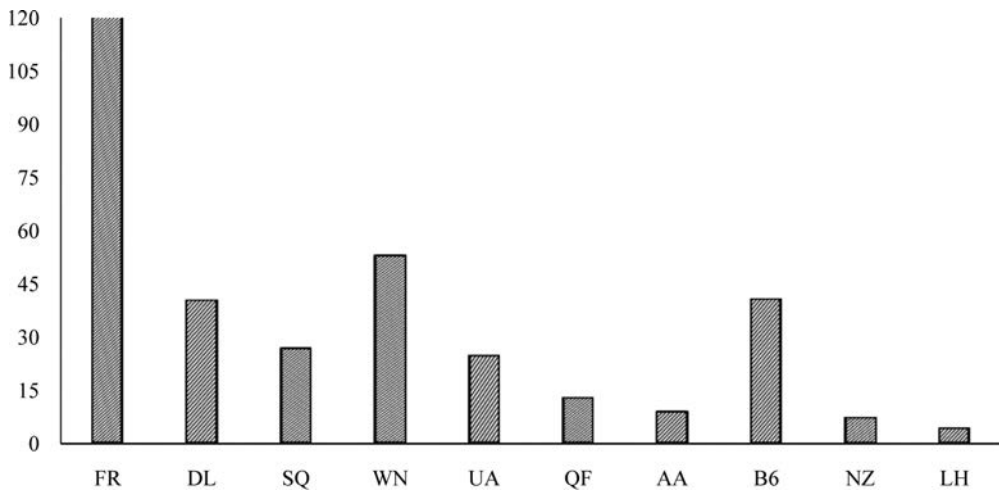


Figure 14.5 is a comparison of the price–earnings ratio of various global airlines for the year 2015. Ryanair (FR) is without a doubt the airline with the highest P/E ratio of 123.44, indicating that it has maximum growth potential. Among the US carriers Southwest Airlines (WN) [52.51] is the front-runner with American Airlines having the least P/E ratio of 8.64. Overall, Lufthansa Airlines (LH) [4.15] has the lowest P/E ratio among the comparison group.

DuPont System Ratios

The idea of the DuPont system was to combine three critical elements of management, namely, operating management, asset management, and the capital structure

(financial management), to calculate the values for two main ratios. The DuPont analysis shows how ROA and ROE can be broken down into subcomponents. By using the DuPont equation, an analyst can easily determine what processes the company does well and what processes can be improved. The equation is called the DuPont analysis, because it was created by the DuPont Corporation back in the 1920s.

Return on Assets (ROA)

The usual way to calculate return on assets as stated previously is:

$$ROA = \frac{\text{Net Income}}{\text{Total Assets}}$$

$$ROA = \frac{NI}{TA}$$

Using the DuPont system, this can be transformed as follows:

$$ROA = NPM \left(\frac{NI}{Sales} \right) \times TAT \left(\frac{Sales}{TA} \right)$$

Consider the example of Delta Air Lines for the year ending December 31, 2015.

$$ROA = NPM \left(\frac{NI}{Sales} \right) \times TAT \left(\frac{Sales}{TA} \right)$$

$$ROA = 0.1112 \times 0.76$$

$$ROA = 0.0845$$

$$ROA = 8.45\%$$

The product of net profit margin (0.1112) and total asset turnover ratio (0.76) results in a return on assets of 0.0845 or 8.45 percent. Table 14.5 provides an analysis of the calculation of the ROA using the DuPont system.

Return on Equity (ROE)

If two companies have the same ROE, but the first is well managed (high net-profit margin), with its assets managed efficiently (high asset turnover), but has a low equity multiplier compared to the other company, then an investor is better off investing in the first company, because the capital structure can be changed easily, but changing management is difficult.

The usual way to calculate return on equity as stated previously is:

$$ROE = \frac{\text{Net Income}}{\text{Shareholders' Equity}}$$

$$ROE = \frac{NI}{\text{Shareholders' Equity}}$$

Using the DuPont system, this can be transformed as follows:

$$ROE = NPM \left(\frac{NI}{Sales} \right) \times TAT \left(\frac{Sales}{TA} \right) \times Equity\ Multiplier \left(\frac{TA}{Shareholders' Equity} \right)$$

Consider the example of Delta Air Lines for the year ending December 31, 2015:

$$ROE = NPM \left(\frac{NI}{Sales} \right) \times TAT \left(\frac{Sales}{TA} \right) \times Equity\ Multiplier \left(\frac{TA}{Shareholders' Equity} \right)$$

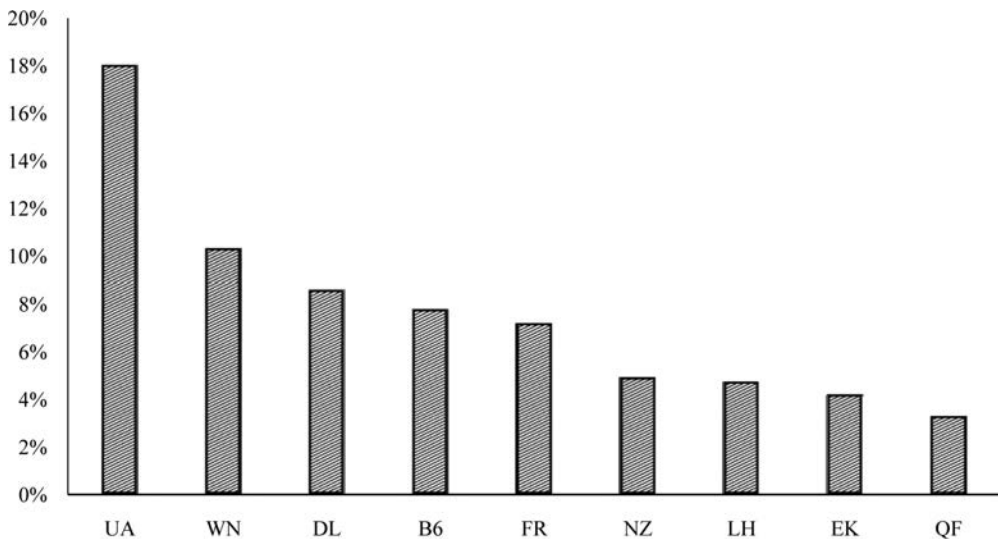
Table 14.5 ROA Using DuPont System for Various Airlines

Airline Name	Net Income (NI)	Total Revenue (TR)	Total Assets (TA)	NI/TR	TR/TA	ROA
UA	7,340	37,864	40,861	19%	93%	17.96%
WN	2,181	19,820	21,312	11%	93%	10.23%
DL	4,526	40,704	53,134	11%	77%	8.52%
B6	667	6,416	8,660	10%	74%	7.70%
FR	867	5,654	12,185	15%	46%	7.11%
NZ	327	4,925	6,775	7%	73%	4.83%
LH	1,034	15,669	22,249	7%	70%	4.65%
EK	4,555	86,728	111,362	5%	78%	4.09%
QF	560	15,816	17,530	4%	90%	3.19%

Source: Compiled from respective 10k reports. Reporting periods may slightly vary.

Note: Numbers are in millions of respective local currencies

Figure 14.6 ROA Using DuPont System for Various Airlines (2015)



$$ROE = 0.1112 \times 0.76 \times 4.9$$

$$ROE = 0.41$$

$$ROE = 41\%$$

In the DuPont system, return on equity is calculated as the product between net profit margin (0.1112), total asset turnover ratio (0.76) and equity multiplier (4.9), which is the ratio between total assets (\$53,134 million) and shareholders' equity (\$10,850 million). Hence the ROE from the DuPont system is found to be 0.41 or 41 percent.

Example 14.12

The following information is taken from the most recent financial statements of DirectJet.

Balance Sheet	Beginning of Year	End of Year
Current assets	\$21,875,000	\$24,500,000
Total assets	\$36,750,000	\$42,000,000
Current liabilities	\$9,625,000	\$6,125,000
Total liabilities	17,500,000	\$21,000,000
Total shareholders' equity	\$20,125,000	\$25,375,000

Income Statement	
Net sales	\$66,500,000
Gross profit	\$31,500,000
Operating income	\$15,750,000
Profit	\$10,500,000

Compute the following:

- Current ratio at year-end;
- Working capital at the beginning of the year;
- Gross profit rate for the year;
- The rate of return on average total assets for the year;
- The rate of return on average total equity for the year.

Solutions:

$$a. \quad CR = \frac{\$24,500,000}{\$6,125,000}$$

$$CR = 4$$

$$b. \quad \text{Working capital} = \$21,875,000 - \$9,625,000$$

$$\text{Working capital} = \$12,250,000$$

$$c. \quad \text{Gross Profit Margin} = \frac{\$31,500,000}{\$66,500,000}$$

$$GPM = 47.4\%$$

Table 14.6 ROE Using DuPont System for Various Airlines

Airline Name	Net Income (NI)	Total Revenue (TR)	Total Assets (TA)	Shareholders' Equity (SE)	NI/TR	TR/TA	TA/SE	ROE
UA	7,340	37,864	40,861	8,966	19%	93%	4.557328	82%
DL	4,526	40,704	53,134	10,850	11%	77%	4.897143	42%
WN	2,181	19,820	21,312	7,358	11%	93%	2.896439	30%
LH	1,034	15,669	22,249	4,539	7%	70%	4.90174	23%
B6	667	6,416	8,660	3,210	10%	74%	2.697819	21%
FR	867	5,654	12,185	5,035	15%	46%	3.019826	21%
NZ	327	4,925	6,775	1,965	7%	73%	3.447837	17%
QF	560	15,816	17,530	3,447	4%	90%	5.085582	16%
EK	4,555	86,728	111,362	28,286	5%	78%	3.937001	16%

Source: Compiled from respective 10k reports. Reporting periods may slightly vary.

Note: Numbers are in millions of respective local currencies

$$d. \text{ ROA} = \frac{\$15,750,00}{\$42,000,000}$$

$$\text{ROA} = 37.5\%$$

$$\text{ROE} = \frac{\$10,500,000}{\frac{\$20,125,000 + \$25,375,000}{2}}$$

$$\text{ROE} = 46.20\%$$

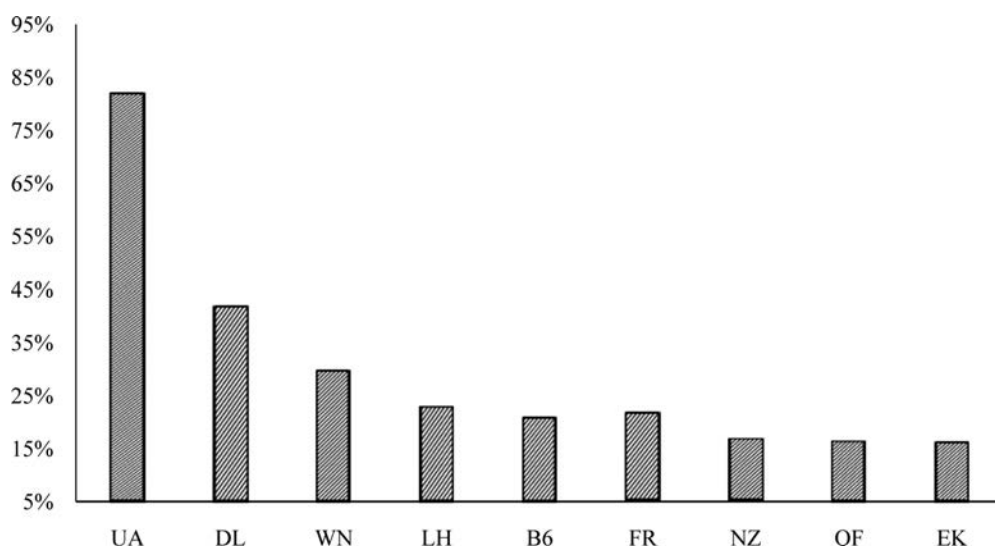
A high and stable ROE can signal that the company has a competitive advantage over its competitors.

Figure 14.7 is an illustration of the ROEs for the various airlines. It can be seen that United Airlines has a very high return on equity along with Delta Air Lines and Southwest. Qantas Airways and Emirates Airlines have the lowest ROE of 16 percent.

Window Dressing

Window dressing is an action taken by the management to make financial statements and financial results look better. To improve liquidity ratio, firms often delay payments for equipment, loans, etc. to build up their liquid accounts and thus

Off-balance-sheet activities refers to the fact that for many years neither leased assets nor the liabilities under lease contracts appeared on the lessees' balance sheets.

Figure 14.7 ROE Using DuPont System for Various Airlines (2015)

their liquidity ratios. If possible, it is often more accurate to use other than year-end financial statements to conduct performance analysis. Let us assume that DirectJet wants its balance sheet to look attractive to a potential investor. DirectJet might do some window dressing by disposing of debt from its balance sheet by leasing aircraft rather than debt financing. Because an operating lease is not considered a long-term debt or liability, it does not appear as debt on the financial statement. In some countries, accounting rules encourage airlines to intentionally structure lease obligations as operating leases, thus excluding the obligations from their balance sheets, essentially a form of window dressing.

Table 14.7 Direct Jet Current Balance Sheet

Current assets	\$30,000,00	Debt	\$40,000,000
Fixed assets	50,000,000	Equity	40,000,000
Total assets	\$80,000,000	Total claims	\$80,000,000

Example 14.13

Assume Table 14.7 presents the current balance sheet of DirectJet.

Therefore, the company's debt ratio = $\$40,000,000 / \$80,000,000 = 50\%$.

DirectJet has recently decided to acquire an aircraft for \$10,000,000 and has capitalized the lease. Hence the new balance sheet (Table 14.8):

Table 14.8 Direct Jet New Balance Sheet

Current assets	\$30,000,00	Debt	\$50,000,000
Fixed assets	60,000,00	Equity	40,000,000
Total assets	\$90,000,000	Total claims	\$90,000,000

Therefore, DirectJet's debt ratio = $\$50,000,000 / \$90,000,000 = 55.6\%$. In reality, the airline's financial risk is not different whether the equipment is leased or purchased.

Altman Insolvency Index

Airline industry capital structure and solvency ratio are risky. Both total debt to equity and long-term debt to equity ratios are very high. The Altman Z-score is an analytical representation created by Edward Altman in the 1960s involving a combination of five distinctive financial ratios used for determining the odds of bankruptcy amongst companies. Altman argued that no single ratio can thoroughly signify the financial status of a firm as it is multi-dimensional; thus a multivariate approach is needed to ensure that all of the dimensions relevant to insolvency are captured. Most commonly, a lower score reflects higher odds of bankruptcy. There are several variations of the formulation depending on whether the companies are private or publicly held and also on whether the companies are manufacturing or non-manufacturing.

The formulation for Z-score for public manufacturing companies is as follows:

$$Z\text{-score} = 1.2 * T1 + 1.4 * T2 + 3.3 * T3 + 0.6 * T4 + 1.0 * T5$$

Where:

$$T1 = \frac{\text{Working Capital}}{\text{Total Assets}}$$

$$T2 = \frac{\text{Retained Earnings}}{\text{Total Assets}}$$

$$T3 = \frac{\text{Earnings Before Interest \& Tax}}{\text{Total Assets}}$$

$$T4 = \frac{\text{Market Value of Equity}}{\text{Book Value of Total Liabilities}}$$

$$T5 = \frac{\text{Sales}}{\text{Total Assets}}$$

The zones of discrimination for public manufacturing companies for the values obtained as Z-score are as follows:

- ◆ "Distress" Zone: 1.8 or less
- ◆ "Grey" Zone: 1.8 to 2.99
- ◆ "Safe" Zone: 2.99 or more

Table 14.9 Altman Z-score for Selected Manufacturing Companies

Manufacturing Company	2015	2014
Lockheed Martin	2.628	3.419
General Dynamics	3.725	3.366
Textron Aviation	2.605	2.507
United Technologies	2.621	2.826
Boeing	2.736	2.694
Rolls-Royce	1.527	1.838
Airbus	1.099	1.079
Bombardier	−0.2778	0.6634

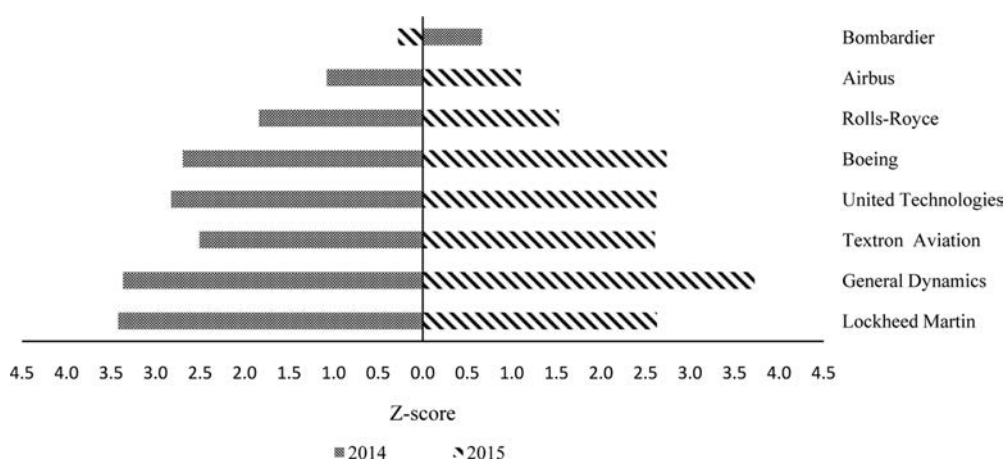
Source: <https://ycharts.com>**Figure 14.8** Z-scores for Manufacturing Companies in Aviation

Table 14.9 shows Altman's Z-score model for public manufacturing companies applied to some of the major manufacturing companies in the aviation industry.

As seen from Table 14.9, four of the eight chosen manufacturing companies are in the "Grey" zone while three of them appear to be in the "Distress" zone in the year 2015. The aviation industry is not among the most liquid industries and it can be seen that the relatively smaller companies such as Lockheed Martin are in a better position with respect to bankruptcy than some of the big companies such as Boeing and Airbus.

Key Financial Ratios for the Airline Industry

Operational efficiency is another important aspect of any company, particularly an airline. A set of metrics has been devised which can sufficiently provide an insight into the operational efficiency of an airline.

Available Seat Mile (ASM)⁸

Available seat mile (ASM) is a measure of an airline's passenger carrying capacity. ASM is calculated as the total number of seats multiplied by the total distance traveled. Assuming a 200-seater flying 1,000 miles the aircraft generates 200,000 ASMs for that particular flight.



Revenue Passenger Mile (RPM)

Revenue passenger mile (or RPM) is a measure of production for an airline. RPM is calculated as the number of revenue passengers multiplied by the total distance travelled. For example, if 500 passengers flew 8,000 miles, it would generate 4,000,000 RPMs. RPMs are used as the basic measure of airline demand when compared to ASMs. RPMs show how many available seats the airline actually sold relative to its overall capacity. While revenue passenger mile measures demand, available seat mile measures supply.

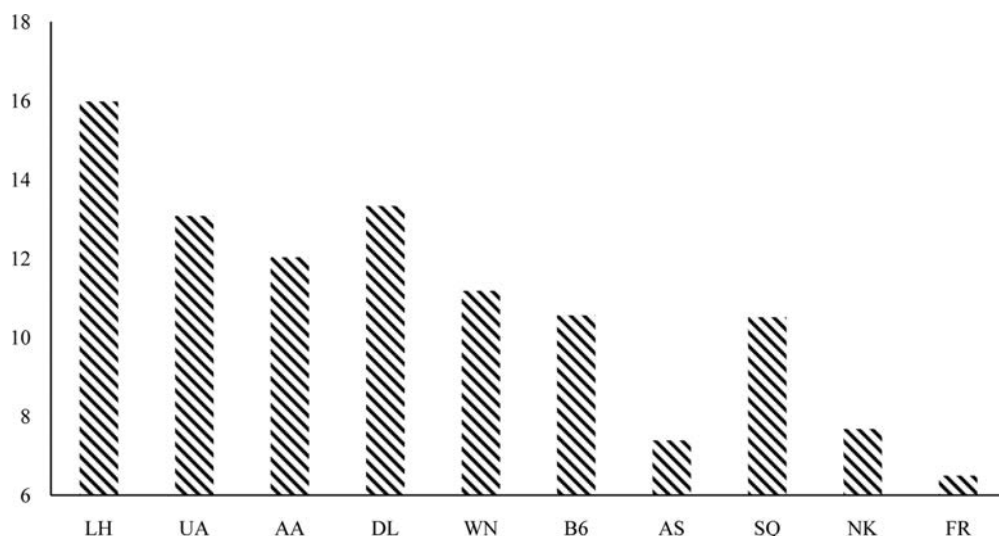


Cost per Available Seat Mile (CASM)

The primary metric used to measure the cost structure of an airline is cost per available seat mile (CASM or unit cost). CASM is the cost to fly one seat one mile whether it's sold or not, and is calculated by dividing the total operating cost (TOC) by the total available seat miles (ASM), which is given by the product of the number of seats in a plane and the distance flown in miles. The airline industry has successfully reduced its unit costs since 2001. Operating CASM standardizes the direct operational costs incurred in a particular flight. Airlines must reduce CASM to remain competitive and earn profit.

$$\text{CASM} = \frac{\text{Total Operating Cost}}{\text{Available Seat Miles}}$$

In 2013, Alaska and Horizon Airlines reduced CASM to remain competitive with lower cost airlines in the long-run. Air Canada is cutting costs by replacing paper manuals with iPads.⁹ United completed the installation of a newly designed wingtip

Figure 14.9 CASM (Cents) for Various Airlines (2015)

curved extension on one of its Boeing 787–800 aircraft. The airline said that it aims to reduce its fuel costs by \$1 billion annually by 2017.¹⁰ American Airlines claims that by switching from paper to iPads, they will save 1.2 million dollars in fuel over the course of a year.

Airlines may also be able to reduce the unit cost by:¹¹

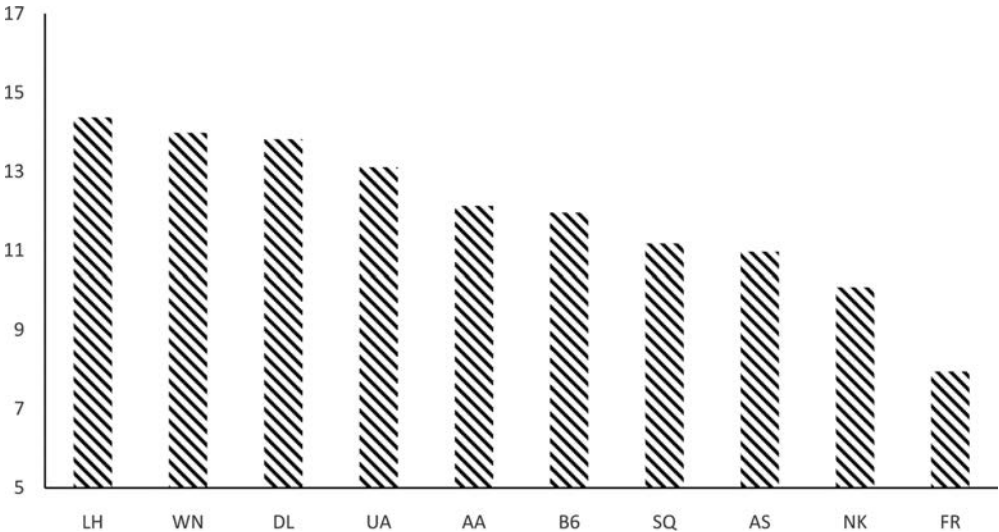
- ♦ A winglet equipped aircraft which can reduce fuel consumption over the non-winglet equivalent aircraft
- ♦ Ditching life rafts on over-land flights
- ♦ Redesigning the economy class to accommodate more seating
- ♦ One engine to taxi in and to taxi out
- ♦ Narrower aisles
- ♦ Reducing the number of flights and filling up the aircraft
- ♦ Smaller toilets

Figure 14.9¹² gives a fair idea of the standing of various global airlines with respect to CASM. It can be seen that all the US carriers have comparable CASM while some of the international carriers like Lufthansa have a very high CASM and Ryanair has a very low CASM. Generally, a low CASM is preferred as it indicates that cost is minimal.

Revenue per Available Seat Mile (RASM)

RASM or unit revenue is a ratio used to depict the revenue side of an airline. It measures the total passenger revenue as compared to the overall available seat miles. Figure 14.10¹³ is a comparison of RASM for various airlines and it is clearly seen that Lufthansa Airlines (LH) along with Southwest and Delta Air Lines have higher RASM compared to the rest of the carriers in the sample. Four major US carriers have similar values for RASM indicating similar choice of routes and hence similar profits.

Figure 14.10 RASM (Cents) for Various Airlines (2015)



Revenue per Revenue Passenger Mile (RRPM) or Passenger Yield

Another typical aviation based ratio, RRPM, also referred to as yield, measures passenger revenue as a ratio of the revenue passenger mile. Revenue passenger miles refer to the total miles flown by fare-paying customers, not taking into account the empty seats in a particular flight. This is a more accurate measure of revenue generated by an airline as it incorporates only those passengers who are contributing to the revenue.

Figure 14.11¹⁴ is another illustration showing how various airlines performed with regard to this particular metric. Similar to RASM, almost all the US carriers

Figure 14.11 RRPM (Cents) for Various Airlines (2015)

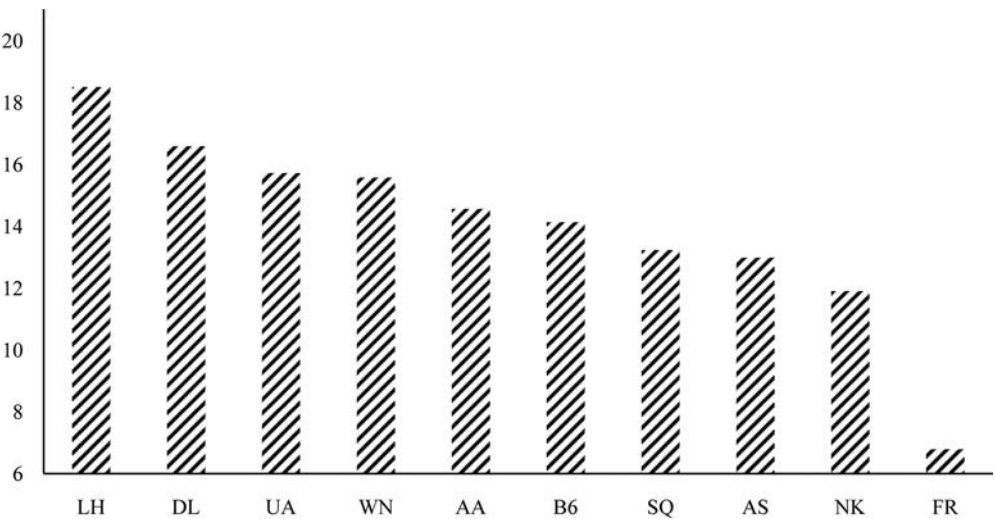
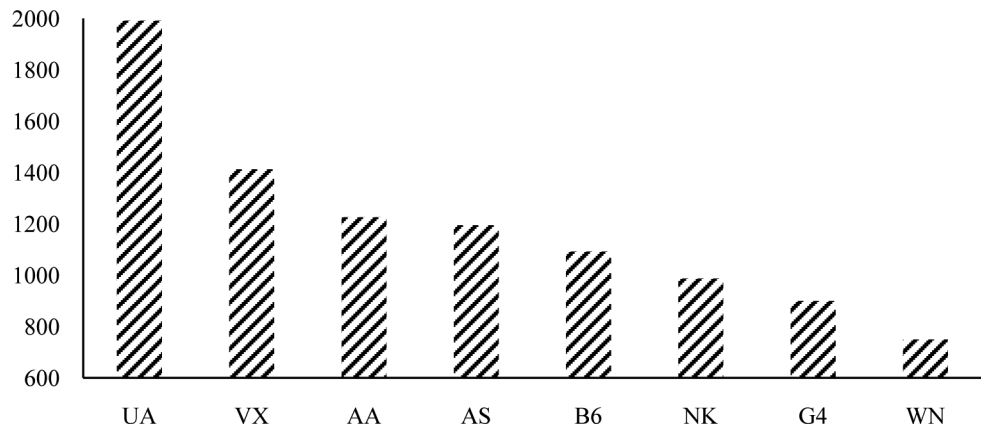


Figure 14.12 Average Stage Length (Miles), US Carriers (2015)



have the same level of RRPM while the international carriers such as Lufthansa Airlines (LH) outperform them.

Average Stage Length (ASL)

Average stage length refers to the actual length of a particular flight, measured in miles. An increased stage length generally implies a decrease in CASM, due to a smaller number of take-offs and landings, and, hence, fewer costs incurred.

It can be clearly noted from Figure 14.12 that United Airlines flies the most international or long haul flights, while, consistent with its cost structure and planning strategies, Southwest Airlines has the least average stage length.

CASM (Stage Length Adjusted)

Comparing one airline's CASM with that of another offers inaccurate information about which airline is truly more cost efficient. Some airlines serve short haul distance, and others vastly different route structures. Operating costs are significantly impacted by the distance flown, and stage length adjusted CASM is devised to compare results as if all airlines fly the same distance.

$$CASM_{SLA} = \frac{\text{Length of haul}}{\sqrt{\text{market average (stage length)}}} \times CASM_i$$

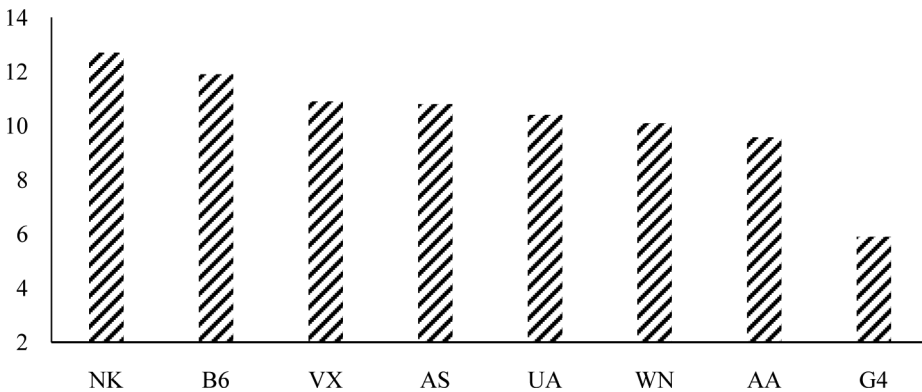
Where:

$CASM_i$ = Respective Carrier Unit Cost

$CASM_{SLA}$ = Stage Length Adjustment CASM

Average Daily Block Hour Utilization

Aircraft utilization refers to the average number of block hours that each aircraft flies. Block hours refer to the time between door close (blocks away from the wheels) and door open (blocks under wheels). It is generally measured on a daily basis and serves

Figure 14.13 Average Daily Block Hour Utilization, US Carriers (2015)

as a measure of the productivity of an airline. Average daily block hours is a parameter closely related to average stage length (ASL).

Figure 14.13 shows that Spirit Airlines (NK) had the maximum average daily block hour utilization of 12.7 hours, while Allegiant Air (G4) had the minimum value of 5.9 hours. Southwest Airlines (WN) had a reasonable utilization of average block hour per day (10.09 hours), even though it had the least average stage length (ASL).

Load Factor (LF)

Load factor measures the percentage of total capacity that an airline utilizes, and can be calculated using the following formula:

$$LF = \frac{RPM}{ASM}$$

Global airline industry achieved a record load factor of 80.6 percent in the year 2015,¹⁵ while US airlines enjoyed 83.78 percent average load factor.¹⁶

Break-even Load Factor (LF_{b-e})

Break-even Load Factor is the percentage of the seats an airline has in service that it must sell at a given yield, or price level, to cover its costs. The formula for break-even load factor LF_{b-e} is

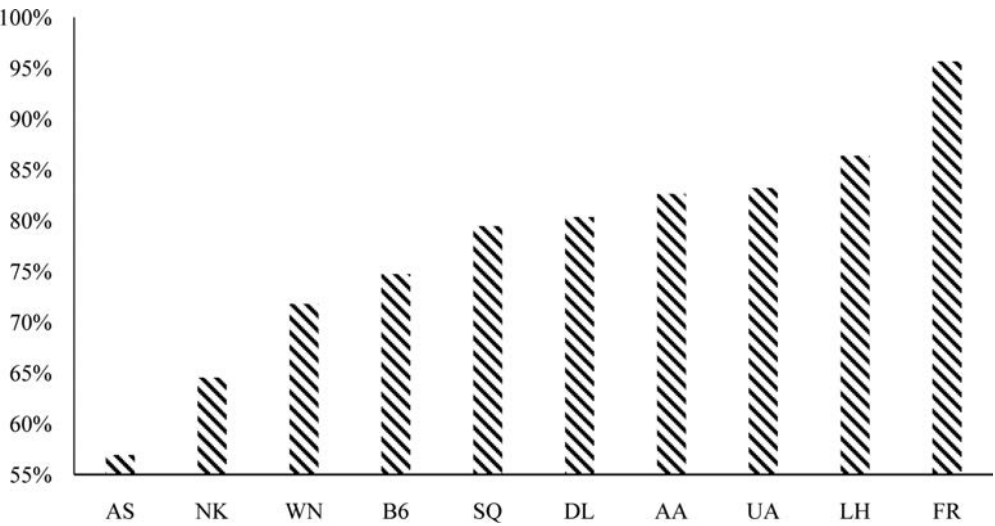
$$LF_{b-e} = \frac{CASM}{RRPM}$$

The lower the break-even load factor, the better it is for an airline, because every seat sold after achieving break-even solely contributes toward profits generated. It can be seen from Figure 14.14¹⁷ that among the US carriers Spirit Airlines has the lowest break-even load factor, while among the international airlines Singapore Airlines has the least break-even load factor. United Airlines, Lufthansa, and Ryanair have considerably higher break-even load factors for the year 2015.

Table 14.10¹⁸ Break-even Load Factors for Various Airlines

Carrier Name	Carrier Code	CASM	Yield (R/RPM)	Break-even Load Factor
Alaska Airlines Inc.	AS	7.39	12.98	56.93%
Spirit Air Lines	NK	7.68	11.9	64.54%
Southwest Airlines	WN	11.18	15.57	71.80%
JetBlue Airways	B6	10.56	14.13	74.73%
Singapore Airlines	SQ	10.512	13.23	79.46%
Delta Air Lines Inc.	DL	13.33	16.59	80.35%
American Airlines Inc.	AA	12.03	14.56	82.62%
United Air Lines Inc.	UA	13.08	15.72	83.21%
Lufthansa Airlines	LH	15.98	18.5	86.38%
Ryan Air	FR	6.4956	6.79	95.66%

Figure 14.14 Break-even Load Factors (%) for Various Airlines (2015)



Summary

This chapter explains in detail all the financial ratios required to assess the current financial standing of any organization. Using financial ratios, stockholders can develop a feel for a company’s attractiveness based on its competitive position, financial strength, and profitability. Broad classifications of different types of ratios have been made, such as short-term solvency ratios, debt management ratios, asset management ratios, profitability ratios, and market value ratios. A liquidity ratio is

a measure that shows the relationship of a firm's cash and other liquid assets to its current liabilities, and it is divided into current ratio and quick ratio. The current ratio is a commonly used measure of short-run solvency and presents the ability of a firm to meet its short-term debt requirements as they come due. The quick ratio is a more conservative measure of liquidity. That is, the quick ratio assumes inventory is not very liquid; therefore, it should not be counted. Financial leverage ratios measure the use of debt financing. The debt ratio is the ratio of total debt to total assets; it measures the percentage of funds provided by creditors. Analysis of leverage ratios provides insight on a company's capital structure as well as the level of financial solvency a firm has. The debt-to-assets ratio is the most basic leverage ratio, measuring the percentage of a company's total assets that is financed by debt. The debt to equity ratio measures the amount of debt capital a firm uses compared to the amount of equity capital it uses. Each of these different types has specific sub-types which cater for the measurement and analysis of ratios using only the values provided in the financial statements of firms. Toward the end of the chapter, a specific index, known as the Altman insolvency index is discussed, which is a measure used to analyze the bankruptcy status of a firm. The last section in the chapter covers aviation based ratios which convey the financial and managerial standpoint of airlines. CASM, RASM, and RRPM are financial standpoint ratios, while ASL and block hour utilization refer to the efficiency in operations of airlines.

Discussion Questions and Problems

- 14.1 Fuel performance for a given flight can be influenced by many factors. Explain what might be some variables that impact airlines' fuel efficiency.
- 14.2 How do you calculate working capital?
- 14.3 Your airline has an equity multiplier of 2.50. What is its debt-to-equity ratio?
- 14.4 An airline has an ROE of 30 percent. The industry average ROE is 15 percent. Is this an indicator of a good or a poor management of the airline?
- 14.5 What metrics are most commonly used to evaluate companies in the airline industry?
- 14.6 Why does an aircraft manufacturer report times interest earned?
- 14.7 What does it mean when a firm window dresses its financial statements?
- 14.8 A jet engine manufacturer has \$100,000,000 in inventories, a current ratio equal to 1.2, and a quick ratio equal to 1.1; what is the firm's Net Working Capital?
- 14.9 Jet-Tech is a leading commercial aircraft maintenance, repair and overhaul (MRO) company. If Jet-Tech's inventory level of \$3,000,000 represents 30 days' sales, what is the annual cost of goods sold? What is the inventory turnover ratio?
- 14.10 As a credit agency you are requested to evaluate the balance sheet for PARTS-R-US. From the balance sheet you acquire the following information:

- ◆ Cash and marketable securities = \$500,000
- ◆ Accounts receivable = \$1,000,000
- ◆ Inventory = \$2,000,000
- ◆ Accrued wages and taxes = \$400,000
- ◆ Accounts payable = \$800,000
- ◆ Notes payable = \$600,000

Calculate PARTS-R-US's current ratio, quick ratio, and cash ratio.

- 14.11** According to the financial statements provided by DirectJet Airlines, the airline presented the following results:

- ◆ Net sales = \$12.0 million
- ◆ EBIT = \$5.6 million
- ◆ Net income available to common stockholders = \$3.0 million
- ◆ Common stock dividends = \$1.0 million
- ◆ Total assets = \$50 million
- ◆ Common stockholders' equity = \$20 million with 2 million shares outstanding.

Calculate the profit margin, basic earnings power ratio, ROA, ROE, and dividend payout ratio.

- 14.12** DirectJet reported a debt-to-equity ratio of 1.75 times at the end of 2016. If the airline's total debt at year-end was \$250 million, how much equity does DirectJet have?

- 14.13** AVJET provides a full menu of airport services to air carriers at small to medium hub airports, and at general aviation airports. The recent financial analysis of the company's financial statements revealed that cash makes up 10 percent of the current assets and accounts receivable makes up another 40 percent of current assets. AVJET's current ratio = 2.0 times, and it has \$15 million in current liabilities. Calculate the value of inventory listed on the company's balance sheet.

- 14.15** Southwest Airlines is a major US airline and the largest low-cost carrier in the world. As of December 2015, Southwest celebrated 43 consecutive years of profitable operation. Using the most recent financial statements of Southwest Airlines, work out:

- a. The current ratio
- b. The quick ratio
- c. The airline's times interest earned ratio
- d. The average collection period
- e. The inventory turnover
- f. The airline's fixed asset turnover ratio
- g. The airline's total asset turnover ratio
- h. The airline's return on sales ratio
- i. The company's return on equity ratio
- j. The company's P/E ratio
- k. The company's market to book value
- l. The Altman Z-score.

- 14.16** Using Southwest Airlines most recent financial statements, compute the following ratios:

- a. Current ratio
- b. Quick ratio

- c. Inventory turnover
- d. Average collection period
- e. Fixed asset turnover
- f. Return on total asset
- g. Return on equity
- h. Time interest rate
- i. Whether the common stockholders are receiving a good return on their investment
- j. CASM (you need to get ASM data)
- k. RPM (you need to find the number of passengers)
- l. How Southwest Airlines is financing its assets.

Notes

- 1 Refer to tables 13.11a and 13.12b
- 2 Data compiled by the authors from respective annual reports. Reporting periods may slightly vary.
- 3 Why “Dynamic” Pricing Based on Real-time Supply and Demand is Rapidly Spreading, *Los Angeles Times*, March 14, 2016.
- 4 Data compiled by the authors from respective annual reports. Reporting periods may slightly vary.
- 5 *The Washington Post*, May 14, 1982.
- 6 CAPA, July 5, 2013.
- 7 Data compiled by the authors from respective annual reports. Reporting periods may slightly vary.
- 8 Sometimes measured as an available seat kilometer (ASK).
- 9 *The Motley Fool*, October 30, 2013.
- 10 *Forbes*, February 21, 2014.
- 11 *Daily Mail*, November 5, 2014.
- 12 Delta Air Lines reported consolidated operations data only as of December 31, 2015.
- 13 Ibid.
- 14 Delta Air Lines reported consolidated operations data only as of December 31, 2015.
- 15 IATA, Airlines Continue to Improve Profitability, December 2010.
- 16 Bureau of Transportation Statistics T-100 Segment data.
- 17 Delta Air Lines reported consolidated operations data only as of December 31, 2015.
- 18 Ibid.

15

Fundamentals of Decision Theory

We will get additional customers. . . We are fully booked for the next three years, where is the problem? Airbus will one day launch an A380neo and will one day launch an A380 stretch.

F. Bregier, CEO Airbus



Business organizations make decisions on a daily basis, focusing on anything from day-to-day operational issues to long-term strategic planning. The majority of business failures, or lack of corporate success, can be attributed to poorly made decisions without proper evaluation of the future outcomes. Understanding of decision theory can help managers make better decisions. Generally, all business decisions involve some degree of risk, and there is no guarantee of a positive outcome. The decision to build a new aircraft, to develop a new jet engine, or to construct a mega airport begins years before the first phase actually happens. In 1956, the British government began research into the development of a supersonic aircraft for commercial services (Hooks et al., 2009). The high fuel consumption created a significant price premium for Concorde's success. The aircraft became very expensive to operate, and as a result did not gain interest among the airline companies. The Arab Oil Embargo of 1973 created a steep rise in fuel prices and Concorde looked less attractive. While carrying a full load, Concorde achieved 15.8 passenger miles per gallon (PMPG) of fuel, compared to the Boeing 747's 46.4 PMPG, the Boeing 707's 33.3 PMPG, and the McDonnell Douglas DC-10's 53.6 PMPG.¹ Concorde failed to gain support among traveling public, and, as a result, the 2003 decommissioning of the Concorde represented the end of supersonic commercial transport.²

Even with proven technology, well-developed capital market, and highly capable labor force, the air transport industry is continually faced with numerous risks and uncertainty. Managers often have to make decisions when many critical factors are evolving or unknown. For instance, the decision to build a new aircraft must be made in the absence of critical information on demand, air traffic growth, potential

mechanical problems, fuel cost, political issues, and the competitive forces of the market.³ Decision theory is a step toward understanding those conditions and knowing how it can influence the process of decision making. Decision theory is interpreted in the form of decision analysis, which shows how best to acquire and understand information before making a decision. This chapter gives a basic idea of how this is achieved and provides a critical review of modern decision making by analyzing selected decision-making environments.

- ◆ Decision-making Process
- ◆ Decision Trees
 - Nodes
 - Branches
- ◆ Decision Environment
 - Decisions under Certainty
 - Decisions under Uncertainty
 - The Optimistic Approach
 - The Conservative Approach
 - The Minimax Regret Approach
 - Equally Likely Approach
 - Decisions under Risk
- ◆ The Economic Theory of Information
 - Expected Value of Perfect Information (EVPI)
 - EV with PI
- ◆ Queueing Theory
 - Customer Arrivals
 - Distribution of Arrivals
 - Arrival Characteristics
 - Arrival Patterns
 - Size of Arrival Units
 - Degree of Patience
 - Waiting Line Characteristics
 - Line Length
 - Number of Lines
 - Queue Discipline
 - First Come, First Served (FCFS)
 - Last Come, First Served (LCFS)
 - Service Facility Characteristics
 - Cost–Capacity Tradeoff
- ◆ Summary
- ◆ Discussion Questions and Problems

Decision-making Process

A wise man makes his own decisions; an ignorant man follows public opinion.

Chinese Proverb

Decision making is the process of making selections by setting objectives, collecting data, and evaluating potential outcomes. To make a good decision, a manager must evaluate all the potential positives and negatives of each option, and consider all conceivable opportunities. Decision making requires comprehensive knowledge of the subject, transparency of judgment, communication skills, firmness of decision, and successful execution. The practical application of decision making is aimed at finding tools, methodologies, and software to help business managers make the best decisions. The decision-making process includes the followings steps:

♦ *Recognize and identify the problem*

The first step in decision making is recognizing that a problem exists. Between 1968 and 1984, Lockheed manufactured a total of 250 L-1011, wide-body commercial aircraft.⁴ Lockheed needed to sell 500 planes to break even; failing to achieve profitability, Lockheed ceased production in 1984. The aircraft's sales were hampered by two years of delays due to financial and operational issues.⁵

Lockheed lost \$2.5 billion on this project, and the TriStar was to be Lockheed's last commercial aircraft.⁶

♦ *Define the objectives*

The goal or objective can be an overall purpose of a business initiative, or a special project. For example, the primary objective for international strategic alliances in the airline industry is to access the foreign markets, and to benefit from the economies of scale resulting from an increased scope of operations. Airline alliances are marketing, financial, and operational collaborations between two or more airlines. Many airlines have been able to reduce costs through the sharing of facilities and services, aircraft maintenance and ground handling by joining airline alliances.⁷ Boeing for many years held a monopoly position in very large aircraft in terms of its 747. Airbus didn't have any airplanes to compete with the 747s and that is why Airbus was motivated to enter this new segment. In June

Aviation Snippets

After the disappearance of Malaysia Airlines' flight MH370, the aviation industry has considered a number of established aircraft tracking solutions but most airlines are still undecided. Singapore Airlines, Royal Brunei and Norwegian Air Shuttle are the latest airlines to decide on this system of aircraft tracking, which will give them oversight over the status of their aircraft both over ground and when crossing the oceans.

Marisa Garcia, Skift,
June 26, 2015

1994, Airbus announced its plan to develop the A380 and the first aircraft was delivered to Singapore Airlines on October 15, 2007.⁸

◆ *Collect the needed data and information*

To make a good decision, we need to collect and gather credible and reliable data. In order to perform successful decision making in the aviation industry, various data are required. In engineering decision making, in addition to the following sources, a major source of useful information is in the company's own financial and operational data.

- Airline for America (A4A)
- Airline Monitor
- Airports Council International (ACI)
- Air Transport Intelligence (ATI)
- Bureau of Economic Analysis (BEA)
- Bureau of Transportation Statistics (BTS)
- Euro-control
- Federal Aviation Administration (FAA)
- International Air Transport Association (IATA)
- International Civil Aviation Organization (ICAO)
- Official Airline Guide (OAG)
- Organization for Economic Cooperation and Development (OECD)
- Security and Exchange Committee (SEC)
- Transport Canada (TC)
- The Aircraft Owners and Pilots Association (AOPA)
- UK Civil Aviation Authority (CAA)
- US Department of Transportation (DOT)

◆ *Identify and evaluate different alternatives*

The project managers should regularly evaluate projects based on their profitability and effectiveness. When evaluating different alternatives, managers generally use either quantitative or qualitative methods.

◆ *Implement and follow up on the solution*

After alternatives have been evaluated, the next step is to make decisions as to which alternatives are the most acceptable to the company.

Decision Trees

It doesn't matter which side of the fence you get off on sometimes. What matters most is getting off. You cannot make progress without making decisions.

Jim Rohn

A decision tree is a graphical tool that uses a branching method to illustrate sequential decision processes. A decision tree is a description of the decisions that are to be made, the events that may occur, and the outcomes associated with combinations of decisions and events. Probabilities are assigned to the events, and respective values

are determined for each outcome. The primary objective of a decision tree is to determine the best decisions.

Decision trees are excellent tools which aid in choosing between several courses of action.

This method provides a highly effective structure within which a decision maker can lay out options and investigate the possible outcomes of choosing those options (Berger, 1985). A balanced picture of the risks and rewards associated with each possible course of action can also be neatly studied.

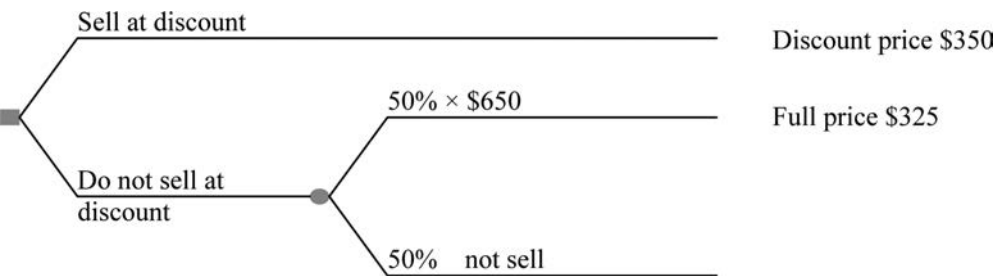
Example 15.1

Assume that DirectJet Airlines has only one seat left in its flight between Philadelphia International Airport (PHL) and Orlando International Airport (MCO). The airline's goal is to make as much as it possibly can from that single seat. DJ can either sell the seat now at \$350, or take the chance of selling the seat between now and the date of departure (DOD) at \$650.

DirectJet's dilemma is that if they set the price at \$650, they have a chance of selling the seat, or the seat may go unsold. On the other hand, if the airline sells the seat at \$350, they may lose the opportunity of selling it at \$650, an opportunity cost of $\$650 - \$350 = \$300$.

The key to determining which situation DirectJet must choose is finding the actual likelihood of selling the ticket at the higher price. In our scenario, the probability of selling the discounted ticket at \$350 is 100 percent, and the probability of selling the full price ticket at \$650 is 50 percent. This of course means that there is also a 50 percent chance that the ticket will not be sold at the full price. In order to make a decision, the expected revenue needs to be computed. The expected revenue is simply calculated by multiplying the probability of selling the ticket by the ticket price. This process provides expected revenue of \$350 for the discounted ticket and expected revenue of \$325 for the full price ticket; therefore, the discounted ticket should be sold.

Figure 15.1 Decision Tree



The main components of decision trees are as follows:

Nodes

There are three main types of nodes in decision trees, namely, square nodes, round nodes, and terminal nodes.

Square nodes are mainly known as “decision nodes.” Decision nodes are points where choices are to be made. All the possible choices or alternatives arising from a square node must be mutually exclusive and collectively exhaustive.⁹

Round nodes are known as “event nodes.” An event node is a point where an uncertainty is resolved. It is the point where the decision maker learns about the occurrence of an event. Similar to choices arising from a square node, all the possible choices or alternatives arising from an event node as well must be mutually exclusive and collectively exhaustive. An event node is sometimes called a “chance node.” Each event is assigned a subjective probability and it is ensured that the sum of all the probabilities is one (North, 1968).

The third kind of node is a **terminal node**, which represents the final result of a combination of decisions and events. Terminal nodes are the endpoints of a decision tree. They are denoted by a triangle or vertical bar on computer-generated diagrams.

Branches

The two types of branches in a decision tree are decision branches and event branches.

Decision branches are the branches extending from the decision nodes, with each branch representing one possible alternative or course of action available at that point.

Event branches are the branches extending from event nodes, and each branch represents one of the possible events that could occur at that point.

A major understanding of decision tree components is that the decision nodes and branches represent the controllable factors in a decision problem, while the event nodes and branches represent the uncontrollable factors.

Example 15.2

The AviaTech Motor Manufacturing Institute (AMMI) is a British jet engine manufacturer established in 1985. AMMI discovers that a general aviation aircraft manufacturer wants to develop a highly efficient fuel injector for a proposed new jet engine. Because the manufacturers do not have enough research personnel available to develop the new fuel injectors, they will subcontract the development to an independent research firm. The aircraft manufacturers have offered a fixed fee of \$250,000 for the best proposal for developing the new fuel injectors. The contract will go to the firm with the best technical plan and the highest reputation for technical competence. AviaTech Research Institute wants to enter the competition.

Management estimates a moderate cost of \$50,000 for preparing a proposal, but they are concerned that they may not win the contract. If AviaTech decides to prepare a proposal, and if they win the contract, their engineers are not sure about how they will develop the fuel injectors. They are considering three alternative approaches. The first approach is a very expensive mechanical method, and the engineers are certain they can develop a successful model with this approach. The mechanical approach

Table 15.1 Nodes and Branches

Type of Node	Written Symbol	Computer Symbol	Node Successor
Decision	Square	Square	Decision branches
Event	Circle	Circle	Event branches
Terminal	Endpoint	Triangle or vertical bar	Terminal value

is estimated to have a cost of \$120,000. A second approach involves electronic components. The engineers think that the electronic approach is a relatively inexpensive method, costing around \$50,000 for developing a model of the proposed fuel injectors, but they are not sure that the results will be satisfactory for the contract. A third inexpensive approach uses magnetic components. The magnetic method costs \$80,000, which is more than the electronic method, and the engineers think that it has a higher chance of success. AviaTech Research can work on only one approach at a time and has time to try only two approaches. If it tries either the magnetic or electronic method and the attempt fails, the second choice must be the mechanical method to guarantee a successful model. Create an example decision tree.

The decision tree for this example, incorporating all the values associated with each choice or event, has been depicted in Figure 15.2.

Terminal value (*TV*) is calculated by determining the sum of all previous choices and events, which have led to one particular terminal node. For example, consider the terminal node that follows electronic failure. The terminal value is obtained by the summation of all the previous expenses or rewards, incurred or gained, in order to reach that result.

$$TV \text{ at Electronic Failure} = -\$50,000 + \$250,000 - \$50,000 - \$120,000 = \$30,000$$

Now, let us consider the various probabilities associated with a certain decision or choice, in order to predict the expected values for the decisions. AviaTech management thinks there is a 50–50 chance of winning the contract. The engineers think that

Figure 15.2 Decision Tree with Terminal Values

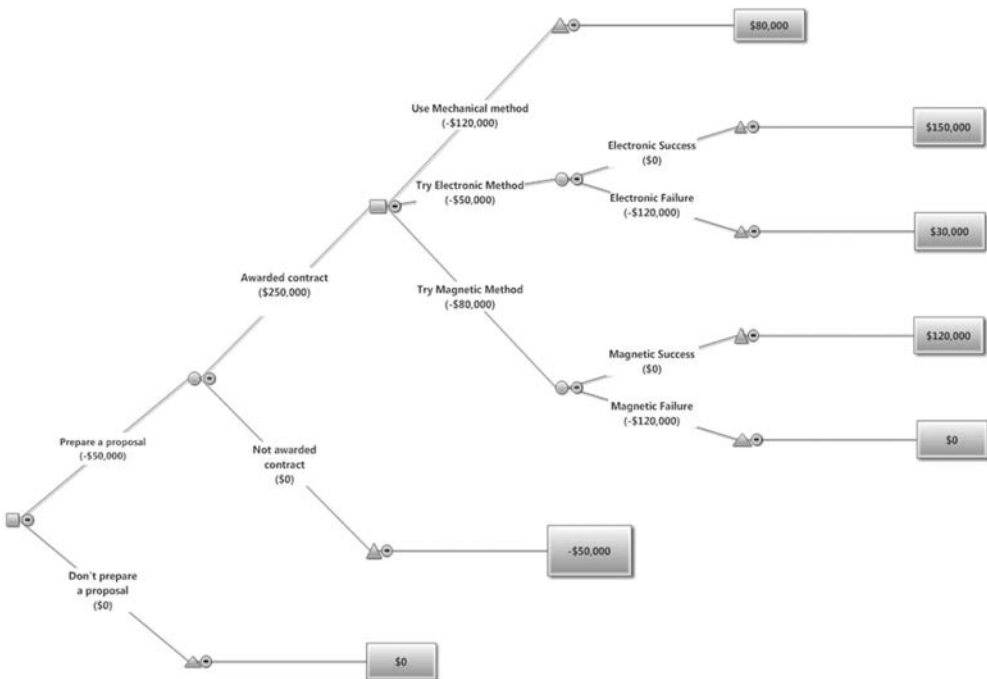
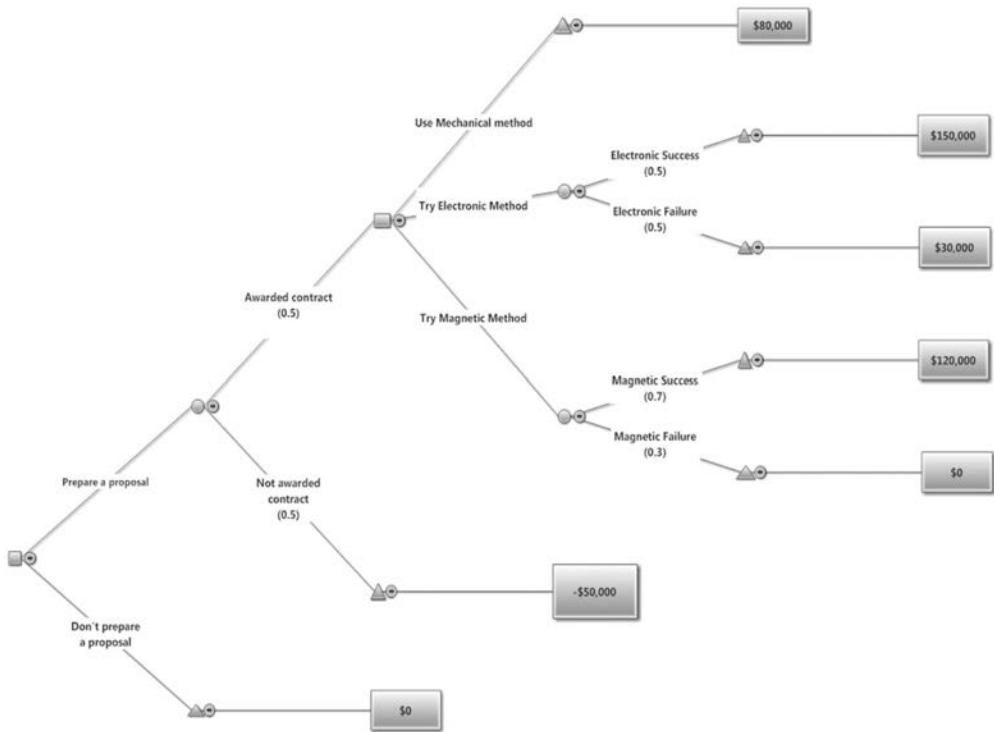


Figure 15.3 Decision Tree with Probabilities and Terminal Values (Complete Tree)

the inexpensive electronic method has only a 50 percent chance of satisfactory results. In their opinion the somewhat more costly magnetic method has 70 percent chance of success.

Expected value (*EV*) is the weighted average of the terminal values, which are determined by multiplying the respective probabilities with the terminal values.

The expected value for the electronic method is given by:

$$EV(\text{electronic method}) = 0.5 \times \$150,000 + 0.5 \times \$30,000 = \$90,000$$

The expected value for the magnetic method is given by:

$$EV(\text{magnetic method}) = 0.7 \times \$120,000 + 0.3 \times \$0 = \$84,000$$

The expected value for the use of the mechanical method is directly the terminal value, which is \$80,000.

Because the expected value for the electronic method is highest, the best decision at that node would be to choose the electronic method.

Decision Environments

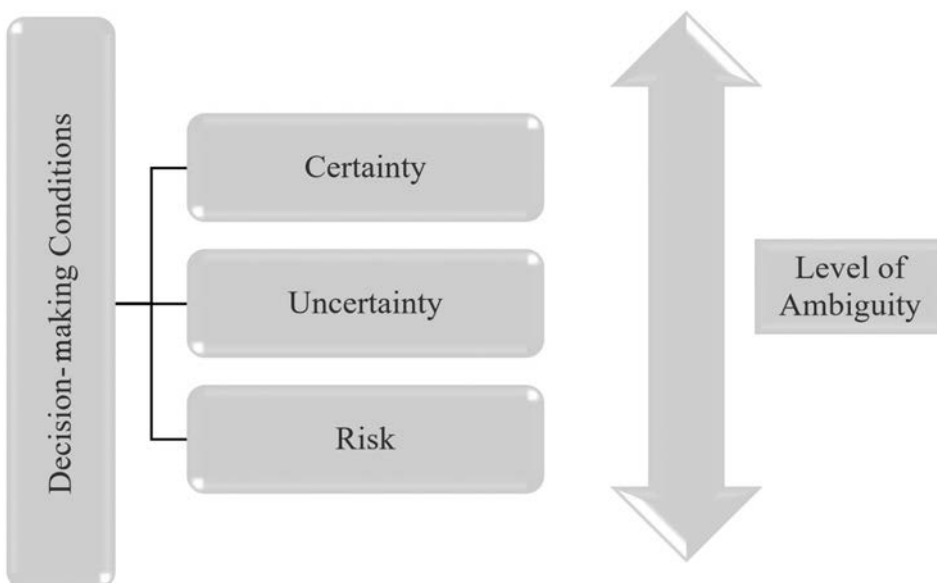
A decision environment is defined as the collection of information, alternatives, values, and preferences available at the time of the decision. An extensive body of literature

has also recognized the importance of decision making in aviation, with the focus predominantly on airlines and airframe manufacturers. The decisions of aviation management are very much affected by these other players. An ideal decision environment would include all available information, as accurate as possible, about every plausible alternative. Managing uncertainty in decision-making relies on identifying, quantifying, and analyzing the factors that can affect outcomes. However, there are two major constraints involved in gaining information or identifying alternatives, namely, time constraint and effort constraint. Time constraint implies that a decision must be made by a certain time. The effort constraint reflects the limits of manpower, money, and priorities. Because decisions must be made within this constrained environment, the major challenge of decision making is uncertainty, and a major goal of decision analysis is to reduce uncertainty (Keeney, 1992). We can almost never have all the information needed to make a decision with certainty, so most decisions involve an undeniable risk and uncertainty.

Aviation Snippets

Boeing enthusiastically embraced outsourcing, both locally and internationally, as a way of lowering costs and accelerating development. The approach was intended to reduce the 787's development time from six to four years and development cost from \$10 to \$6 billion. . . . Even with proven technology, there are major risks in outsourcing that components won't fit together when the plane is being assembled. Not surprisingly, the tool failed. Suppliers did not input accurate and timely information, in part due to cultural differences and lack of trust. As a result, neither Tier-1 suppliers nor Boeing became aware of problems in a timely fashion. Boeing sent hundreds of its engineers to the sites of various Tier-1, Tier-2, or Tier-3 suppliers worldwide to solve various technical problems that appeared to be the root cause of the delay in the 787's development. Ultimately, Boeing had to redesign the entire aircraft sub-assembly process.

Forbes, January 21, 2013



There are three types of environments in which decisions are made:

- ◆ Decisions under certainty
- ◆ Decisions under uncertainty
- ◆ Decisions under risk

Decisions under Certainty

In the decision-making environment under certainty, there is only one type of event that can take place. It is very difficult to find complete certainty in most business decisions. However, almost complete certainty can be noticed in many routine types of decisions (Raiffa, 1997).

Decisions made under certainty are generally of very little significance to the success of a particular business.

Decision making under certainty implies that the future state of nature is certain. (This generally implies that the probability of the desired future occurrence is 1.0, and all other future events have zero probability). The solution, naturally, is to choose the alternative that gives us the most favorable outcome. However, many situations are extremely complex, requiring sophisticated mathematical techniques to find the best solution.

One common technique for decision making under certainty is called linear programming. In this method, a desired outcome (such as profit) can be expressed as a mathematical function (the objective function) of several variables. The solution obtained from the linear programming is in the form of independent variables, also known as decision variables, which serve to maximize the outcome or, in some cases, minimize the cost, subject to certain limits (constraints). There are two major ways to solve a linear programming model, namely, the graphical method and the computer method, which makes use of specific software.

An example for decisions under certainty could be investing in a savings account, buying a certificate of deposit (CD), or the purchase of short-term government bonds, because the payoff for all is insured and guaranteed.

A decision under certainty is the one in which a decision maker knows with certainty which state of event will occur.

Decisions under Uncertainty

In the environment of uncertainty, more than one type of event can take place and the decision maker has no knowledge or control regarding the event that is likely to take place.

Such situations generally arise in cases where the happening of the event is determined by external factors. For example, demand for the product, alterations by competitors, etc. are the factors that involve uncertainty. Under uncertainty, the scope to assign probabilities to the events is extremely minimal as well, ensuring that decisions made in this environment are highly volatile.

Under uncertainty, the decision maker has no knowledge of current conditions or the risks associated with each future outcome. Business decision making is almost always accompanied by conditions of uncertainty.

In certain situations, a decision maker cannot assess the probability of occurrence for the various future states of nature. Uncertainty occurs when there exists more than one future state of nature but the probabilities of each of the states occurring are not known. The decision maker is forced to choose among several possible approaches for making the decision, because there is no concrete way of arriving at a conclusion.

Example 15.3

In order to explain the types of decision making under uncertainty, consider the payoff table for DirectJet Airlines against MyJet Airlines, depicting the payoffs DirectJet is expected to receive based on the pricing behavior of both airlines. The payoff table is shown in Table 15.2 and each section describes the various types of decisions which could be made. A payoff matrix is a simplified representation that shows the possible payouts to each airline based on each potential outcome. The two airlines want to decide whether to increase the price, cut the price or keep the same price. The number in each cell is the payoff (profits) to DirectJet Airlines when both airlines start a new pricing policy. For example, if both airlines decide to increase the ticket prices, DirectJet makes \$12,000,000 profit. If both airlines opt for cutting the price (lower right-hand quadrant) DirectJet loses \$5,000,000. Since DirectJet and MyJet are choosing their price levels at the same time, they are unable to detect beforehand the decision of the other.

The types of decisions made under uncertainty are primarily based on attitudes toward risk.

The Optimistic Approach (Maximax Approach)

The maximax principle suggests the player choose the strategy that yields the best of the best possible outcomes. The assumption is that only the best possible scenarios will occur and only the large payoffs are considered, thereby ignoring all the probabilities. The possible outcomes are normally presented in a table format called payoff table. A payoff table

Under Maximax approach, the best payoff for each alternative should be identified, then the alternative with the maximum payoff is selected.

Table 15.2 DirectJet Payoff Table

	Pricing Behavior		
	DirectJet		
MyJet	Increase	Keep It the Same	Cut
Increase price	\$12,000,000	\$10,000,000	\$5,000,000
Keep it the same	\$2,500,000	\$6,000,000	\$3,000,000
Cut price	\$1,000,000	\$4,000,000	−\$5,000,000

Note: These values are DirectJet’s estimates of profit associated with each alternative.

Table 15.3 DirectJet Payoff Table—Maximax Approach

	Pricing Behavior		
	DirectJet		
MyJet	Increase	Keep It the Same	Cut
Increase price	\$12,000,000	\$10,000,000	\$5,000,000
Keep it the same	\$2,500,000	\$6,000,000	\$3,000,000
Cut price	\$1,000,000	\$4,000,000	−\$5,000,000

Note: These values are DirectJet's estimates of profit associated with each alternative.

consists of columns and rows. Major airlines operate under conditions of oligopoly, where strategy is very important. Because of competition from MyJet Airways, DirectJet has been forced to offer discount flights.

For example, suppose DirectJet is faced with the payoff table given as Table 15.3. The manager has to choose the pricing strategy: increase the price, keep the same price, or cut the price before he knows the actual demand. So in this case if the manager employs the maximax rule, he would look at the first row and pick the first course of action, and increase the price since this gives the largest possible return (\$12,000,000). To summarize, the airline would optimistically assume that competitors would raise their prices, resulting in maximum payoffs, and decide accordingly. For DirectJet, using maximax would mean increasing average ticket price, because that was the alternative with the highest possible payoff, \$12,000,000 per year.

The Conservative Approach (Maximin Approach)

A pessimistic decision maker chooses the conservative approach, which guarantees that the worst outcome would be the “least bad.” The assumption is that the worst that can happen will, and hence, a decision to choose the best worst case scenario is made. The maximin criterion results in the maximum of the minimum payoffs. If the DirectJet management is not optimistic, the airline might select the alternative where

Table 15.4 DirectJet Payoff Table—Maximin Approach

	Pricing Behavior		
	DirectJet		
MyJet	Increase	Keep It the Same	Cut
Increase price	\$12,000,000	\$10,000,000	\$5,000,000
Keep it the same	\$2,500,000	\$6,000,000	\$3,000,000
Cut price	\$1,000,000	\$4,000,000	−\$5,000,000

Note: These values are DirectJet's estimates of profit associated with each alternative.

the worst possible outcome is better than the worst possible outcome from any other alternative. Hence, under the maximin criterion, DirectJet should keep the price the same. Because this worst outcome, \$4,000,000, is better than any other alternative's worst outcome.

The Minimax Regret Approach

Another decision maker may choose the alternative that has the smallest difference between the best and worst outcomes. Regret is considered proportional to the difference between what we actually get, and the better position that we could have received if a different course of action had been chosen. Regret is also referred to as opportunity cost. The minimax regret rule is derived from the behavior of individuals who spend their post-decision time regretting their choices. To solve for the value, Table 15.5, showing the size of the regret, needs to be constructed. This means we need to find the biggest payoff for each demand row, then subtract all other numbers in this row from the largest number. Look what will happen if the competition increases the ticket price: DirectJet will make a maximum profit of \$12,000,000 if they increase the price. If DirectJet cuts the price, the opportunity cost will be (\$12,000,000 – \$10,000,000 = \$2,000,000). Likewise, the regret value for cutting the price is equal to \$7,000,000.

If DirectJet uses the minimax regret criterion, it will want to minimize that maximum regret, and therefore keep the price the same.

Equally Likely Approach

A different type of decision maker may assume that all possible states of nature are equally likely, and hence set all the probability values to be equal to (1/n), where n is the number of possible states of nature, and maximize expected value based on that assumption.

For example, if there are three possible states of nature, as in the case of DirectJet, and each of the possible outcomes is equally likely, then:

$$P_1 = P_2 = P_3 = \frac{1}{3}$$

Table 15.5 DirectJet Payoff Table—Minimax Approach

	Pricing Behavior		
	DirectJet		
MyJet	Increase	Keep It the Same	Cut
Increase price	\$0	\$2,000,000	\$7,000,000
Keep it the same	\$4,500,000	0	\$3,000,000
Cut price	\$3,000,000	\$0	\$9,000,000

Note: These values are DirectJet's estimates of opportunity cost associated with each alternative.

The equal likelihood criterion multiplies the decision payoff for each pricing behavior by an equal weight, thus assuming that the states of nature are equally likely to occur:

$$\begin{aligned}\text{Payoff (Increase Price)} &= \$12,000,000 \times \frac{1}{3} + \$2,500,000 \times \frac{1}{3} + \$1,000,000 \times \frac{1}{3} \\ &= \$5,166,666.67\end{aligned}$$

$$\begin{aligned}\text{Payoff (Same Price)} &= \$10,000,000 \times \frac{1}{3} + \$6,000,000 \times \frac{1}{3} + \$4,000,000 \times \frac{1}{3} \\ &= \$6,666,666.67\end{aligned}$$

$$\begin{aligned}\text{Payoff (Cut Price)} &= \$5,000,000 \times \frac{1}{3} + \$3,000,000 \times \frac{1}{3} - \$5,000,000 \times \frac{1}{3} \\ &= \$1,000,000\end{aligned}$$

Hence, the option of keeping the price the same is preferred with respect to the equally likely approach because it returns the maximum expected value (Table 15.6).

Example 15.4

Consider an example of an aircraft manufacturing company, AvParts, trying to decide whether they need to research and build a new aircraft to compete in a market which is currently controlled by Airbus. The three possible decisions are: don't build; research and build the AvParts 7E7; or contract out the research and manufacturing to a third party to whom a share of the profit needs to be given. The above decisions along with the three possible market outcomes are illustrated in Table 15.7.

The three possible outcomes could be a result of a reaction from a competitor, in this case Airbus, or could even arise from an economic crisis or improvement in the market at the time of the launch. For example, if AvParts contracted the research and manufacturing to a third party, agreeing to pay them a share of the profits, and this resulted in only a small market claim, the very small profit made would have to be shared with the third party. This implies AvParts would only get a meagre \$20,000 as

Table 15.6 DirectJet Payoff Table—Equally Likely Approach

	Pricing Behavior			
	DirectJet			
MyJet	Increase	Keep It the Same	Cut	Probability
Increase price	\$12,000,000	\$10,000,000	\$5,000,000	(1/3)
Keep it the same	\$2,500,000	\$6,000,000	\$3,000,000	(1/3)
Cut price	\$1,000,000	\$4,000,000	−\$5,000,000	(1/3)
Equally likely payoff	\$5,166,667	\$6,666,667	\$1,000,000	

Note: These values are DirectJet's estimates of profit associated with each alternative.

Table 15.7 AvParts Example—Decision under Uncertainty

	Small Market Claim	Moderate Market Claim	Large Market Claim
Don't build	\$0	\$0	\$0
Research and build 7E7	−\$2,000,000	\$26,000,000	\$145,000,000
Contract it to third party	\$20,000	\$12,000,000	\$70,000,000

Table 15.8 AvParts Example—Decision under Uncertainty, Maximax Approach

	Maximum
Research and build 7E7	\$145,000,000
Contract it to third party	\$70,000,000

Table 15.9 AvParts Example—Decision under Uncertainty, Maximin Approach

	Minimum
Research and build 7E7	−\$2,000,000
Contract it to third party	\$20,000

profit. On the other hand, if AvParts themselves did the research and built the aircraft, and managed only a small market claim, there would be an overall loss of about \$2 million because the money spent on research and manufacturing would not be recovered. On the contrary, if AvParts managed a moderate or a large market claim, the profits would be considerably larger because there would be no requirement to share the profits with a third party.

The decision an optimistic decision maker would make in such a situation would use the “maximax approach,” which measures the maximum possible profit in each choice among the possible outcomes and selects the maximum among the maximum profits as the best outcome to be chosen. The maximum profits are given in Table 15.8.

Hence under this approach, AvParts would choose to research and build the 7E7 by themselves.

The decision a pessimistic decision maker would make in such a situation would use the “maximin approach,” which measures the minimum profits made in each choice among the three outcomes and selects the maximum of these minimum profits as the best possible outcome to be chosen. The minimum profits are given in Table 15.9.

Hence under this approach, AvParts would choose to contract the research and manufacturing to a third party.

Table 15.10 AvParts Example—Decision under Uncertainty, Equally Likely Approach

	Equally Likely (Expected Values)
Research and build 7E7 (E_1)	\$56,333,333
Contract it to third party (E_2)	\$27,340,000

A more neutral decision maker would choose to use the “equally likely approach,” which assigns an equal probability for all the three outcomes in all the cases, and chooses the one which gives the maximum expected value in this approach. The values obtained from the approach are illustrated in Table 15.10.

$$E_1 = \frac{-2,000,000 + 26,000,000 + 145,000,000}{3}$$

$$E_1 = \$56,333,333$$

$$E_2 = \frac{20,000 + 12,000,000 + 70,000,000}{3}$$

$$E_2 = \$27,340,000$$

Hence, the option of AvParts doing the research and manufacturing by themselves, gives the highest expected value of \$56,333,333, implying that the best choice in this approach is for AvParts to research and build the 7E7 themselves.

The last course of action a decision maker could take under uncertainty involves the “minimax regret approach.” Regret is defined as the difference between the profit from the best alternative and the profit from the chosen alternative, given a particular outcome is fixed. Also, regret can never be negative. The maximum regrets are calculated in a separate matrix and the minimum among the maximums is chosen as the best alternative because it is the option with the least amount of money lost. The regret analysis for the above example is shown in Table 15.11.

Under risk, the decision maker has some knowledge regarding probability of future outcomes.

Table 15.11 AvParts Example—Decision under Uncertainty, Regret Analysis

	Small Market Claim	Moderate Market Claim	Large Market Claim	Maximum Regret
Don't build	\$20,000	\$26,000,000	\$145,000,000	\$145,000,000
Research and build 7E7	\$2,020,000	\$0	\$0	\$2,020,000
Contract it to third party	\$0	\$14,000,000	\$75,000,000	\$75,000,000

$$\text{Regret} = \{\text{Profit from best alternative}\} - \{\text{Profit from chosen alternative}\}$$

Among the maximum regrets, the minimum is obtained when AvParts chooses to research and build the 7E7 by themselves. Hence, using the minimax regret approach, AvParts must decide to research and build the 7E7 aircraft by itself.

Decisions under Risk

The main difference between decisions under uncertainty and decisions under risk is that the decision maker under risk has some information about the probability of the possibility of each of the possible outcomes. Under the condition of risk, there is more than one possible event that can take place. However, the decision maker has adequate information to assign probabilities to the happening or non-happening of each possible event. Such information is generally based on prior experience (French and Insua, 2000). Let us start with an easy question such as: what is the probability of drawing an ace from a shuffled pack of cards? Well, there are four aces so the probability of getting an ace would be $4/52$, or $1/13$. Virtually, every decision in a modern business enterprise is based on interplay of a number of factors and the risks associated with them.

The most common tool used to determine the best decision under risk is calculating the expected value.

Example 15.5

In the AvParts example illustrated in the previous section, consider Table 15.12, with the probability of occurrence of the outcomes assigned as follows. The probability of a small market claim is assumed to be 0.6, moderate market claim to be 0.3, and large market claim to be 0.1.

It can be noted that alternative A_3 has a better value than A_1 in all the outcomes and hence A_3 can be considered dominant over A_1 , thereby, eliminating A_1 . The expected monetary value (EMV) for A_2 and A_3 is calculated as follows:

$$EMV_2 = (0.6 \times -2,000,000) + (0.3 \times 26,000,000) + (0.1 \times 145,000,000)$$

$$EMV_2 = \$21,100,000$$

$$EMV_3 = (0.6 \times 20,000) + (0.3 \times 12,000,000) + (0.1 \times 70,000,000)$$

$$EMV_3 = \$10,612,000$$

Table 15.12 AvParts Example—Decision under Risk

	Small Market Claim $P_1 = 0.6$	Moderate Market Claim $P_2 = 0.3$	Large Market Claim $P_3 = 0.1$
Don't build (A_1)	\$0	\$0	\$0
Research and build 7E7 (A_2)	−\$2,000,000	\$26,000,000	\$145,000,000
Contract it to third party (A_3)	\$20,000	\$12,000,000	\$70,000,000

Expected monetary value is the weighted average of a certain outcome based on the probabilities of occurrence of all the events. Purely based on expected monetary value, alternative A_2 is better as it has a higher expected value than A_3 . But a decision maker will choose alternative A_2 only if he or she is ready to accept the risk of losing \$2,000,000.

Example 15.6

Another common meaning of risk is variability of outcome, measured by the variance or (more often) its square root, the standard deviation. Consider two investment projects, X and Y, having the discrete probability distribution of expected cash flows in each of the next several years as shown in Table 15.13.

Expected value of cash flows for X and Y are as follows:

$$E_X = (0.1 \times 7,000) + (0.2 \times 7,500) + (0.4 \times 8,000) + (0.2 \times 8,500) + (0.1 \times 9,000)$$

$$E_X = \$8,000$$

$$E_Y = (0.1 \times 6,000) + (0.25 \times 7,000) + (0.3 \times 8,000) + (0.25 \times 9,000) + (0.1 \times 10,000)$$

$$E_Y = \$8,000$$

Although both projects have the same expected cash flows, the expected values of the variances (squares of the deviations from the expected cash flows) differ as follows:

$$V_X = 0.1 \times (7,000 - 8,000)^2 + 0.2 \times (7,500 - 8,000)^2 + 0.4 \times (8,000 - 8,000)^2 + 0.2 \times (8,500 - 8,000)^2 + 0.1 \times (9,000 - 8,000)^2$$

$$V_X = 300,000$$

$$V_Y = 0.1 \times (6,000 - 8,000)^2 + 0.25 \times (7,000 - 8,000)^2 + 0.3 \times (8,000 - 8,000)^2 + 0.25 \times (9,000 - 8,000)^2 + 0.1 \times (10,000 - 8,000)^2$$

$$V_Y = 1,300,000$$

The standard deviations are square roots of the above variance values.

$$\sigma_X = 547.72$$

$$\sigma_Y = 1140.18$$

Table 15.13 Risk as Variance Example

X		Y	
Probability	Cash Flow	Probability	Cash Flow
0.10	\$7,000	0.10	\$6,000
0.20	\$7,500	0.25	\$7,000
0.40	\$8,000	0.30	\$8,000
0.20	\$8,500	0.25	\$9,000
0.10	\$9,000	0.10	\$10,000

Because project Y has the greater variability (whether measured in variance or in standard deviation), it must be considered to offer greater risk than does project X. Hence, it is a better option to choose project X over project Y.

The Economic Theory of Information

The economics of information is a subdivision of microeconomic theory that analyzes how information and information systems can affect the economic decision-making process (Allen, 1990). Information has some special characteristics, such as easy to create but hard to trust and easy to spread but hard to control. These special characteristics (as compared with other types of goods) complicate many standard economic theories. The starting point for economic analysis is the understanding that information has economic value as it enables individuals to make choices that yield higher expected returns compared to a return obtained from a decision or choice made under the absence of information.

Trading of information is not the same as buying and selling most other goods. A number of factors are responsible for making the economics of buying and selling information different from common types of goods. Information is non-rivalrous, implying that consuming information does not exclude another entity from also consuming it (Arrow, 1999). A related characteristic that alters information markets is that information has almost zero marginal cost. This means that duplicating already existing information costs absolutely nothing or close to nothing. Also, **exclusion** is not a natural property of information goods. The nature of information is that if it is known, it is difficult to exclude other individuals from its use. Another factor which affects the economics of information is that the degrees of **transparency** exhibited by the information market are not explicably high, i.e., to evaluate the information, the information must be known, and this forces one to invest in learning it, in order to evaluate it.

Expected Value of Perfect Information (EVPI)

EVPI measures the amount of benefit one can obtain on a decision if the occurrence of each state of nature is known at all times. EVPI gives a measure of how much an individual is willing to pay for perfect information. It is mathematically given by the difference between the expected value with perfect information ($EV|PI$) and the maximum of the expected monetary values among the possible outcomes.

$$EVPI = EV|PI - EMV$$

EV with PI

$EV|PI$ is the expected value, given the knowledge of perfect information, regarding which outcomes will provide maximum profits based on the choices made. Perfect information is said to be available when a 100 percent accurate prediction can be made about the future. This concept can be better explained using an example.

Example 15.7

Consider an investment decision that needs to be made between three alternatives, namely, a stock, a mutual fund or a certificate of deposit (CD). Further assume that

the market has a 50 percent chance of increasing, a 30 percent chance of staying even, and a 20 percent chance of decreasing. If the market increases, the stock investment will earn \$1,700 and the mutual fund will earn \$1,100. If the market stays even, the stock investment will earn \$500 and the mutual fund will earn \$800. If the market decreases, the stock investment will lose \$1,000 and the mutual fund will lose \$400. The certificate of deposit will earn \$700 independent of the market fluctuation.

The first step is to calculate the expected monetary values for all the three alternatives based on the conditions provided.

$$EMV_{stock} = 0.5 \times 1,700 + 0.3 \times 500 + 0.2 \times (-1,000)$$

$$EMV_{stock} = \$800$$

$$EMV_{MF} = 0.5 \times 1,100 + 0.3 \times 800 + 0.2 \times (-400)$$

$$EMV_{MF} = \$710$$

$$EMV_{CD} = 0.5 \times 700 + 0.3 \times 700 + 0.2 \times 700$$

$$EMV_{CD} = \$700$$

The maximum of the calculated expected monetary values is obtained for the stock. Hence, in this example, $EMV = \$800$

$EV \mid PI$ can be calculated by assuming the maximum profit in each of the choices given the state of the market is known in advance, i.e., knowledge of perfect information is already given. Hence the maximum profit option under each market state is used to calculate the $EV \mid PI$.

$$EV \mid PI = 0.5 \times 1,700 + 0.3 \times 800 + 0.2 \times 700$$

$$EV \mid PI = \$1,230$$

From the formula mentioned earlier:

$$EVPI = EV \mid PI - EMV$$

$$EVPI = 1,230 - 800$$

$$EVPI = \$430$$

The conclusion drawn from this is that if any individual is willing to purchase the perfect information regarding the direction the market will take, the cost of such an information should be less than \$430. If it is priced at anything over the $EVPI$, it will only result in a loss.

Assume the price for the information was \$410 and the individual purchased it. Then he would expect to make $\$1,230 - \$410 = \$820 > \$800$ (EMV). Therefore, by purchasing the information, the individual was able to make \$20 more than if he hadn't purchased the information.

Suppose the price for the information was \$440 and the individual purchased it. Then he would expect to make $\$1,230 - \$440 = \$790 < \$800$ (EMV). Therefore, by purchasing the information, he lost \$10 when compared to not having purchased the information.

The third possibility is that the price for the information was \$430 and the individual purchased it. Then he would expect to make $\$1,230 - \$430 = \$800 = \$800$ (EMV). Therefore, by purchasing the information, he neither gained nor lost any money when compared to not purchasing the information.

Queueing Theory

Queueing theory is the mathematical study of waiting lines, or queues. The primary objective is to construct a model, so that queue lengths and waiting time can be predicted. Queueing theory is normally considered a branch of operations research, as the results are often used when making business decisions about the resources needed to provide a service. A queueing model has been used to predict how long a plane will wait before takeoff, how long a plane will wait before landing given runway traffic, and weather traffic. The model may help air traffic controllers to minimize runway congestion. The aviation industry as a whole can be viewed as a queueing network:

- ♦ Aircraft de-icing
- ♦ Aircraft landing queue
- ♦ Aircraft waiting for fuel
- ♦ Aircraft waiting for parts
- ♦ Airport check-in counters
- ♦ Airport security checks
- ♦ Airport border control and immigrations
- ♦ Baggage claim
- ♦ Passengers waiting for boarding

The queueing theory was constructed so that the waiting time in the queue can be predicted. Queueing theory was introduced for the first time with the work of Erlang in 1909 (Post et al., 2008). Erlang's work with the Copenhagen Telephone Company is what prompted his initial foray into the field. Erlang studied the problem of determining how many telephone circuits were necessary to provide a phone service that would prevent customers from waiting too long for an available circuit. In developing a solution to this problem, he began to realize that the problem of minimizing waiting time was applicable to many fields, and began developing the theory further.¹⁰

Aviation Snippets

While both Airbus and Boeing have a strong order book, Airbus is running ahead in the race. This year Airbus has received 708 net new plane orders as compared to 447 for Boeing. Overall the backlog for Airbus currently stands at 6,697 planes as compared to 5,710 for Boeing. It will take 10 years for Airbus and more than 8 years for Boeing to complete these orders at current production rate. While this is good for both the companies as it supports long-term cash flows, the former would seem to have greater momentum. As we have noted previously, however, Airbus has greater market share in the single-aisle segment of the market, largely on the strong position of the A320. Boeing has the strong position in the wide-body segment, where lower unit counts and higher selling prices prevail. Also, it's not just Airbus it is competing with, there are more new players, such as China's Comac, Russia's Irkut and Mitsubishi's Regional Jet. All these aircraft already have an order book to cater to, reflecting the high demand for aircraft and impatience of airlines for long lead times to delivery. If the order cancellations intensify for Boeing going forward, it will result in lower global market share of airplane deliveries. While we believe that Boeing will be able to maintain its current market share of 45%, if Boeing's market share falls to 40%, then there could be a potential downside of around 10% to our price estimate. This is possible as the waiting period for airlines is very long with Boeing as well as Airbus.

Forbes, September 11, 2015

Queues are an imminent day-to-day occurrence in a number of different places. The choice to provide too much service involves incurring excessive costs, while not providing enough service capacity causes waiting lines to become excessively long. The ultimate goal or outcome expected from queueing theory is to achieve an economic balance between the cost of service and the cost associated with the waiting for that service. Queueing theory describes probabilistically and mathematically, the interaction between the arrival process of customers and the service provided to them in order to manage the system in an efficient manner.

The queueing system consists essentially of three major components:

- ◆ the source population and the customer arrival system,
- ◆ the servicing system or queue discipline, and
- ◆ the condition of the customer's exiting the system.

A detailed discussion of arrival patterns and distribution of arrivals along with some waiting line characteristics and service facility characteristics is provided in this section.

Customer Arrivals

Customer arrivals at a service system may be drawn from a finite or an infinite population. It is important to distinguish between the two, because the analyses are based on different premises and require different treatments for an optimum solution.

A finite population refers to the limited-size customer pool that utilizes the service and, at times, forms a line. The number of units in a finite population is symbolized by N , and thus N represents the size of the population. The reason this finite classification is important is that when a customer ceases to be a member of the population, the size of the user group is reduced by one, which reduces the probability of the next requirement of service. An example could be a set of jet engines serviced by a mechanic. The engines form a finite population as customers that require service. If one engine breaks down, the probability of another engine breaking down reduces.

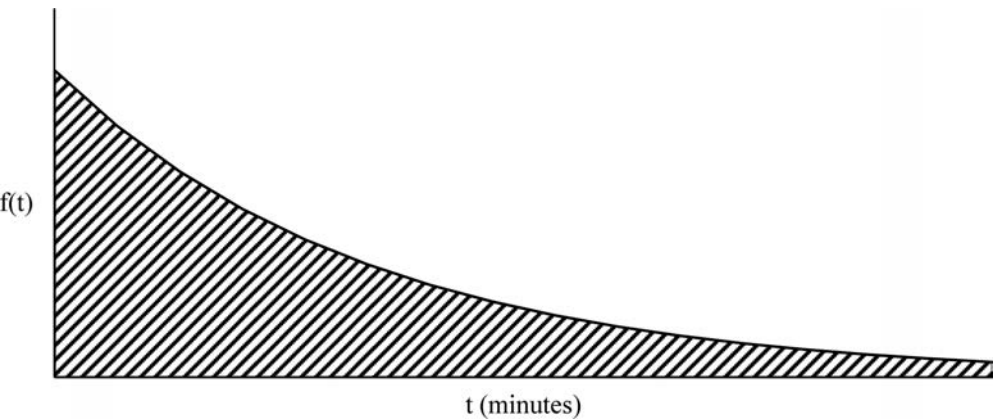
An infinite population is large enough in comparison to the service system, that the population size caused by reductions or additions to the population (a customer requiring service or a serviced customer returning to the population) does not significantly affect the system probabilities. The example for an infinite population, as a modification of the engine example, would be, if there were 100 jet engines serviced by a single mechanic. In this case, the removal of one engine does not much affect the possible breakdown of another.

Distribution of Arrivals

Waiting line formulas generally require an **arrival rate (λ)**, or the number of units per period, arriving at a service system (such as an average of one every five minutes). A constant arrival distribution is periodic, with exactly the same time between successive arrivals. In reality, only systems subject to machine control have a constant interval period. Variable (random) arrival distributions are more practical and common.

Observation of arrivals at a service facility can be done using two methods.

Figure 15.4 Exponential Probability Distribution



First, the time between successive arrivals can be calculated and analyzed to see if the times follow any kind of a statistical distribution. Usually, the time between arrivals is assumed to be exponentially distributed.

Second, a certain time length (T) can be set and the number of arrivals entering the system within T calculated. A typical assumption is that the number of arrivals per time unit is Poisson distributed.

In the first case, when arrivals at a service facility occur in a purely random fashion, a plot of the inter-arrival times yields an **exponential distribution** (Figure 15.4). The probability function for an exponential distribution is as follows:

$$f(t) = \lambda e^{-\lambda t}$$

Where λ is the mean number of arrivals per time period (t).

The summation of the above equation over its positive range is $e^{-\lambda t}$. This integral allows us to compute the probabilities of arrivals within a specified time. For example, for the case of single arrivals to a waiting line ($\lambda = 1$), Table 15.14 can be derived by solving $e^{-\lambda t}$.

Table 15.14 Probabilities for Exponential Distribution

t (minutes)	Probability that the next arrival will occur in t minutes or more [$f(t)$]	Probability that the next arrival will occur in t minutes or less
0.0	1.00	0.00
0.5	0.61	0.39
1.0	0.37	0.63
1.5	0.22	0.78
2.0	0.14	0.86

In the second case, the number of arrivals (occurrences) during some time period T is the primary consideration, and is obtained by determining the probability of exactly n arrivals (occurrences) during T . If the arrival (occurrence) process is random, the distribution is referred to as the **Poisson distribution**, and the formula is as follows:

$$P_T(n) = \frac{(\lambda T)^n e^{-\lambda T}}{n!}$$

Where n is the number of arrivals (occurrences) in a given time period T , for which the probability is to be found.

Example 15.8

The annual worldwide average number of airplane crashes of commercial airlines is 3 ($\lambda = 3$). What is the probability that exactly five accidents occur in a given year ($n = 5$, $T = 1$)?

$$P_1(5) = \frac{(3 \times 1)^5 e^{-3}}{5!}$$

$$P_1(5) = \frac{3^5 \times e^{-3}}{120}$$

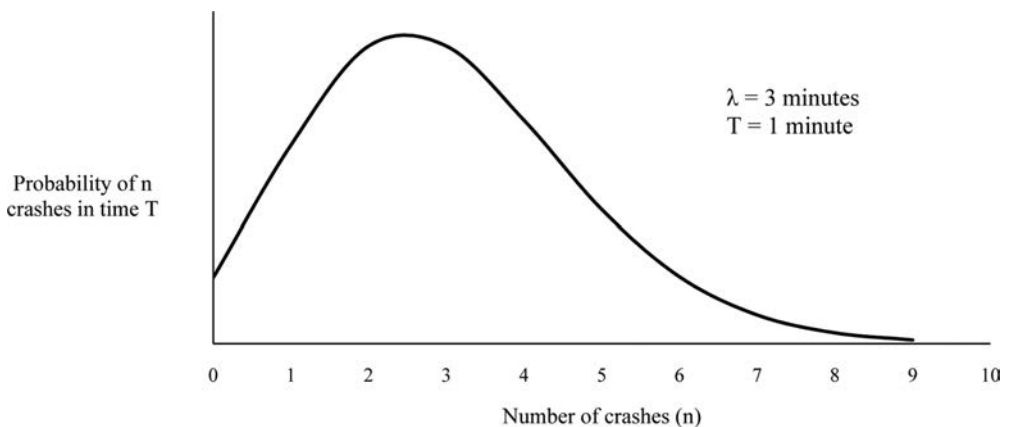
$$P_1(5) = 2.025 e^{-3}$$

$$P_1(5) = 0.101$$

That is, there is a 10.1 percent chance that there will be five accidents in any given year.

Hence, it can be noted that the time between arrivals (occurrences) is exponentially distributed and the number of arrivals (occurrences) per unit of time is Poisson distributed.

Figure 15.5 Probabilities for Poisson Distribution



Another important feature of the waiting structure is the time the customer spends with the server once the service has started. Waiting line formulas generally specify **service rate (μ)** as the capacity of the server in number of units per time period (such as 12 completions per hour). A constant service time rule states that each service takes exactly the same time. This characteristic is similar to constant arrivals, and is generally limited to machine-controlled operations. When service times are random, they can be approximated by the exponential distribution. In this distribution, μ is referred to as the average number of units or customers that can be served per time period.

Arrival Characteristics

Arrival characteristics mainly include:

- ♦ Arrival Patterns
- ♦ Size of Arrival Units
- ♦ Degree of Patience

Arrival Patterns

The arrivals at a system are far more controllable than is generally recognized. Departmental stores primarily hold one-day only sales in part for purposes of control. Airlines offer excursion and off-season rates as well, so as to control the arrival pattern. The simplest among all arrival-control devices is the posting of business hours.

Size of Arrival Units

A single arrival is considered to be one unit. A single arrival on the floor of the London Stock Exchange (LSE) is 100 shares of stock; a single arrival at an egg-processing plant might be a dozen eggs; a single arrival at a restaurant is a single person.

A batch arrival is some multiple of the unit, such as a block of 1,000 shares on the NYSE, a case of eggs at the processing plant, or a party of five at a restaurant.

Degree of Patience

A patient arrival is one who waits as long as necessary until the service facility is ready to serve him or her. There are two types of impatient arrivals. Members of the first type arrive, analyze both the service facility and the length of the line, and then decide to leave. Those belonging to the second type, arrive, view the situation, join the waiting line, and then, after some period of time, depart. The behavior of the first type is termed **balking**, while the second is termed **reneging**.

Waiting Line Characteristics

The three important waiting line characteristics are:

- ♦ Line Length
- ♦ Number of Lines
- ♦ Queue Discipline

Line Length

A line length is considered *infinite*, if the line is very long in terms of the capacity of the service system. Examples of infinite potential length are a line of vehicles stuck in

traffic for miles at a certain crossing and theatre enthusiasts who form a line around the block as they wait to purchase tickets, for example, on Broadway.

Parking lots, loading docks, and gas stations have limited line capacity caused by legal restrictions or physical space restrictions. This complicates the waiting line problem not only in service system utilization and waiting line computations but also in the shape of the actual arrival distribution.

A particular arrival denied entry into the waiting line because of lack of space, may rejoin the population for a later try or may seek service elsewhere. Either of the two actions can have a significant effect on the queue functions for the finite case.

Number of Lines

A single line or single file refers to the existence of one line only. The term “multiple lines” refers to the single lines that form in front of two or more servers (counters) or to single lines that converge at some central redistribution point. The disadvantage of multiple lines in a busy facility is that arrivals often shift lines if several previous services have been of short duration or if those customers currently in other lines appear to require a short service time.

Queue Discipline

A queue discipline is a priority rule or set of rules for determining the order of service to customers in a waiting line. The discipline selected can have a vast effect on the system’s overall performance. The number of customers in line, the range of variability in waiting time, the average waiting time and the efficiency of the service facility are just a few of the factors affected by the choice of queue discipline.

The major types of queue disciplines are as follows:

- ◆ First Come, First Served (FCFS)
- ◆ Last Come, First Served (LCFS)

First Come, First Served (FCFS)

The most common queue discipline followed is the first come, first served (FCFS) or first in, first out (FIFO) model. This rule states that customers in line are served purely on the basis of their chronological arrival. This is popularly accepted as the fairest rule, although in practice it discriminates against the arrival requiring a short service time. When checking out at a store that is using first come, first served as a priority rule, a customer waiting behind another customer with a large number of items waits longer than a customer waiting behind a second customer with only a few items. Although processing is sequential, the wait times vary because of the preceding customer. Check-in process at the airport can be another example of first come, first served model: when no seat reservation option is available, passengers who check-in earlier get into the airplane first. However, there are some situations where the first come, first served rule does not serve as the most effective one. For example, a first come, first served rule doesn’t make sense in a hospital emergency room and in fact, could cause unnecessary deaths.

Last Come, First Served (LCFS)

Another major type of queue discipline is the last come, first served (LCFS) or last in, first out (LIFO) model. This rule follows a policy where the last customer in the

queue is served first. A practical example of LIFO is an elevator, where the last person entering the elevator becomes the first person to leave it, assuming all the members get on and get off the elevator at the same floors. Similarly, at the airport passengers who get on the bus last, will be the first to board the airplane.

Reservations first, emergencies first, highest-profit customer first, largest orders first, best customers first, longest waiting time in line, and soonest promised date are other examples of service disciplines or priority rules. There are two major practical problems in using any rule or discipline: one is ensuring that customers know and follow the rule, and the other is ensuring that a system exists to enable employees to manage the line.

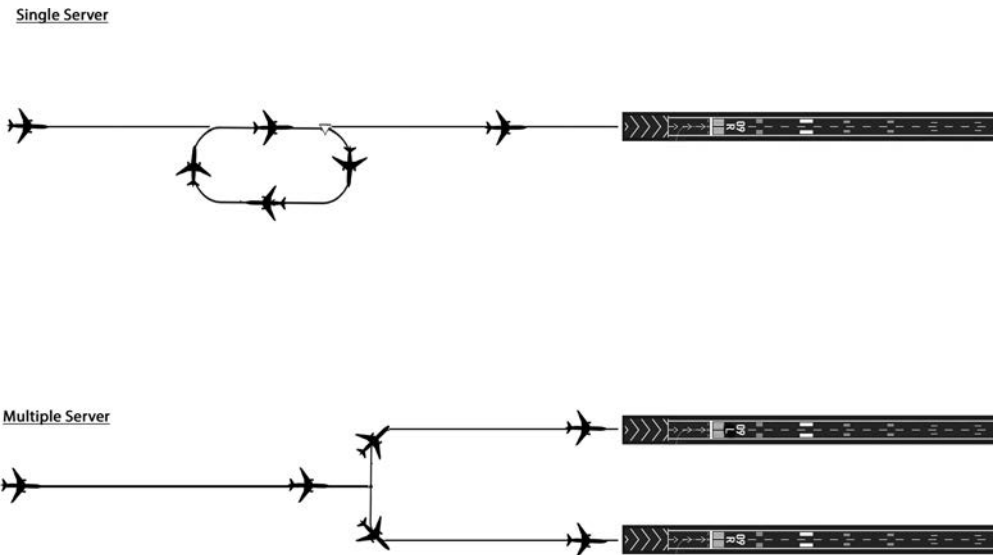
Service Facility Characteristics

A queue may be a single queue or a multiple queue, and the service facility can have one server, or multiple servers. A single service channel may have a single server, or multiple servers. Airport securities may have multiple lines and multiple servers. Aircraft landing at an airport with a single landing strip is an example of a single line and single server.

At busy airports there is usually a long line of aircraft descending toward a runway. If the runway is busy, the air traffic controllers maintain the aircraft in a holding pattern awaiting their respective landings.

The most important terms associated with determining the different aspects of a single line, single server waiting line (such as the average number of items in the system, the average time spent by an item at the server, the average number of items in the queue, and the average time spent by an item in the queue) are illustrated with the help of the following example.

Figure 15.6 Single and Multiple Server Aircraft Landing at the Airport



Example 15.9

In Allen Town Regional Airport (ATR), aircraft arrive at an average rate of 20 aircraft per hour and follow a normal distribution. The runway can clear 30 aircraft per hour. Determine the following:

- a. The average number of aircraft in the system (the number in the landing area plus the number of aircraft served)

$$\lambda = 20$$

$$\mu = 30$$

L is the number of aircraft in the system

$$L = \frac{\lambda}{\mu - \lambda}$$

$$L = \frac{20}{30 - 20}$$

$$L = 2 \text{ aircraft}$$

- b. The average time an aircraft spends on the runway (W)

$$W = \frac{1}{\mu - \lambda}$$

$$W = \frac{1}{30 - 20}$$

$$W = 6 \text{ minutes}$$

- c. The average number of aircraft in the queue (L_q)

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$L_q = \frac{20^2}{30(30 - 20)}$$

$$L_q = 1\frac{1}{3} \text{ aircraft}$$

- d. The average time an aircraft spends waiting for clearance (W_q)

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$W_q = \frac{20}{30(30 - 20)}$$

$$W_q = 4 \text{ minutes}$$

A further extension of waiting line characteristics could be the cost/benefit analysis of adding an additional server to an existing single server system, making it a multi-server system. An illustration of a similar case is presented in the example below.

Example 15.10

In the Allen Town Regional Airport (ATR), aircrafts requiring take-off request permission from air traffic control and have to wait for confirmation. Aircrafts arrive for take-off in a random (Poisson) fashion at the rate of 40 aircraft per hour, and a single runway can accommodate take-offs at the rate of 20 per hour (exponential). If the cost to operate a new runway is assumed to be \$6,000 per hour and the cost of an aircraft waiting to take off is \$12,000 per hour, determine the optimum number of runways to run the airport efficiently. (Because of the high arrival rate, an infinite source may be assumed.)

- ♦ λ = Arrival rate
- ♦ μ = Service rate
- ♦ L_q = Average number in waiting line
- $\lambda = 40$
- $\mu = 20$

In this case we assume the number of servers, i.e., runways to be three because two or less would mean the system is inefficient and would never be satisfied.

Given the number of servers (S) equals 3, and λ/μ equals 2, L_q can be found from the table which gives the average number in waiting line for a single queue, multi-server model.

From that table, L_q is found to be 0.8888 aircrafts waiting all through the day.

Considering a 16-hour day, at \$12,000 per hour, there is a loss of aircraft time worth:

$$0.888 \times 12,000 \times 16 = \$170,496$$

The next step is to assume the number of servers, i.e., runways to be four and recalculate the L_q from the table and compare the cost saved for the aircrafts by adding the runway to the cost of adding the new runway itself.

In this case, with $S = 4$ and $\lambda/\mu = 2$, L_q is found to be 0.1730.

Considering the same 16-hour day, the loss of aircraft time is now worth:

$$0.1730 \times 12,000 \times 16 = \$33,216$$

The amount of cost reduced, with respect to the aircraft, by adding a fourth runway would be:

$$\$170,496 - \$33,216, \text{ which equals } \$137,280.$$

The cost of adding a fourth runway would be $16 \times 6,000$ which equals \$96,000. Hence by adding the fourth runway, the cost of operation has been saved by:

$$\$137,280 - \$96,000 = \$41,280 \text{ per day.}$$

Therefore, the optimum number of runways required under the given conditions, would be four runways.

Cost–Capacity Tradeoff

A further extension of waiting line and service facility characteristics would be to determine the optimal service capacity that could minimize the aggregate cost (sum of waiting line cost and cost of service capacity). Consider Table 15.15 as an example to demonstrate this cost analysis.

Traditionally, the aggregate cost is assumed to be minimum when the waiting line cost equals the cost of service capacity. However, this is true only when the rate of decline of waiting line cost equals the rate of increase of the cost of service capacity, relative to the increase in service capacity.

Table 15.15 Cost–Capacity Tradeoff

Service Facility Capacity	Waiting Line Cost	Cost of Service Capacity	Aggregate Cost
0.0	\$3,000	\$ –	\$3,000
0.5	\$2,525	\$ 60	\$2,585
1.0	\$2,125	\$ 120	\$2,245
1.5	\$1,788	\$ 180	\$1,968
2.0	\$1,505	\$ 240	\$1,745
2.5	\$1,267	\$ 300	\$1,567
3.0	\$1,066	\$ 360	\$1,426
3.5	\$ 897	\$ 420	\$1,317
4.0	\$ 755	\$ 480	\$1,235
4.5	\$ 636	\$ 540	\$1,176
4.8	\$ 575	\$ 575	\$1,150
5.0	\$ 535	\$ 600	\$1,135
5.5	\$ 450	\$ 660	\$1,110
6.0	\$ 379	\$ 720	\$1,099
6.2	\$ 354	\$ 744	\$1,098
6.5	\$ 319	\$ 780	\$1,099
7.0	\$ 268	\$ 840	\$1,108
7.5	\$ 226	\$ 900	\$1,126
8.0	\$ 190	\$ 960	\$1,150
8.5	\$ 160	\$1,020	\$1,180
9.0	\$ 135	\$1,080	\$1,215
9.5	\$ 113	\$1,140	\$1,253
10.0	\$ 95	\$1,200	\$1,295

Figure 15.7 Cost vs Capacity

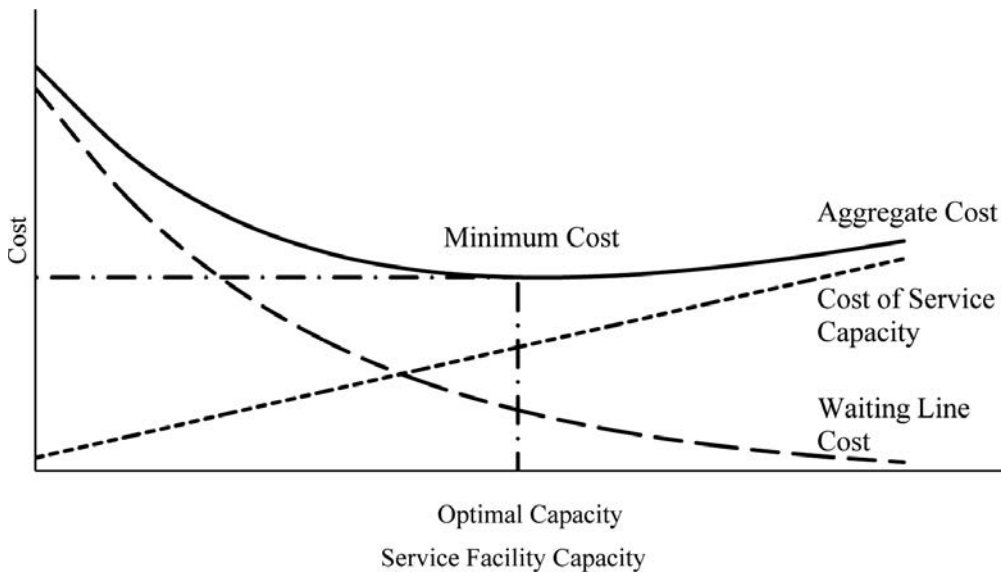


Figure 15.7 provides a significant idea of how the tradeoff balance needs to be achieved. As the number of servers is increased, the cost of service capacity keeps increasing, whereas the waiting line costs keep decreasing.

In Table 15.15, however, the waiting line cost decreases in an exponential manner while the cost of service capacity is assumed to increase linearly. In such a scenario, as can be seen from both the table and the figure, the optimal capacity is not obtained at the point where the waiting line cost equals the cost of service capacity. It is obtained at a point beyond that due to the fact that the rate of increase in the cost of service capacity is greater than the rate of decrease in the waiting line cost and so the aggregate cost is lower than the aggregate cost obtained when the two costs are equal. The point where the aggregate cost (sum of the cost of service capacity and the waiting line cost) is found to be minimum is considered the optimal capacity. This is true for any process which utilizes a waiting line or queueing theory model.

Summary

This chapter has set out to introduce the students and future industry managers to the decision-making environment. Decision making and risk management are important components of the air transport industry, an industry where the margin for error is very limited. The managerial decision-making models help explain how the decision-making process is used to choose the best alternative. Most decision makers rely on emotions in making judgments concerning risky decisions, and the decisions are made based on the decision maker's intuition. Intuitive decision making is making decisions on the basis of experience, feelings, and accumulated judgment. This chapter has discussed several concepts that prevail in an organization's decision making. These include: decision making under certainty, decision making under uncertainty, and decision making under risk. Consider Airbus's decision to develop Airbus 380. Airbus

did not have a very large aircraft product to compete with Boeing's 747. In the early 1990s, Airbus developed an aircraft with 500 passenger capacity and a 9,000 mile range to compete with the Boeing 747. The aircraft cost \$25B to develop, but the program has struggled to chalk up the large orders Airbus had envisioned, at \$440m each.¹¹ In real-life business situations, decisions can often fail because the best alternatives are not clear at the outset, or key factors are not considered as part of the process. Examples may include Lockheed L-1011, and Concorde. The last part of the chapter provides a brief introduction to queueing theory, a mathematical study of waiting lines or queues, the primary objective of which is to construct a model so that the queue length and waiting time can be predicted. Queueing theory is often used when making business decisions about the resources needed to provide a service. A queueing model has been used to predict how long a plane will wait before take-off, how long a plane will wait before landing given runway traffic, and weather traffic. The model may: help air traffic controllers to minimize runway congestion; aid airlines to estimate queue time and length at the airport check-in counters or at boarding; help airports predict border control, immigration, and security queue waiting time and length; assist MROs in predicting the time a particular aircraft will have to wait to receive parts or service, and so on. The ultimate goal or outcome expected from queueing theory is to achieve an economic balance between the cost of service and the cost associated with the waiting for that service. Queueing theory describes, probabilistically and mathematically, the interaction between the arrival process of customers and the service provided to them in order to manage the system in an efficient manner.

Discussion Questions and Problems

- 15.1 Discuss the differences between decision making under certainty, risk, and uncertainty and provide examples.
- 15.2 What is Queueing Theory?
- 15.3 Describe the basics of other decision-making techniques.
- 15.4 SkyJet Services' (SJS) operations manager believes that the probability of a favorable market is exactly the same as that of an unfavorable market; that is, each state of nature has a 0.50 chance of occurring. Can you now determine the EMV for each alternative?

Alternative	State of Nature	
	Favorable Market	Unfavorable Market
Build a large plant	\$200,000	−\$180,000
Build a small plant	\$100	−\$20,000
Do nothing	\$0	\$0

- 15.5 What is a decision tree?
- 15.6 What is the expected value of perfect information (EVPI)?
- 15.7 What is the expected value under certainty?

- 15.8** Why are the maximax and maximin strategies considered to be optimistic and pessimistic, respectively?
- 15.9** ServiceJet (SJ), Inc. is a leading business aviation services company in Hong Kong. SJ offers cost-effective maintenance solutions at major airports throughout the Asian Pacific area with the ability to respond quickly to the needs of customers. SJ's demand has increased, aided by the growth of transportation in Asia and Asia Pacific regions. To meet the demand, SJ has the following alternatives:

	Demand		
	Increases	Remains the Same	Decreases
Build new plant	\$122,500,000	\$84,000,000	−\$105,000,000
Subcontract	\$63,000,000	\$31,500,000	−\$7,000,000
Overtime	\$38,500,000	\$21,000,000	−\$3,500,000
Do nothing	\$0	\$0	\$0

Given the above information, determine the appropriate decision under uncertainty using:

- a) Maximax
 - b) Maximin
 - c) Equally likely
- 15.10** MyJet Airline, a new low-cost airline, started operations in 2015. MyJet has entered several new markets with a new low price. Incumbents may ignore direct price competition and carry on with their own strategy as they see fit, or competitors may lower prices and increase services. MyJet has created a table of conditional values for the various alternatives (pricing decisions) and states of nature (reactions by competitors).

	Competitive Reaction		
	Increase the Price	Keep the Same Price	Reduce the Price
Increase the price	\$5,500,000	\$3,000,000	(\$500,000)
Keep the price unchanged	\$3,500,000	\$2,500,000	\$1,500,000
Reduce the price	\$2,250,000	\$2,000,000	\$1,000,000

If the probabilities associated with the states of nature are 0.3 for a big demand, 0.5 for an average demand, and 0.2 for a small demand, determine the alternative that provides MyJet the greatest expected monetary value (EMV).

- 15.11** Service Jet (SJ), is considering opening a maintenance base in Incheon International airport. SJ is faced with significant competition from the

regional aircraft maintenance companies. Service Jet can open a small shop, a large shop, or no shop at all. Because there will be a five-year lease on the building that Service Jet is thinking about using, it wants to make sure it makes the correct decision. Service Jet is also thinking about hiring a consulting company to conduct a marketing research study to see if there is a market for its services. The results of such a study could be either favorable or unfavorable. Develop a decision tree for Service Jet.

- 15.12** Geneva Airport Authorities (GAA), a private airport company, is planning on buying a regional airport for \$500 million. GAA can finance the investment by borrowing the funding, with an expected annual cost of \$50,000,000 to service the debt, plus an additional cost of \$200,000,000 per year to upgrade the airport facilities. At a contemplated landing fee of \$1,000, the probabilities are 0.10, 0.50, and 0.40, respectively, that 600,000 planes, 700,000 planes, or 800,000 planes will land per year. Use a decision tree to determine GAA's optimal action, given its desire to maximize expected monetary value and given the option not to make the investment at all.

Notes

- 1 Is This the End of the Concorde Dream? *The Telegraph*, August 16, 2000.
- 2 Concorde and Supersonic Civil Aviation, Kumar, S., ICFAI Business School Case Development Centre.
- 3 The Federal Aviation Administration issued a warning about the Boeing 787 Dreamliner's computer system, which has a software counter that could periodically shut down all electrical power and could result in loss of control of the airplane.
- 4 The New Steel. *Flight International*, October 24, 1968.
- 5 Greenwald, J., J. Hannifin and J. Kane, Catch a Falling TriStar, *The Times*, December 21, 1981.
- 6 Ibid.
- 7 Airline alliances tend to lessen competition, increase prices, and naturally help airlines' profits.
- 8 Norris, G. Creating a Titan, *Flight International*, June 14, 2005.
- 9 Only one choice can be made at a time and all the possible choices are included.
- 10 *Queueing Theory*, Ryan Berry.
- 11 *The Guardian*, December 28, 2014.

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Appendix A: Tables for Compound Interest Factors

Appendix A includes tables that present compound interest factors at various interest rates for different time periods.

Column 1 n Time periods

Columns 2–3 present pre-calculated values for single payment

Column 2 F/P Future value of \$1 at the end of n periods
 Column 3 P/F Present value of \$1 due at the end of n periods

Columns 4–7 present pre-calculated values for annuity or uniform series

Column 4 A/F Annuity of \$1 due at the end of n periods (Sinking fund)
 Column 5 F/A Future value of an annuity of \$1 at the end of n periods
 Column 6 A/P Annuity of present amount of \$1 over n periods (Capital Recovery)
 Column 7 P/A Present value of an annuity of \$1 over n periods

Columns 8–9 present pre-calculated values for arithmetic gradient series

Column 8 P/G Present value of an arithmetic gradient series of \$1 in the base period increasing by \$1 per period for n periods
 Column 9 A/G Annuity or uniform series of an arithmetic gradient series of \$1 in the base period increasing by \$1 per period for n periods

Table A1 Compound Interest Factors 0.25%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.00250	0.99751	1.00000	1.00000	1.00250	0.99751		
2	1.00501	0.99502	0.49938	2.00250	0.50188	1.99252	0.99502	0.49938
3	1.00752	0.99254	0.33250	3.00751	0.33500	2.98506	2.98009	0.99834
4	1.01004	0.99006	0.24906	4.01503	0.25156	3.97512	5.95028	1.49688

(Continued)

Table A1 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
5	1.01256	0.98759	0.19900	5.02506	0.20150	4.96272	9.90065	1.99501
6	1.01509	0.98513	0.16563	6.03763	0.16813	5.94785	14.82630	2.49272
7	1.01763	0.98267	0.14179	7.05272	0.14429	6.93052	20.72235	2.99001
8	1.02018	0.98022	0.12391	8.07035	0.12641	7.91074	27.58391	3.48689
9	1.02273	0.97778	0.11000	9.09053	0.11250	8.88852	35.40614	3.98335
10	1.02528	0.97534	0.09888	10.11325	0.10138	9.86386	44.18420	4.47940
11	1.02785	0.97291	0.08978	11.13854	0.09228	10.83677	53.91328	4.97503
12	1.03042	0.97048	0.08219	12.16638	0.08469	11.80725	64.58858	5.47025
13	1.03299	0.96806	0.07578	13.19680	0.07828	12.77532	76.20532	5.96504
14	1.03557	0.96565	0.07028	14.22979	0.07278	13.74096	88.75874	6.45943
15	1.03816	0.96324	0.06551	15.26537	0.06801	14.70420	102.24409	6.95339
16	1.04076	0.96084	0.06134	16.30353	0.06384	15.66504	116.65666	7.44694
17	1.04336	0.95844	0.05766	17.34429	0.06016	16.62348	131.99172	7.94008
18	1.04597	0.95605	0.05438	18.38765	0.05688	17.57953	148.24459	8.43279
19	1.04858	0.95367	0.05146	19.43362	0.05396	18.53320	165.41059	8.92510
20	1.05121	0.95129	0.04882	20.48220	0.05132	19.48449	183.48508	9.41698
21	1.05383	0.94892	0.04644	21.53341	0.04894	20.43340	202.46341	9.90845
22	1.05647	0.94655	0.04427	22.58724	0.04677	21.37995	222.34096	10.39951
23	1.05911	0.94419	0.04229	23.64371	0.04479	22.32414	243.11313	10.89014
24	1.06176	0.94184	0.04048	24.70282	0.04298	23.26598	264.77534	11.38036
25	1.06441	0.93949	0.03881	25.76457	0.04131	24.20547	287.32301	11.87017
26	1.06707	0.93714	0.03727	26.82899	0.03977	25.14261	310.75160	12.35956
27	1.06974	0.93481	0.03585	27.89606	0.03835	26.07742	335.05657	12.84853
28	1.07241	0.93248	0.03452	28.96580	0.03702	27.00989	360.23340	13.33709
29	1.07510	0.93015	0.03329	30.03821	0.03579	27.94004	386.27760	13.82523
30	1.07778	0.92783	0.03214	31.11331	0.03464	28.86787	413.18468	14.31296
36	1.09405	0.91403	0.02658	37.62056	0.02908	34.38647	592.49878	17.23058
40	1.10503	0.90495	0.02380	42.01320	0.02630	38.01986	728.73988	19.16735
48	1.12733	0.88705	0.01963	50.93121	0.02213	45.17869	1040.0552	23.02092
50	1.13297	0.88263	0.01880	53.18868	0.02130	46.94617	1125.7767	23.98016
52	1.13864	0.87824	0.01803	55.45746	0.02053	48.70484	1214.5885	24.93774
55	1.14720	0.87168	0.01698	58.88194	0.01948	51.32644	1353.5286	26.37098
60	1.16162	0.86087	0.01547	64.64671	0.01797	55.65236	1600.0845	28.75142
72	1.19695	0.83546	0.01269	78.77939	0.01519	65.81686	2265.5569	34.42214

75	1.20595	0.82922	0.01214	82.37922	0.01464	68.31075	2447.6069	35.83048
84	1.23335	0.81080	0.01071	93.34192	0.01321	75.68132	3029.7592	40.03312
90	1.25197	0.79874	0.00992	100.78845	0.01242	80.50382	3446.8700	42.81623
96	1.27087	0.78686	0.00923	108.34739	0.01173	85.25460	3886.2832	45.58444
100	1.28362	0.77904	0.00881	113.44996	0.01131	88.38248	4191.2417	47.42163
108	1.30952	0.76364	0.00808	123.80926	0.01058	94.54530	4829.0125	51.07618
120	1.34935	0.74110	0.00716	139.74142	0.00966	103.5618	5852.1116	56.50843
132	1.39040	0.71922	0.00640	156.15817	0.00890	112.3121	6950.0144	61.88128
144	1.43269	0.69799	0.00578	173.07425	0.00828	120.8041	8117.4133	67.19487
240	1.82075	0.54922	0.00305	328.30200	0.00555	180.3109	19398.9852	107.58631
360	2.45684	0.40703	0.00172	582.73688	0.00422	237.1894	36263.9299	152.89019
480	3.31515	0.30165	0.00108	926.05950	0.00358	279.3418	53820.7525	192.66991

Table A2 Compound Interest Factors 0.50%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.00500	0.99502	1.00000	1.00000	1.00500	0.99502		
2	1.01003	0.99007	0.49875	2.00500	0.50375	1.98510	0.99007	0.49875
3	1.01508	0.98515	0.33167	3.01502	0.33667	2.97025	2.96037	0.99667
4	1.02015	0.98025	0.24813	4.03010	0.25313	3.95050	5.90111	1.49377
5	1.02525	0.97537	0.19801	5.05025	0.20301	4.92587	9.80260	1.99003
6	1.03038	0.97052	0.16460	6.07550	0.16960	5.89638	14.65519	2.48545
7	1.03553	0.96569	0.14073	7.10588	0.14573	6.86207	20.44933	2.98005
8	1.04071	0.96089	0.12283	8.14141	0.12783	7.82296	27.17552	3.47382
9	1.04591	0.95610	0.10891	9.18212	0.11391	8.77906	34.82436	3.96675
10	1.05114	0.95135	0.09777	10.22803	0.10277	9.73041	43.38649	4.45885
11	1.05640	0.94661	0.08866	11.27917	0.09366	10.67703	52.85264	4.95013
12	1.06168	0.94191	0.08107	12.33556	0.08607	11.61893	63.21360	5.44057
13	1.06699	0.93722	0.07464	13.39724	0.07964	12.55615	74.46023	5.93018
14	1.07232	0.93256	0.06914	14.46423	0.07414	13.48871	86.58346	6.41896
15	1.07768	0.92792	0.06436	15.53655	0.06936	14.41662	99.57430	6.90691
16	1.08307	0.92330	0.06019	16.61423	0.06519	15.33993	113.42380	7.39403
17	1.08849	0.91871	0.05651	17.69730	0.06151	16.25863	128.12311	7.88031
18	1.09393	0.91414	0.05323	18.78579	0.05823	17.17277	143.66343	8.36577
19	1.09940	0.90959	0.05030	19.87972	0.05530	18.08236	160.03602	8.85040
20	1.10490	0.90506	0.04767	20.97912	0.05267	18.98742	177.23221	9.33419

(Continued)

Table A2 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
21	1.11042	0.90056	0.04528	22.08401	0.05028	19.88798	195.24341	9.81716
22	1.11597	0.89608	0.04311	23.19443	0.04811	20.78406	214.06109	10.29929
23	1.12155	0.89162	0.04113	24.31040	0.04613	21.67568	233.67676	10.78060
24	1.12716	0.88719	0.03932	25.43196	0.04432	22.56287	254.08203	11.26107
25	1.13280	0.88277	0.03765	26.55912	0.04265	23.44564	275.26856	11.74072
26	1.13846	0.87838	0.03611	27.69191	0.04111	24.32402	297.22805	12.21953
27	1.14415	0.87401	0.03469	28.83037	0.03969	25.19803	319.95231	12.69751
28	1.14987	0.86966	0.03336	29.97452	0.03836	26.06769	343.43317	13.17467
29	1.15562	0.86533	0.03213	31.12439	0.03713	26.93302	367.66255	13.65099
30	1.16140	0.86103	0.03098	32.28002	0.03598	27.79405	392.63241	14.12649
36	1.19668	0.83564	0.02542	39.33610	0.03042	32.87102	557.55983	16.96205
40	1.22079	0.81914	0.02265	44.15885	0.02765	36.17223	681.33469	18.83585
48	1.27049	0.78710	0.01849	54.09783	0.02349	42.58032	959.91881	22.54372
50	1.28323	0.77929	0.01765	56.64516	0.02265	44.14279	1035.69659	23.46242
52	1.29609	0.77155	0.01689	59.21803	0.02189	45.68975	1113.81615	24.37781
55	1.31563	0.76009	0.01584	63.12577	0.02084	47.98145	1235.26857	25.74471
60	1.34885	0.74137	0.01433	69.77003	0.01933	51.72556	1448.64580	28.00638
72	1.43204	0.69830	0.01157	86.40886	0.01657	60.33951	2012.34779	33.35041
75	1.45363	0.68793	0.01102	90.72650	0.01602	62.41365	2163.75249	34.66794
84	1.52037	0.65773	0.00961	104.07393	0.01461	68.45304	2640.66405	38.57628
90	1.56655	0.63834	0.00883	113.31094	0.01383	72.33130	2976.07688	41.14508
96	1.61414	0.61952	0.00814	122.82854	0.01314	76.09522	3324.18460	43.68454
100	1.64667	0.60729	0.00773	129.33370	0.01273	78.54264	3562.79343	45.36126
108	1.71370	0.58353	0.00701	142.73990	0.01201	83.29342	4054.37473	48.67581
120	1.81940	0.54963	0.00610	163.87935	0.01110	90.07345	4823.50506	53.55080
132	1.93161	0.51770	0.00537	186.32263	0.01037	96.45960	5624.58677	58.31029
144	2.05075	0.48763	0.00476	210.15016	0.00976	102.47474	6451.31165	62.95514
240	3.31020	0.30210	0.00216	462.04090	0.00716	139.58077	13415.5395	96.11309
360	6.02258	0.16604	0.00100	1004.51504	0.00600	166.79161	21403.3041	128.32362
480	10.95745	0.09126	0.00050	1991.49073	0.00550	181.74758	27588.3573	151.79491

Table A3 Compound Interest Factors 0.75%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.00750	0.99256	1.00000	1.00000	1.00750	0.99256		
2	1.01506	0.98517	0.49813	2.00750	0.50563	1.97772	0.98517	0.49813
3	1.02267	0.97783	0.33085	3.02256	0.33835	2.95556	2.94083	0.99502
4	1.03034	0.97055	0.24721	4.04523	0.25471	3.92611	5.85250	1.49066
5	1.03807	0.96333	0.19702	5.07556	0.20452	4.88944	9.70581	1.98506
6	1.04585	0.95616	0.16357	6.11363	0.17107	5.84560	14.48660	2.47821
7	1.05370	0.94904	0.13967	7.15948	0.14717	6.79464	20.18084	2.97011
8	1.06160	0.94198	0.12176	8.21318	0.12926	7.73661	26.77467	3.46077
9	1.06956	0.93496	0.10782	9.27478	0.11532	8.67158	34.25438	3.95019
10	1.07758	0.92800	0.09667	10.34434	0.10417	9.59958	42.60641	4.43836
11	1.08566	0.92109	0.08755	11.42192	0.09505	10.52067	51.81736	4.92529
12	1.09381	0.91424	0.07995	12.50759	0.08745	11.43491	61.87398	5.41097
13	1.10201	0.90743	0.07352	13.60139	0.08102	12.34235	72.76316	5.89541
14	1.11028	0.90068	0.06801	14.70340	0.07551	13.24302	84.47197	6.37860
15	1.11860	0.89397	0.06324	15.81368	0.07074	14.13699	96.98758	6.86055
16	1.12699	0.88732	0.05906	16.93228	0.06656	15.02431	110.29735	7.34126
17	1.13544	0.88071	0.05537	18.05927	0.06287	15.90502	124.38875	7.82072
18	1.14396	0.87416	0.05210	19.19472	0.05960	16.77918	139.24940	8.29894
19	1.15254	0.86765	0.04917	20.33868	0.05667	17.64683	154.86708	8.77592
20	1.16118	0.86119	0.04653	21.49122	0.05403	18.50802	171.22969	9.25165
21	1.16989	0.85478	0.04415	22.65240	0.05165	19.36280	188.32527	9.72614
22	1.17867	0.84842	0.04198	23.82230	0.04948	20.21121	206.14200	10.19939
23	1.18751	0.84210	0.04000	25.00096	0.04750	21.05331	224.66820	10.67139
24	1.19641	0.83583	0.03818	26.18847	0.04568	21.88915	243.89233	11.14216
25	1.20539	0.82961	0.03652	27.38488	0.04402	22.71876	263.80295	11.61168
26	1.21443	0.82343	0.03498	28.59027	0.04248	23.54219	284.38879	12.07996
27	1.22354	0.81730	0.03355	29.80470	0.04105	24.35949	305.63869	12.54701
28	1.23271	0.81122	0.03223	31.02823	0.03973	25.17071	327.54162	13.01281
29	1.24196	0.80518	0.03100	32.26094	0.03850	25.97589	350.08668	13.47737

(Continued)

Table A3 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
30	1.25127	0.79919	0.02985	33.50290	0.03735	26.77508	373.26310	13.94069
36	1.30865	0.76415	0.02430	41.15272	0.03180	31.44681	524.99236	16.69462
40	1.34835	0.74165	0.02153	46.44648	0.02903	34.44694	637.46933	18.50583
48	1.43141	0.69861	0.01739	57.52071	0.02489	40.18478	886.84045	22.06906
50	1.45296	0.68825	0.01656	60.39426	0.02406	41.56645	953.84863	22.94756
52	1.47483	0.67804	0.01580	63.31107	0.02330	42.92762	1022.58522	23.82115
55	1.50827	0.66301	0.01476	67.76883	0.02226	44.93161	1128.78691	25.12233
60	1.56568	0.63870	0.01326	75.42414	0.02076	48.17337	1313.51888	27.26649
72	1.71255	0.58392	0.01053	95.00703	0.01803	55.47685	1791.24629	32.28818
75	1.75137	0.57098	0.00998	100.18331	0.01748	57.20267	1917.22249	33.51631
84	1.87320	0.53385	0.00859	116.42693	0.01609	62.15396	2308.12830	37.13566
90	1.95909	0.51044	0.00782	127.87899	0.01532	65.27461	2577.99605	39.49462
96	2.04892	0.48806	0.00715	139.85616	0.01465	68.25844	2853.93524	41.81073
100	2.11108	0.47369	0.00675	148.14451	0.01425	70.17462	3040.74530	43.33112
108	2.24112	0.44620	0.00604	165.48322	0.01354	73.83938	3419.90409	46.31545
120	2.45136	0.40794	0.00517	193.51428	0.01267	78.94169	3998.56214	50.65210
132	2.68131	0.37295	0.00446	224.17484	0.01196	83.60642	4583.57014	54.82318
144	2.93284	0.34097	0.00388	257.71157	0.01138	87.87109	5169.58283	58.83144
240	6.00915	0.16641	0.00150	667.88687	0.00900	111.14495	9494.11617	85.42103
360	14.73058	0.06789	0.00055	1830.74348	0.00805	124.28187	13312.3871	107.11448
480	36.10990	0.02769	0.00021	4681.32027	0.00771	129.64090	15513.0866	119.66198

Table A4 Compound Interest Factors 1%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.01000	0.99010	1.00000	1.00000	1.01000	0.99010		
2	1.02010	0.98030	0.49751	2.01000	0.50751	1.97040	0.98030	0.49751
3	1.03030	0.97059	0.33002	3.03010	0.34002	2.94099	2.92148	0.99337
4	1.04060	0.96098	0.24628	4.06040	0.25628	3.90197	5.80442	1.48756

5	1.05101	0.95147	0.19604	5.10101	0.20604	4.85343	9.61028	1.98010
6	1.06152	0.94205	0.16255	6.15202	0.17255	5.79548	14.32051	2.47098
7	1.07214	0.93272	0.13863	7.21354	0.14863	6.72819	19.91681	2.96020
8	1.08286	0.92348	0.12069	8.28567	0.13069	7.65168	26.38120	3.44777
9	1.09369	0.91434	0.10674	9.36853	0.11674	8.56602	33.69592	3.93367
10	1.10462	0.90529	0.09558	10.46221	0.10558	9.47130	41.84350	4.41792
11	1.11567	0.89632	0.08645	11.56683	0.09645	10.36763	50.80674	4.90052
12	1.12683	0.88745	0.07885	12.68250	0.08885	11.25508	60.56868	5.38145
13	1.13809	0.87866	0.07241	13.80933	0.08241	12.13374	71.11263	5.86073
14	1.14947	0.86996	0.06690	14.94742	0.07690	13.00370	82.42215	6.33836
15	1.16097	0.86135	0.06212	16.09690	0.07212	13.86505	94.48104	6.81433
16	1.17258	0.85282	0.05794	17.25786	0.06794	14.71787	107.27336	7.28865
17	1.18430	0.84438	0.05426	18.43044	0.06426	15.56225	120.78340	7.76131
18	1.19615	0.83602	0.05098	19.61475	0.06098	16.39827	134.99569	8.23231
19	1.20811	0.82774	0.04805	20.81090	0.05805	17.22601	149.89501	8.70167
20	1.22019	0.81954	0.04542	22.01900	0.05542	18.04555	165.46636	9.16937
21	1.23239	0.81143	0.04303	23.23919	0.05303	18.85698	181.69496	9.63542
22	1.24472	0.80340	0.04086	24.47159	0.05086	19.66038	198.56628	10.09982
23	1.25716	0.79544	0.03889	25.71630	0.04889	20.45582	216.06600	10.56257
24	1.26973	0.78757	0.03707	26.97346	0.04707	21.24339	234.18002	11.02367
25	1.28243	0.77977	0.03541	28.24320	0.04541	22.02316	252.89446	11.48312
26	1.29526	0.77205	0.03387	29.52563	0.04387	22.79520	272.19566	11.94092
27	1.30821	0.76440	0.03245	30.82089	0.04245	23.55961	292.07016	12.39707
28	1.32129	0.75684	0.03112	32.12910	0.04112	24.31644	312.50472	12.85158
29	1.33450	0.74934	0.02990	33.45039	0.03990	25.06579	333.48630	13.30444
30	1.34785	0.74192	0.02875	34.78489	0.03875	25.80771	355.00207	13.75566
36	1.43077	0.69892	0.02321	43.07688	0.03321	30.10751	494.62069	16.42848
40	1.48886	0.67165	0.02046	48.88637	0.03046	32.83469	596.85606	18.17761
48	1.61223	0.62026	0.01633	61.22261	0.02633	37.97396	820.14601	21.59759
50	1.64463	0.60804	0.01551	64.46318	0.02551	39.19612	879.41763	22.43635
52	1.67769	0.59606	0.01476	67.76889	0.02476	40.39419	939.91752	23.26863
55	1.72852	0.57853	0.01373	72.85246	0.02373	42.14719	1032.81478	24.50495
60	1.81670	0.55045	0.01224	81.66967	0.02224	44.95504	1192.80614	26.53331

(Continued)

Table A4 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
72	2.04710	0.48850	0.00955	104.70993	0.01955	51.15039	1597.86733	31.23861
75	2.10913	0.47413	0.00902	110.91285	0.01902	52.58705	1702.73397	32.37934
84	2.30672	0.43352	0.00765	130.67227	0.01765	56.64845	2023.31531	35.71704
90	2.44863	0.40839	0.00690	144.86327	0.01690	59.16088	2240.56748	37.87245
96	2.59927	0.38472	0.00625	159.92729	0.01625	61.52770	2459.42979	39.97272
100	2.70481	0.36971	0.00587	170.48138	0.01587	63.02888	2605.77575	41.34257
108	2.92893	0.34142	0.00518	192.89258	0.01518	65.85779	2898.42028	44.01029
120	3.30039	0.30299	0.00435	230.03869	0.01435	69.70052	3334.11485	47.83486
132	3.71896	0.26889	0.00368	271.89586	0.01368	73.11075	3761.69441	51.45200
144	4.19062	0.23863	0.00313	319.06156	0.01313	76.13716	4177.46642	54.86764
240	10.89255	0.09181	0.00101	989.25537	0.01101	90.81942	6878.60156	75.73933
360	35.94964	0.02782	0.00029	3494.96413	0.01029	97.21833	8720.43230	89.69947
480	118.64773	0.00843	0.00008	11764.7725	0.01008	99.15717	9511.15793	95.92002

Table A5 Compound Interest Factors 1.25%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.01250	0.98765	1.00000	1.00000	1.01250	0.98765		
2	1.02516	0.97546	0.49689	2.01250	0.50939	1.96312	0.97546	0.49689
3	1.03797	0.96342	0.32920	3.03766	0.34170	2.92653	2.90230	0.99172
4	1.05095	0.95152	0.24536	4.07563	0.25786	3.87806	5.75687	1.48447
5	1.06408	0.93978	0.19506	5.12657	0.20756	4.81784	9.51598	1.97516
6	1.07738	0.92817	0.16153	6.19065	0.17403	5.74601	14.15685	2.46377
7	1.09085	0.91672	0.13759	7.26804	0.15009	6.66273	19.65715	2.95032
8	1.10449	0.90540	0.11963	8.35889	0.13213	7.56812	25.99494	3.43479
9	1.11829	0.89422	0.10567	9.46337	0.11817	8.46234	33.14870	3.91720
10	1.13227	0.88318	0.09450	10.58167	0.10700	9.34553	41.09733	4.39754
11	1.14642	0.87228	0.08537	11.71394	0.09787	10.21780	49.82011	4.87581
12	1.16075	0.86151	0.07776	12.86036	0.09026	11.07931	59.29670	5.35202

13	1.17526	0.85087	0.07132	14.02112	0.08382	11.93018	69.50717	5.82616
14	1.18995	0.84037	0.06581	15.19638	0.07831	12.77055	80.43196	6.29824
15	1.20483	0.82999	0.06103	16.38633	0.07353	13.60055	92.05186	6.76825
16	1.21989	0.81975	0.05685	17.59116	0.06935	14.42029	104.34806	7.23620
17	1.23514	0.80963	0.05316	18.81105	0.06566	15.22992	117.30207	7.70208
18	1.25058	0.79963	0.04988	20.04619	0.06238	16.02955	130.89580	8.16591
19	1.26621	0.78976	0.04696	21.29677	0.05946	16.81931	145.11145	8.62767
20	1.28204	0.78001	0.04432	22.56298	0.05682	17.59932	159.93161	9.08738
21	1.29806	0.77038	0.04194	23.84502	0.05444	18.36969	175.33919	9.54502
22	1.31429	0.76087	0.03977	25.14308	0.05227	19.13056	191.31742	10.00062
23	1.33072	0.75147	0.03780	26.45737	0.05030	19.88204	207.84986	10.45415
24	1.34735	0.74220	0.03599	27.78808	0.04849	20.62423	224.92039	10.90564
25	1.36419	0.73303	0.03432	29.13544	0.04682	21.35727	242.51321	11.35507
26	1.38125	0.72398	0.03279	30.49963	0.04529	22.08125	260.61282	11.80245
27	1.39851	0.71505	0.03137	31.88087	0.04387	22.79630	279.20402	12.24778
28	1.41599	0.70622	0.03005	33.27938	0.04255	23.50252	298.27192	12.69106
29	1.43369	0.69750	0.02882	34.69538	0.04132	24.20002	317.80191	13.13230
30	1.45161	0.68889	0.02768	36.12907	0.04018	24.88891	337.77969	13.57150
36	1.56394	0.63941	0.02217	45.11551	0.03467	28.84727	466.28302	16.16385
40	1.64362	0.60841	0.01942	51.48956	0.03192	31.32693	559.23198	17.85148
48	1.81535	0.55086	0.01533	65.22839	0.02783	35.93148	759.22956	21.12993
50	1.86102	0.53734	0.01452	68.88179	0.02702	37.01288	811.67385	21.92950
52	1.90784	0.52415	0.01377	72.62710	0.02627	38.06773	864.94093	22.72110
55	1.98028	0.50498	0.01275	78.42246	0.02525	39.60169	946.22770	23.89362
60	2.10718	0.47457	0.01129	88.57451	0.02379	42.03459	1084.84285	25.80834
72	2.44592	0.40884	0.00865	115.67362	0.02115	47.29247	1428.45610	30.20472
75	2.53879	0.39389	0.00812	123.10349	0.02062	48.48897	1515.79039	31.26052
84	2.83911	0.35222	0.00680	147.12904	0.01930	51.82219	1778.83839	34.32581
90	3.05881	0.32692	0.00607	164.70501	0.01857	53.84606	1953.83026	36.28548
96	3.29551	0.30344	0.00545	183.64106	0.01795	55.72457	2127.52438	38.17929
100	3.46340	0.28873	0.00507	197.07234	0.01757	56.90134	2242.24109	39.40577
108	3.82528	0.26142	0.00442	226.02255	0.01692	59.08651	2468.26361	41.77373
120	4.44021	0.22521	0.00363	275.21706	0.01613	61.98285	2796.56945	45.11844

(Continued)

Table A5 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
132	5.15400	0.19402	0.00301	332.31981	0.01551	64.47807	3109.35041	48.22338
144	5.98253	0.16715	0.00251	398.60208	0.01501	66.62772	3404.60974	51.09900
240	19.71549	0.05072	0.00067	1497.23948	0.01317	75.94228	5101.52883	67.17640
360	87.54100	0.01142	0.00014	6923.27961	0.01264	79.08614	5997.90267	75.84012
480	388.70068	0.00257	0.00003	31016.0548	0.01253	79.79419	6284.74422	78.76193

Table A6 Compound Interest Factors 1.5%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.01500	0.98522	1.00000	1.00000	1.01500	0.98522		
2	1.03023	0.97066	0.49628	2.01500	0.51128	1.95588	0.97066	0.49628
3	1.04568	0.95632	0.32838	3.04522	0.34338	2.91220	2.88330	0.99007
4	1.06136	0.94218	0.24444	4.09090	0.25944	3.85438	5.70985	1.48139
5	1.07728	0.92826	0.19409	5.15227	0.20909	4.78264	9.42289	1.97023
6	1.09344	0.91454	0.16053	6.22955	0.17553	5.69719	13.99560	2.45658
7	1.10984	0.90103	0.13656	7.32299	0.15156	6.59821	19.40176	2.94046
8	1.12649	0.88771	0.11858	8.43284	0.13358	7.48593	25.61574	3.42185
9	1.14339	0.87459	0.10461	9.55933	0.11961	8.36052	32.61248	3.90077
10	1.16054	0.86167	0.09343	10.70272	0.10843	9.22218	40.36748	4.37721
11	1.17795	0.84893	0.08429	11.86326	0.09929	10.07112	48.85681	4.85118
12	1.19562	0.83639	0.07668	13.04121	0.09168	10.90751	58.05708	5.32267
13	1.21355	0.82403	0.07024	14.23683	0.08524	11.73153	67.94540	5.79169
14	1.23176	0.81185	0.06472	15.45038	0.07972	12.54338	78.49944	6.25824
15	1.25023	0.79985	0.05994	16.68214	0.07494	13.34323	89.69736	6.72231
16	1.26899	0.78803	0.05577	17.93237	0.07077	14.13126	101.51783	7.18392
17	1.28802	0.77639	0.05208	19.20136	0.06708	14.90765	113.93999	7.64306
18	1.30734	0.76491	0.04881	20.48938	0.06381	15.67256	126.94349	8.09973
19	1.32695	0.75361	0.04588	21.79672	0.06088	16.42617	140.50842	8.55394
20	1.34686	0.74247	0.04325	23.12367	0.05825	17.16864	154.61536	9.00569

21	1.36706	0.73150	0.04087	24.47052	0.05587	17.90014	169.24532	9.45497
22	1.38756	0.72069	0.03870	25.83758	0.05370	18.62082	184.37976	9.90180
23	1.40838	0.71004	0.03673	27.22514	0.05173	19.33086	200.00058	10.34618
24	1.42950	0.69954	0.03492	28.63352	0.04992	20.03041	216.09009	10.78810
25	1.45095	0.68921	0.03326	30.06302	0.04826	20.71961	232.63103	11.22758
26	1.47271	0.67902	0.03173	31.51397	0.04673	21.39863	249.60654	11.66460
27	1.49480	0.66899	0.03032	32.98668	0.04532	22.06762	267.00017	12.09918
28	1.51722	0.65910	0.02900	34.48148	0.04400	22.72672	284.79585	12.53132
29	1.53998	0.64936	0.02778	35.99870	0.04278	23.37608	302.97790	12.96102
30	1.56308	0.63976	0.02664	37.53868	0.04164	24.01584	321.53101	13.38829
36	1.70914	0.58509	0.02115	47.27597	0.03615	27.66068	439.83026	15.90092
40	1.81402	0.55126	0.01843	54.26789	0.03343	29.91585	524.35682	17.52773
48	2.04348	0.48936	0.01437	69.56522	0.02937	34.04255	703.54615	20.66667
50	2.10524	0.47500	0.01357	73.68283	0.02857	34.99969	749.96361	21.42772
52	2.16887	0.46107	0.01283	77.92489	0.02783	35.92874	796.87737	22.17938
55	2.26794	0.44093	0.01183	84.52960	0.02683	37.27147	868.02846	23.28936
60	2.44322	0.40930	0.01039	96.21465	0.02539	39.38027	988.16739	25.09296
72	2.92116	0.34233	0.00781	128.07720	0.02281	43.84467	1279.79379	29.18927
75	3.05459	0.32738	0.00730	136.97278	0.02230	44.84160	1352.56005	30.16306
84	3.49259	0.28632	0.00602	166.17264	0.02102	47.57863	1568.51404	32.96677
90	3.81895	0.26185	0.00532	187.92990	0.02032	49.20985	1709.54387	34.73987
96	4.17580	0.23947	0.00472	211.72023	0.01972	50.70168	1847.47253	36.43810
100	4.43205	0.22563	0.00437	228.80304	0.01937	51.62470	1937.45061	37.52953
108	4.99267	0.20029	0.00376	266.17777	0.01876	53.31375	2112.13479	39.61708
120	5.96932	0.16752	0.00302	331.28819	0.01802	55.49845	2359.71143	42.51851
132	7.13703	0.14011	0.00244	409.13539	0.01744	57.32571	2588.70855	45.15789
144	8.53316	0.11719	0.00199	502.21092	0.01699	58.85401	2798.57842	47.55119
240	35.63282	0.02806	0.00043	2308.85437	0.01543	64.79573	3870.69117	59.73682
360	212.70378	0.00470	0.00007	14113.5854	0.01507	66.35324	4310.71648	64.96618
480	1269.69754	0.00079	0.00001	84579.8363	0.01501	66.61416	4415.74120	66.28833

Table A7 Compound Interest Factors 1.75%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.01750	0.98280	1.00000	1.00000	1.01750	0.98280		
2	1.03531	0.96590	0.49566	2.01750	0.51316	1.94870	0.96590	0.49566
3	1.05342	0.94929	0.32757	3.05281	0.34507	2.89798	2.86447	0.98843
4	1.07186	0.93296	0.24353	4.10623	0.26103	3.83094	5.66334	1.47832
5	1.09062	0.91691	0.19312	5.17809	0.21062	4.74786	9.33099	1.96531
6	1.10970	0.90114	0.15952	6.26871	0.17702	5.64900	13.83671	2.44941
7	1.12912	0.88564	0.13553	7.37841	0.15303	6.53464	19.15057	2.93062
8	1.14888	0.87041	0.11754	8.50753	0.13504	7.40505	25.24345	3.40895
9	1.16899	0.85544	0.10356	9.65641	0.12106	8.26049	32.08698	3.88439
10	1.18944	0.84073	0.09238	10.82540	0.10988	9.10122	39.65354	4.35695
11	1.21026	0.82627	0.08323	12.01484	0.10073	9.92749	47.91623	4.82662
12	1.23144	0.81206	0.07561	13.22510	0.09311	10.73955	56.84886	5.29341
13	1.25299	0.79809	0.06917	14.45654	0.08667	11.53764	66.42596	5.75733
14	1.27492	0.78436	0.06366	15.70953	0.08116	12.32201	76.62270	6.21836
15	1.29723	0.77087	0.05888	16.98445	0.07638	13.09288	87.41495	6.67653
16	1.31993	0.75762	0.05470	18.28168	0.07220	13.85050	98.77919	7.13182
17	1.34303	0.74459	0.05102	19.60161	0.06852	14.59508	110.69257	7.58424
18	1.36653	0.73178	0.04774	20.94463	0.06524	15.32686	123.13283	8.03379
19	1.39045	0.71919	0.04482	22.31117	0.06232	16.04606	136.07832	8.48048
20	1.41478	0.70682	0.04219	23.70161	0.05969	16.75288	149.50799	8.92431
21	1.43954	0.69467	0.03981	25.11639	0.05731	17.44755	163.40134	9.36529
22	1.46473	0.68272	0.03766	26.55593	0.05516	18.13027	177.73847	9.80341
23	1.49036	0.67098	0.03569	28.02065	0.05319	18.80125	192.49999	10.23868
24	1.51644	0.65944	0.03389	29.51102	0.05139	19.46069	207.66706	10.67111
25	1.54298	0.64810	0.03223	31.02746	0.04973	20.10878	223.22138	11.10069
26	1.56998	0.63695	0.03070	32.57044	0.04820	20.74573	239.14512	11.52744
27	1.59746	0.62599	0.02929	34.14042	0.04679	21.37173	255.42098	11.95135
28	1.62541	0.61523	0.02798	35.73788	0.04548	21.98695	272.03215	12.37243
29	1.65386	0.60465	0.02676	37.36329	0.04426	22.59160	288.96226	12.79069
30	1.68280	0.59425	0.02563	39.01715	0.04313	23.18585	306.19544	13.20613

36	1.86741	0.53550	0.02018	49.56613	0.03768	26.54275	415.12498	15.63986
40	2.00160	0.49960	0.01747	57.23413	0.03497	28.59423	492.01087	17.20665
48	2.29960	0.43486	0.01347	74.26278	0.03097	32.29380	652.60539	20.20838
50	2.38079	0.42003	0.01267	78.90222	0.03017	33.14121	693.70101	20.93167
52	2.46485	0.40570	0.01195	83.70547	0.02945	33.95972	735.03220	21.64424
55	2.59653	0.38513	0.01096	91.23016	0.02846	35.13545	797.33210	22.69310
60	2.83182	0.35313	0.00955	104.67522	0.02705	36.96399	901.49545	24.38848
72	3.48721	0.28676	0.00704	142.12628	0.02454	40.75645	1149.11809	28.19476
75	3.67351	0.27222	0.00655	152.77206	0.02405	41.58748	1209.77384	29.08986
84	4.29429	0.23287	0.00531	188.24499	0.02281	43.83614	1387.15838	31.64417
90	4.76538	0.20985	0.00465	215.16462	0.02215	45.15161	1500.87981	33.24089
96	5.28815	0.18910	0.00408	245.03739	0.02158	46.33703	1610.47158	34.75560
100	5.66816	0.17642	0.00375	266.75177	0.02125	47.06147	1681.08862	35.72112
108	6.51204	0.15356	0.00317	314.97378	0.02067	48.36790	1816.18525	37.54939
120	8.01918	0.12470	0.00249	401.09620	0.01999	50.01709	2003.02686	40.04685
132	9.87514	0.10126	0.00197	507.15073	0.01947	51.35632	2170.82384	42.26985
144	12.16063	0.08223	0.00157	637.75045	0.01907	52.44385	2320.13512	44.24036
240	64.30730	0.01555	0.00028	3617.56017	0.01778	56.25427	3001.26781	53.35183
360	515.69206	0.00194	0.00003	29410.9747	0.01753	57.03205	3219.08332	56.44341
480	4135.42921	0.00024	0.00000	236253.098	0.01750	57.12904	3257.88395	57.02676

Table A8 Compound Interest Factors 2%

n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.02000	0.98039	1.00000	1.00000	1.02000	0.98039		
2	1.04040	0.96117	0.49505	2.02000	0.51505	1.94156	0.96117	0.49505
3	1.06121	0.94232	0.32675	3.06040	0.34675	2.88388	2.84581	0.98680
4	1.08243	0.92385	0.24262	4.12161	0.26262	3.80773	5.61735	1.47525
5	1.10408	0.90573	0.19216	5.20404	0.21216	4.71346	9.24027	1.96040
6	1.12616	0.88797	0.15853	6.30812	0.17853	5.60143	13.68013	2.44226
7	1.14869	0.87056	0.13451	7.43428	0.15451	6.47199	18.90349	2.92082
8	1.17166	0.85349	0.11651	8.58297	0.13651	7.32548	24.87792	3.39608

(Continued)

Table A8 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
9	1.19509	0.83676	0.10252	9.75463	0.12252	8.16224	31.57197	3.86805
10	1.21899	0.82035	0.09133	10.94972	0.11133	8.98259	38.95510	4.33674
11	1.24337	0.80426	0.08218	12.16872	0.10218	9.78685	46.99773	4.80213
12	1.26824	0.78849	0.07456	13.41209	0.09456	10.57534	55.67116	5.26424
13	1.29361	0.77303	0.06812	14.68033	0.08812	11.34837	64.94755	5.72307
14	1.31948	0.75788	0.06260	15.97394	0.08260	12.10625	74.79992	6.17862
15	1.34587	0.74301	0.05783	17.29342	0.07783	12.84926	85.20213	6.63090
16	1.37279	0.72845	0.05365	18.63929	0.07365	13.57771	96.12881	7.07990
17	1.40024	0.71416	0.04997	20.01207	0.06997	14.29187	107.55542	7.52564
18	1.42825	0.70016	0.04670	21.41231	0.06670	14.99203	119.45813	7.96811
19	1.45681	0.68643	0.04378	22.84056	0.06378	15.67846	131.81388	8.40732
20	1.48595	0.67297	0.04116	24.29737	0.06116	16.35143	144.60033	8.84328
21	1.51567	0.65978	0.03878	25.78332	0.05878	17.01121	157.79585	9.27599
22	1.54598	0.64684	0.03663	27.29898	0.05663	17.65805	171.37947	9.70546
23	1.57690	0.63416	0.03467	28.84496	0.05467	18.29220	185.33090	10.13169
24	1.60844	0.62172	0.03287	30.42186	0.05287	18.91393	199.63049	10.55468
25	1.64061	0.60953	0.03122	32.03030	0.05122	19.52346	214.25924	10.97445
26	1.67342	0.59758	0.02970	33.67091	0.04970	20.12104	229.19872	11.39100
27	1.70689	0.58586	0.02829	35.34432	0.04829	20.70690	244.43113	11.80433
28	1.74102	0.57437	0.02699	37.05121	0.04699	21.28127	259.93924	12.21446
29	1.77584	0.56311	0.02578	38.79223	0.04578	21.84438	275.70639	12.62138
30	1.81136	0.55207	0.02465	40.56808	0.04465	22.39646	291.71644	13.02512
36	2.03989	0.49022	0.01923	51.99437	0.03923	25.48884	392.04045	15.38087
40	2.20804	0.45289	0.01656	60.40198	0.03656	27.35548	461.99313	16.88850
48	2.58707	0.38654	0.01260	79.35352	0.03260	30.67312	605.96572	19.75559
50	2.69159	0.37153	0.01182	84.57940	0.03182	31.42361	642.36059	20.44198
52	2.80033	0.35710	0.01111	90.01641	0.03111	32.14495	678.78489	21.11638
55	2.97173	0.33650	0.01014	98.58653	0.03014	33.17479	733.35269	22.10572
60	3.28103	0.30478	0.00877	114.05154	0.02877	34.76089	823.69753	23.69610
72	4.16114	0.24032	0.00633	158.05702	0.02633	37.98406	1034.05570	27.22341

75	4.41584	0.22646	0.00586	170.79177	0.02586	38.67711	1084.63929	28.04344
84	5.27733	0.18949	0.00468	213.86661	0.02468	40.52552	1230.41912	30.36159
90	5.94313	0.16826	0.00405	247.15666	0.02405	41.58693	1322.17008	31.79292
96	6.69293	0.14941	0.00351	284.64666	0.02351	42.52943	1409.29734	33.13699
100	7.24465	0.13803	0.00320	312.23231	0.02320	43.09835	1464.75275	33.98628
108	8.48826	0.11781	0.00267	374.41288	0.02267	44.10951	1569.30251	35.57742
120	10.76516	0.09289	0.00205	488.25815	0.02205	45.35539	1710.41605	37.71142
132	13.65283	0.07324	0.00158	632.64148	0.02158	46.33776	1833.47151	39.56755
144	17.31509	0.05775	0.00123	815.75446	0.02123	47.11235	1939.79497	41.17381
240	115.88874	0.00863	0.00017	5744.43676	0.02017	49.56855	2374.87999	47.91102
360	1247.56113	0.00080	0.00002	62328.056	0.02002	49.95992	2483.56794	49.71121
480	13430.1989	0.00007	0.00000	671459.947	0.02000	49.99628	2498.02683	49.96426

Table A9 Compound Interest Factors 3%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.03000	0.97087	1.00000	1.00000	1.03000	0.97087		
2	1.06090	0.94260	0.49261	2.03000	0.52261	1.91347	0.94260	0.49261
3	1.09273	0.91514	0.32353	3.09090	0.35353	2.82861	2.77288	0.98030
4	1.12551	0.88849	0.23903	4.18363	0.26903	3.71710	5.43834	1.46306
5	1.15927	0.86261	0.18835	5.30914	0.21835	4.57971	8.88878	1.94090
6	1.19405	0.83748	0.15460	6.46841	0.18460	5.41719	13.07620	2.41383
7	1.22987	0.81309	0.13051	7.66246	0.16051	6.23028	17.95475	2.88185
8	1.26677	0.78941	0.11246	8.89234	0.14246	7.01969	23.48061	3.34496
9	1.30477	0.76642	0.09843	10.15911	0.12843	7.78611	29.61194	3.80318
10	1.34392	0.74409	0.08723	11.46388	0.11723	8.53020	36.30879	4.25650
11	1.38423	0.72242	0.07808	12.80780	0.10808	9.25262	43.53300	4.70494
12	1.42576	0.70138	0.07046	14.19203	0.10046	9.95400	51.24818	5.14850
13	1.46853	0.68095	0.06403	15.61779	0.09403	10.63496	59.41960	5.58720
14	1.51259	0.66112	0.05853	17.08632	0.08853	11.29607	68.01413	6.02104
15	1.55797	0.64186	0.05377	18.59891	0.08377	11.93794	77.00020	6.45004
16	1.60471	0.62317	0.04961	20.15688	0.07961	12.56110	86.34770	6.87421

(Continued)

Table A9 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
17	1.65285	0.60502	0.04595	21.76159	0.07595	13.16612	96.02796	7.29357
18	1.70243	0.58739	0.04271	23.41444	0.07271	13.75351	106.01367	7.70812
19	1.75351	0.57029	0.03981	25.11687	0.06981	14.32380	116.27882	8.11788
20	1.80611	0.55368	0.03722	26.87037	0.06722	14.87747	126.79866	8.52286
21	1.86029	0.53755	0.03487	28.67649	0.06487	15.41502	137.54964	8.92309
22	1.91610	0.52189	0.03275	30.53678	0.06275	15.93692	148.50939	9.31858
23	1.97359	0.50669	0.03081	32.45288	0.06081	16.44361	159.65661	9.70934
24	2.03279	0.49193	0.02905	34.42647	0.05905	16.93554	170.97108	10.09540
25	2.09378	0.47761	0.02743	36.45926	0.05743	17.41315	182.43362	10.47677
26	2.15659	0.46369	0.02594	38.55304	0.05594	17.87684	194.02598	10.85348
27	2.22129	0.45019	0.02456	40.70963	0.05456	18.32703	205.73090	11.22554
28	2.28793	0.43708	0.02329	42.93092	0.05329	18.76411	217.53197	11.59298
29	2.35657	0.42435	0.02211	45.21885	0.05211	19.18845	229.41367	11.95582
30	2.42726	0.41199	0.02102	47.57542	0.05102	19.60044	241.36129	12.31407
31	2.50008	0.39999	0.02000	50.00268	0.05000	20.00043	253.36090	12.66777
32	2.57508	0.38834	0.01905	52.50276	0.04905	20.38877	265.39935	13.01694
33	2.65234	0.37703	0.01816	55.07784	0.04816	20.76579	277.46419	13.36160
34	2.73191	0.36604	0.01732	57.73018	0.04732	21.13184	289.54367	13.70177
35	2.81386	0.35538	0.01654	60.46208	0.04654	21.48722	301.62670	14.03749
36	2.89828	0.34503	0.01580	63.27594	0.04580	21.83225	313.70284	14.36878
40	3.26204	0.30656	0.01326	75.40126	0.04326	23.11477	361.74994	15.65016
48	4.13225	0.24200	0.00958	104.40840	0.03958	25.26671	455.02547	18.00890
50	4.38391	0.22811	0.00887	112.79687	0.03887	25.72976	477.48033	18.55751
55	5.08215	0.19677	0.00735	136.07162	0.03735	26.77443	531.74111	19.86004
60	5.89160	0.16973	0.00613	163.05344	0.03613	27.67556	583.05261	21.06742
64	6.63105	0.15081	0.00533	187.70171	0.03533	28.30648	621.83055	21.96778
70	7.91782	0.12630	0.00434	230.59406	0.03434	29.12342	676.08687	23.21454
72	8.40002	0.11905	0.00405	246.66724	0.03405	29.36509	693.12255	23.60363
84	11.97642	0.08350	0.00273	365.88054	0.03273	30.55009	784.54337	25.68056
90	14.30047	0.06993	0.00226	443.34890	0.03226	31.00241	823.63021	26.56665

96	17.07551	0.05856	0.00187	535.85019	0.03187	31.38122	858.63770	27.36151
100	19.21863	0.05203	0.00165	607.28773	0.03165	31.59891	879.85405	27.84445
108	24.34559	0.04108	0.00129	778.18627	0.03129	31.96416	917.60126	28.70719
120	34.71099	0.02881	0.00089	1123.69957	0.03089	32.37302	963.86347	29.77366

Table A10 Compound Interest Factors 4%

n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.04000	0.96154	1.00000	1.00000	1.04000	0.96154		
2	1.08160	0.92456	0.49020	2.04000	0.53020	1.88609	0.92456	0.49020
3	1.12486	0.88900	0.32035	3.12160	0.36035	2.77509	2.70255	0.97386
4	1.16986	0.85480	0.23549	4.24646	0.27549	3.62990	5.26696	1.45100
5	1.21665	0.82193	0.18463	5.41632	0.22463	4.45182	8.55467	1.92161
6	1.26532	0.79031	0.15076	6.63298	0.19076	5.24214	12.50624	2.38571
7	1.31593	0.75992	0.12661	7.89829	0.16661	6.00205	17.06575	2.84332
8	1.36857	0.73069	0.10853	9.21423	0.14853	6.73274	22.18058	3.29443
9	1.42331	0.70259	0.09449	10.58280	0.13449	7.43533	27.80127	3.73908
10	1.48024	0.67556	0.08329	12.00611	0.12329	8.11090	33.88135	4.17726
11	1.53945	0.64958	0.07415	13.48635	0.11415	8.76048	40.37716	4.60901
12	1.60103	0.62460	0.06655	15.02581	0.10655	9.38507	47.24773	5.03435
13	1.66507	0.60057	0.06014	16.62684	0.10014	9.98565	54.45462	5.45329
14	1.73168	0.57748	0.05467	18.29191	0.09467	10.56312	61.96179	5.86586
15	1.80094	0.55526	0.04994	20.02359	0.08994	11.11839	69.73550	6.27209
16	1.87298	0.53391	0.04582	21.82453	0.08582	11.65230	77.74412	6.67200
17	1.94790	0.51337	0.04220	23.69751	0.08220	12.16567	85.95809	7.06563
18	2.02582	0.49363	0.03899	25.64541	0.07899	12.65930	94.34977	7.45300
19	2.10685	0.47464	0.03614	27.67123	0.07614	13.13394	102.89333	7.83416
20	2.19112	0.45639	0.03358	29.77808	0.07358	13.59033	111.56469	8.20912
21	2.27877	0.43883	0.03128	31.96920	0.07128	14.02916	120.34136	8.57794
22	2.36992	0.42196	0.02920	34.24797	0.06920	14.45112	129.20242	8.94065
23	2.46472	0.40573	0.02731	36.61789	0.06731	14.85684	138.12840	9.29729
24	2.56330	0.39012	0.02559	39.08260	0.06559	15.24696	147.10119	9.64790

(Continued)

Table A10 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
25	2.66584	0.37512	0.02401	41.64591	0.06401	15.62208	156.10400	9.99252
26	2.77247	0.36069	0.02257	44.31174	0.06257	15.98277	165.12123	10.33120
27	2.88337	0.34682	0.02124	47.08421	0.06124	16.32959	174.13846	10.66399
28	2.99870	0.33348	0.02001	49.96758	0.06001	16.66306	183.14235	10.99092
29	3.11865	0.32065	0.01888	52.96629	0.05888	16.98371	192.12059	11.31205
30	3.24340	0.30832	0.01783	56.08494	0.05783	17.29203	201.06183	11.62743
31	3.37313	0.29646	0.01686	59.32834	0.05686	17.58849	209.95564	11.93710
32	3.50806	0.28506	0.01595	62.70147	0.05595	17.87355	218.79244	12.24113
33	3.64838	0.27409	0.01510	66.20953	0.05510	18.14765	227.56345	12.53956
34	3.79432	0.26355	0.01431	69.85791	0.05431	18.41120	236.26067	12.83244
35	3.94609	0.25342	0.01358	73.65222	0.05358	18.66461	244.87679	13.11984
36	4.10393	0.24367	0.01289	77.59831	0.05289	18.90828	253.40520	13.40181
40	4.80102	0.20829	0.01052	95.02552	0.05052	19.79277	286.53030	14.47651
48	6.57053	0.15219	0.00718	139.26321	0.04718	21.19513	347.24455	16.38322
50	7.10668	0.14071	0.00655	152.66708	0.04655	21.48218	361.16385	16.81225
55	8.64637	0.11566	0.00523	191.15917	0.04523	22.10861	393.68897	17.80704
60	10.51963	0.09506	0.00420	237.99069	0.04420	22.62349	422.99665	18.69723
64	12.30648	0.08126	0.00354	282.66190	0.04354	22.96855	444.20089	19.33953
70	15.57162	0.06422	0.00275	364.29046	0.04275	23.39451	472.47892	20.19614
72	16.84226	0.05937	0.00252	396.05656	0.04252	23.51564	481.01697	20.45519
84	26.96500	0.03709	0.00154	649.12512	0.04154	24.07287	523.94309	21.76488
90	34.11933	0.02931	0.00121	827.98333	0.04121	24.26728	540.73692	22.28255
96	43.17184	0.02316	0.00095	1054.29603	0.04095	24.42092	554.93118	22.72360
100	50.50495	0.01980	0.00081	1237.62370	0.04081	24.50500	563.12487	22.98000
108	69.11951	0.01447	0.00059	1702.98772	0.04059	24.63831	576.89491	23.41455
120	110.66256	0.00904	0.00036	2741.56402	0.04036	24.77409	592.24276	23.90573

Table A11 Compound Interest Factors 5%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.05000	0.95238	1.00000	1.00000	1.05000	0.95238		
2	1.10250	0.90703	0.48780	2.05000	0.53780	1.85941	0.90703	0.48780
3	1.15763	0.86384	0.31721	3.15250	0.36721	2.72325	2.63470	0.96749
4	1.21551	0.82270	0.23201	4.31013	0.28201	3.54595	5.10281	1.43905
5	1.27628	0.78353	0.18097	5.52563	0.23097	4.32948	8.23692	1.90252
6	1.34010	0.74622	0.14702	6.80191	0.19702	5.07569	11.96799	2.35790
7	1.40710	0.71068	0.12282	8.14201	0.17282	5.78637	16.23208	2.80523
8	1.47746	0.67684	0.10472	9.54911	0.15472	6.46321	20.96996	3.24451
9	1.55133	0.64461	0.09069	11.02656	0.14069	7.10782	26.12683	3.67579
10	1.62889	0.61391	0.07950	12.57789	0.12950	7.72173	31.65205	4.09909
11	1.71034	0.58468	0.07039	14.20679	0.12039	8.30641	37.49884	4.51444
12	1.79586	0.55684	0.06283	15.91713	0.11283	8.86325	43.62405	4.92190
13	1.88565	0.53032	0.05646	17.71298	0.10646	9.39357	49.98791	5.32150
14	1.97993	0.50507	0.05102	19.59863	0.10102	9.89864	56.55379	5.71329
15	2.07893	0.48102	0.04634	21.57856	0.09634	10.37966	63.28803	6.09731
16	2.18287	0.45811	0.04227	23.65749	0.09227	10.83777	70.15970	6.47363
17	2.29202	0.43630	0.03870	25.84037	0.08870	11.27407	77.14045	6.84229
18	2.40662	0.41552	0.03555	28.13238	0.08555	11.68959	84.20430	7.20336
19	2.52695	0.39573	0.03275	30.53900	0.08275	12.08532	91.32751	7.55690
20	2.65330	0.37689	0.03024	33.06595	0.08024	12.46221	98.48841	7.90297
21	2.78596	0.35894	0.02800	35.71925	0.07800	12.82115	105.66726	8.24164
22	2.92526	0.34185	0.02597	38.50521	0.07597	13.16300	112.84611	8.57298
23	3.07152	0.32557	0.02414	41.43048	0.07414	13.48857	120.00868	8.89706
24	3.22510	0.31007	0.02247	44.50200	0.07247	13.79864	127.14024	9.21397
25	3.38635	0.29530	0.02095	47.72710	0.07095	14.09394	134.22751	9.52377
26	3.55567	0.28124	0.01956	51.11345	0.06956	14.37519	141.25852	9.82655
27	3.73346	0.26785	0.01829	54.66913	0.06829	14.64303	148.22258	10.12240
28	3.92013	0.25509	0.01712	58.40258	0.06712	14.89813	155.11011	10.41138
29	4.11614	0.24295	0.01605	62.32271	0.06605	15.14107	161.91261	10.69360
30	4.32194	0.23138	0.01505	66.43885	0.06505	15.37245	168.62255	10.96914

(Continued)

Table A11 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
31	4.53804	0.22036	0.01413	70.76079	0.06413	15.59281	175.23334	11.23809
32	4.76494	0.20987	0.01328	75.29883	0.06328	15.80268	181.73919	11.50053
33	5.00319	0.19987	0.01249	80.06377	0.06249	16.00255	188.13511	11.75657
34	5.25335	0.19035	0.01176	85.06696	0.06176	16.19290	194.41682	12.00630
35	5.51602	0.18129	0.01107	90.32031	0.06107	16.37419	200.58069	12.24980
36	5.79182	0.17266	0.01043	95.83632	0.06043	16.54685	206.62370	12.48719
40	7.03999	0.14205	0.00828	120.79977	0.05828	17.15909	229.54518	13.37747
45	8.98501	0.11130	0.00626	159.70016	0.05626	17.77407	255.31454	14.36444
50	11.46740	0.08720	0.00478	209.34800	0.05478	18.25593	277.91478	15.22326
55	14.63563	0.06833	0.00367	272.71262	0.05367	18.63347	297.51040	15.96645
60	18.67919	0.05354	0.00283	353.58372	0.05283	18.92929	314.34316	16.60618
65	23.83990	0.04195	0.00219	456.79801	0.05219	19.16107	328.69098	17.15410
70	30.42643	0.03287	0.00170	588.52851	0.05170	19.34268	340.84090	17.62119
75	38.83269	0.02575	0.00132	756.65372	0.05132	19.48497	351.07215	18.01759
80	49.56144	0.02018	0.00103	971.22882	0.05103	19.59646	359.64605	18.35260
85	63.25435	0.01581	0.00080	1245.08707	0.05080	19.68382	366.80070	18.63463
90	80.73037	0.01239	0.00063	1594.60730	0.05063	19.75226	372.74879	18.87120
96	108.18641	0.00924	0.00047	2143.72821	0.05047	19.81513	378.55553	19.10436
98	119.27552	0.00838	0.00042	2365.51035	0.05042	19.83232	380.21388	19.17143
100	131.50126	0.00760	0.00038	2610.02516	0.05038	19.84791	381.74922	19.23372

Table A12 Compound Interest Factors 6%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.06000	0.94340	1.00000	1.00000	1.06000	0.94340		
2	1.12360	0.89000	0.48544	2.06000	0.54544	1.83339	0.89000	0.48544
3	1.19102	0.83962	0.31411	3.18360	0.37411	2.67301	2.56924	0.96118
4	1.26248	0.79209	0.22859	4.37462	0.28859	3.46511	4.94552	1.42723
5	1.33823	0.74726	0.17740	5.63709	0.23740	4.21236	7.93455	1.88363

6	1.41852	0.70496	0.14336	6.97532	0.20336	4.91732	11.45935	2.33040
7	1.50363	0.66506	0.11914	8.39384	0.17914	5.58238	15.44969	2.76758
8	1.59385	0.62741	0.10104	9.89747	0.16104	6.20979	19.84158	3.19521
9	1.68948	0.59190	0.08702	11.49132	0.14702	6.80169	24.57677	3.61333
10	1.79085	0.55839	0.07587	13.18079	0.13587	7.36009	29.60232	4.02201
11	1.89830	0.52679	0.06679	14.97164	0.12679	7.88687	34.87020	4.42129
12	2.01220	0.49697	0.05928	16.86994	0.11928	8.38384	40.33686	4.81126
13	2.13293	0.46884	0.05296	18.88214	0.11296	8.85268	45.96293	5.19198
14	2.26090	0.44230	0.04758	21.01507	0.10758	9.29498	51.71284	5.56352
15	2.39656	0.41727	0.04296	23.27597	0.10296	9.71225	57.55455	5.92598
16	2.54035	0.39365	0.03895	25.67253	0.09895	10.10590	63.45925	6.27943
17	2.69277	0.37136	0.03544	28.21288	0.09544	10.47726	69.40108	6.62397
18	2.85434	0.35034	0.03236	30.90565	0.09236	10.82760	75.35692	6.95970
19	3.02560	0.33051	0.02962	33.75999	0.08962	11.15812	81.30615	7.28673
20	3.20714	0.31180	0.02718	36.78559	0.08718	11.46992	87.23044	7.60515
21	3.39956	0.29416	0.02500	39.99273	0.08500	11.76408	93.11355	7.91508
22	3.60354	0.27751	0.02305	43.39229	0.08305	12.04158	98.94116	8.21662
23	3.81975	0.26180	0.02128	46.99583	0.08128	12.30338	104.70070	8.50991
24	4.04893	0.24698	0.01968	50.81558	0.07968	12.55036	110.38121	8.79506
25	4.29187	0.23300	0.01823	54.86451	0.07823	12.78336	115.97317	9.07220
26	4.54938	0.21981	0.01690	59.15638	0.07690	13.00317	121.46842	9.34145
27	4.82235	0.20737	0.01570	63.70577	0.07570	13.21053	126.85999	9.60294
28	5.11169	0.19563	0.01459	68.52811	0.07459	13.40616	132.14200	9.85681
29	5.41839	0.18456	0.01358	73.63980	0.07358	13.59072	137.30959	10.10319
30	5.74349	0.17411	0.01265	79.05819	0.07265	13.76483	142.35879	10.34221
31	6.08810	0.16425	0.01179	84.80168	0.07179	13.92909	147.28643	10.57402
32	6.45339	0.15496	0.01100	90.88978	0.07100	14.08404	152.09011	10.79875
33	6.84059	0.14619	0.01027	97.34316	0.07027	14.23023	156.76807	11.01655
34	7.25103	0.13791	0.00960	104.18375	0.06960	14.36814	161.31915	11.22756
35	7.68609	0.13011	0.00897	111.43478	0.06897	14.49825	165.74273	11.43192
36	8.14725	0.12274	0.00839	119.12087	0.06839	14.62099	170.03866	11.62977
40	10.28572	0.09722	0.00646	154.76197	0.06646	15.04630	185.95682	12.35898
45	13.76461	0.07265	0.00470	212.74351	0.06470	15.45583	203.10965	13.14129

(Continued)

Table A12 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
50	18.42015	0.05429	0.00344	290.33590	0.06344	15.76186	217.45738	13.79643
55	24.65032	0.04057	0.00254	394.17203	0.06254	15.99054	229.32225	14.34112
60	32.98769	0.03031	0.00188	533.12818	0.06188	16.16143	239.04279	14.79095
65	44.14497	0.02265	0.00139	719.08286	0.06139	16.28912	246.94502	15.16012
70	59.07593	0.01693	0.00103	967.93217	0.06103	16.38454	253.32714	15.46135
75	79.05692	0.01265	0.00077	1300.94868	0.06077	16.45585	258.45274	15.70583
80	105.79599	0.00945	0.00057	1746.59989	0.06057	16.50913	262.54931	15.90328
85	141.57890	0.00706	0.00043	2342.98174	0.06043	16.54895	265.80958	16.06202
90	189.46451	0.00528	0.00032	3141.07519	0.06032	16.57870	268.39461	16.18912
96	268.75903	0.00372	0.00022	4462.65050	0.06022	16.60465	270.79093	16.30814
98	301.97765	0.00331	0.00020	5016.29411	0.06020	16.61147	271.44913	16.34106
100	339.30208	0.00295	0.00018	5638.36806	0.06018	16.61755	272.04706	16.37107

Table A13 Compound Interest Factors 7%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.07000	0.93458	1.00000	1.00000	1.07000	0.93458		
2	1.14490	0.87344	0.48309	2.07000	0.55309	1.80802	0.87344	0.48309
3	1.22504	0.81630	0.31105	3.21490	0.38105	2.62432	2.50603	0.95493
4	1.31080	0.76290	0.22523	4.43994	0.29523	3.38721	4.79472	1.41554
5	1.40255	0.71299	0.17389	5.75074	0.24389	4.10020	7.64666	1.86495
6	1.50073	0.66634	0.13980	7.15329	0.20980	4.76654	10.97838	2.30322
7	1.60578	0.62275	0.11555	8.65402	0.18555	5.38929	14.71487	2.73039
8	1.71819	0.58201	0.09747	10.25980	0.16747	5.97130	18.78894	3.14654
9	1.83846	0.54393	0.08349	11.97799	0.15349	6.51523	23.14041	3.55174
10	1.96715	0.50835	0.07238	13.81645	0.14238	7.02358	27.71555	3.94607
11	2.10485	0.47509	0.06336	15.78360	0.13336	7.49867	32.46648	4.32963
12	2.25219	0.44401	0.05590	17.88845	0.12590	7.94269	37.35061	4.70252
13	2.40985	0.41496	0.04965	20.14064	0.11965	8.35765	42.33018	5.06484

14	2.57853	0.38782	0.04434	22.55049	0.11434	8.74547	47.37181	5.41673
15	2.75903	0.36245	0.03979	25.12902	0.10979	9.10791	52.44605	5.75829
16	2.95216	0.33873	0.03586	27.88805	0.10586	9.44665	57.52707	6.08968
17	3.15882	0.31657	0.03243	30.84022	0.10243	9.76322	62.59226	6.41102
18	3.37993	0.29586	0.02941	33.99903	0.09941	10.05909	67.62195	6.72247
19	3.61653	0.27651	0.02675	37.37896	0.09675	10.33560	72.59910	7.02418
20	3.86968	0.25842	0.02439	40.99549	0.09439	10.59401	77.50906	7.31631
21	4.14056	0.24151	0.02229	44.86518	0.09229	10.83553	82.33932	7.59901
22	4.43040	0.22571	0.02041	49.00574	0.09041	11.06124	87.07930	7.87247
23	4.74053	0.21095	0.01871	53.43614	0.08871	11.27219	91.72013	8.13685
24	5.07237	0.19715	0.01719	58.17667	0.08719	11.46933	96.25450	8.39234
25	5.42743	0.18425	0.01581	63.24904	0.08581	11.65358	100.67648	8.63910
26	5.80735	0.17220	0.01456	68.67647	0.08456	11.82578	104.98137	8.87733
27	6.21387	0.16093	0.01343	74.48382	0.08343	11.98671	109.16556	9.10722
28	6.64884	0.15040	0.01239	80.69769	0.08239	12.13711	113.22642	9.32894
29	7.11426	0.14056	0.01145	87.34653	0.08145	12.27767	117.16218	9.54270
30	7.61226	0.13137	0.01059	94.46079	0.08059	12.40904	120.97182	9.74868
31	8.14511	0.12277	0.00980	102.07304	0.07980	12.53181	124.65501	9.94708
32	8.71527	0.11474	0.00907	110.21815	0.07907	12.64656	128.21199	10.13810
33	9.32534	0.10723	0.00841	118.93343	0.07841	12.75379	131.64350	10.32191
34	9.97811	0.10022	0.00780	128.25876	0.07780	12.85401	134.95074	10.49873
35	10.67658	0.09366	0.00723	138.23688	0.07723	12.94767	138.13528	10.66873
36	11.42394	0.08754	0.00672	148.91346	0.07672	13.03521	141.19902	10.83213
40	14.97446	0.06678	0.00501	199.63511	0.07501	13.33171	152.29277	11.42335
45	21.00245	0.04761	0.00350	285.74931	0.07350	13.60552	163.75592	12.03599
50	29.45703	0.03395	0.00246	406.52893	0.07246	13.80075	172.90512	12.52868
55	41.31500	0.02420	0.00174	575.92859	0.07174	13.93994	180.12433	12.92146
60	57.94643	0.01726	0.00123	813.52038	0.07123	14.03918	185.76774	13.23209
65	81.27286	0.01230	0.00087	1146.75516	0.07087	14.10994	190.14521	13.47598
70	113.98939	0.00877	0.00062	1614.13417	0.07062	14.16039	193.51853	13.66619
75	159.87602	0.00625	0.00044	2269.65742	0.07044	14.19636	196.10351	13.81365
80	224.23439	0.00446	0.00031	3189.06268	0.07031	14.22201	198.07480	13.92735
85	314.50033	0.00318	0.00022	4478.57612	0.07022	14.24029	199.57173	14.01458

(Continued)

Table A13 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
90	441.10298	0.00227	0.00016	6287.18543	0.07016	14.25333	200.70420	14.08122
96	661.97663	0.00151	0.00011	9442.52329	0.07011	14.26413	201.70162	14.14047
98	757.89704	0.00132	0.00009	10812.8149	0.07009	14.26687	201.96514	14.15624
100	867.71633	0.00115	0.00008	12381.6618	0.07008	14.26925	202.20008	14.17034

Table A14 Compound Interest Factors 8%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.08000	0.92593	1.00000	1.00000	1.08000	0.92593		
2	1.16640	0.85734	0.48077	2.08000	0.56077	1.78326	0.85734	0.48077
3	1.25971	0.79383	0.30803	3.24640	0.38803	2.57710	2.44500	0.94874
4	1.36049	0.73503	0.22192	4.50611	0.30192	3.31213	4.65009	1.40396
5	1.46933	0.68058	0.17046	5.86660	0.25046	3.99271	7.37243	1.84647
6	1.58687	0.63017	0.13632	7.33593	0.21632	4.62288	10.52327	2.27635
7	1.71382	0.58349	0.11207	8.92280	0.19207	5.20637	14.02422	2.69366
8	1.85093	0.54027	0.09401	10.63663	0.17401	5.74664	17.80610	3.09852
9	1.99900	0.50025	0.08008	12.48756	0.16008	6.24689	21.80809	3.49103
10	2.15892	0.46319	0.06903	14.48656	0.14903	6.71008	25.97683	3.87131
11	2.33164	0.42888	0.06008	16.64549	0.14008	7.13896	30.26566	4.23950
12	2.51817	0.39711	0.05270	18.97713	0.13270	7.53608	34.63391	4.59575
13	2.71962	0.36770	0.04652	21.49530	0.12652	7.90378	39.04629	4.94021
14	2.93719	0.34046	0.04130	24.21492	0.12130	8.24424	43.47228	5.27305
15	3.17217	0.31524	0.03683	27.15211	0.11683	8.55948	47.88566	5.59446
16	3.42594	0.29189	0.03298	30.32428	0.11298	8.85137	52.26402	5.90463
17	3.70002	0.27027	0.02963	33.75023	0.10963	9.12164	56.58832	6.20375
18	3.99602	0.25025	0.02670	37.45024	0.10670	9.37189	60.84256	6.49203
19	4.31570	0.23171	0.02413	41.44626	0.10413	9.60360	65.01337	6.76969
20	4.66096	0.21455	0.02185	45.76196	0.10185	9.81815	69.08979	7.03695
21	5.03383	0.19866	0.01983	50.42292	0.09983	10.01680	73.06291	7.29403

22	5.43654	0.18394	0.01803	55.45676	0.09803	10.20074	76.92566	7.54118
23	5.87146	0.17032	0.01642	60.89330	0.09642	10.37106	80.67259	7.77863
24	6.34118	0.15770	0.01498	66.76476	0.09498	10.52876	84.29968	8.00661
25	6.84848	0.14602	0.01368	73.10594	0.09368	10.67478	87.80411	8.22538
26	7.39635	0.13520	0.01251	79.95442	0.09251	10.80998	91.18415	8.43518
27	7.98806	0.12519	0.01145	87.35077	0.09145	10.93516	94.43901	8.63627
28	8.62711	0.11591	0.01049	95.33883	0.09049	11.05108	97.56868	8.82888
29	9.31727	0.10733	0.00962	103.96594	0.08962	11.15841	100.57385	9.01328
30	10.06266	0.09938	0.00883	113.28321	0.08883	11.25778	103.45579	9.18971
31	10.86767	0.09202	0.00811	123.34587	0.08811	11.34980	106.21627	9.35843
32	11.73708	0.08520	0.00745	134.21354	0.08745	11.43500	108.85747	9.51967
33	12.67605	0.07889	0.00685	145.95062	0.08685	11.51389	111.38192	9.67370
34	13.69013	0.07305	0.00630	158.62667	0.08630	11.58693	113.79242	9.82075
35	14.78534	0.06763	0.00580	172.31680	0.08580	11.65457	116.09199	9.96107
36	15.96817	0.06262	0.00534	187.10215	0.08534	11.71719	118.28385	10.09490
40	21.72452	0.04603	0.00386	259.05652	0.08386	11.92461	126.04220	10.56992
45	31.92045	0.03133	0.00259	386.50562	0.08259	12.10840	133.73309	11.04465
50	46.90161	0.02132	0.00174	573.77016	0.08174	12.23348	139.59279	11.41071
55	68.91386	0.01451	0.00118	848.92320	0.08118	12.31861	144.00645	11.69015
60	101.25706	0.00988	0.00080	1253.21330	0.08080	12.37655	147.30001	11.90154
65	148.77985	0.00672	0.00054	1847.24808	0.08054	12.41598	149.73870	12.06016
70	218.60641	0.00457	0.00037	2720.08007	0.08037	12.44282	151.53262	12.17832
75	321.20453	0.00311	0.00025	4002.55662	0.08025	12.46108	152.84485	12.26577
80	471.95483	0.00212	0.00017	5886.93543	0.08017	12.47351	153.80008	12.33013
85	693.45649	0.00144	0.00012	8655.70611	0.08012	12.48197	154.49250	12.37725
90	1018.91509	0.00098	0.00008	12723.9386	0.08008	12.48773	154.99254	12.41158
96	1616.89019	0.00062	0.00005	20198.6274	0.08005	12.49227	155.41120	12.44059
98	1885.94072	0.00053	0.00004	23561.7590	0.08004	12.49337	155.51761	12.44801
100	2199.76126	0.00045	0.00004	27484.5157	0.08004	12.49432	155.61073	12.45452

Table A15 Compound Interest Factors 9%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.09000	0.91743	1.00000	1.00000	1.09000	0.91743		
2	1.18810	0.84168	0.47847	2.09000	0.56847	1.75911	0.84168	0.47847
3	1.29503	0.77218	0.30505	3.27810	0.39505	2.53129	2.38605	0.94262
4	1.41158	0.70843	0.21867	4.57313	0.30867	3.23972	4.51132	1.39250
5	1.53862	0.64993	0.16709	5.98471	0.25709	3.88965	7.11105	1.82820
6	1.67710	0.59627	0.13292	7.52333	0.22292	4.48592	10.09238	2.24979
7	1.82804	0.54703	0.10869	9.20043	0.19869	5.03295	13.37459	2.65740
8	1.99256	0.50187	0.09067	11.02847	0.18067	5.53482	16.88765	3.05117
9	2.17189	0.46043	0.07680	13.02104	0.16680	5.99525	20.57108	3.43123
10	2.36736	0.42241	0.06582	15.19293	0.15582	6.41766	24.37277	3.79777
11	2.58043	0.38753	0.05695	17.56029	0.14695	6.80519	28.24810	4.15096
12	2.81266	0.35553	0.04965	20.14072	0.13965	7.16073	32.15898	4.49102
13	3.06580	0.32618	0.04357	22.95338	0.13357	7.48690	36.07313	4.81816
14	3.34173	0.29925	0.03843	26.01919	0.12843	7.78615	39.96333	5.13262
15	3.64248	0.27454	0.03406	29.36092	0.12406	8.06069	43.80686	5.43463
16	3.97031	0.25187	0.03030	33.00340	0.12030	8.31256	47.58491	5.72446
17	4.32763	0.23107	0.02705	36.97370	0.11705	8.54363	51.28208	6.00238
18	4.71712	0.21199	0.02421	41.30134	0.11421	8.75563	54.88598	6.26865
19	5.14166	0.19449	0.02173	46.01846	0.11173	8.95011	58.38679	6.52358
20	5.60441	0.17843	0.01955	51.16012	0.10955	9.12855	61.77698	6.76745
21	6.10881	0.16370	0.01762	56.76453	0.10762	9.29224	65.05094	7.00056
22	6.65860	0.15018	0.01590	62.87334	0.10590	9.44243	68.20475	7.22322
23	7.25787	0.13778	0.01438	69.53194	0.10438	9.58021	71.23594	7.43574
24	7.91108	0.12640	0.01302	76.78981	0.10302	9.70661	74.14326	7.63843
25	8.62308	0.11597	0.01181	84.70090	0.10181	9.82258	76.92649	7.83160
26	9.39916	0.10639	0.01072	93.32398	0.10072	9.92897	79.58630	8.01556
27	10.24508	0.09761	0.00973	102.72313	0.09973	10.02658	82.12410	8.19064
28	11.16714	0.08955	0.00885	112.96822	0.09885	10.11613	84.54191	8.35714
29	12.17218	0.08215	0.00806	124.13536	0.09806	10.19828	86.84224	8.51538
30	13.26768	0.07537	0.00734	136.30754	0.09734	10.27365	89.02800	8.66566

31	14.46177	0.06915	0.00669	149.57522	0.09669	10.34280	91.10243	8.80829
32	15.76333	0.06344	0.00610	164.03699	0.09610	10.40624	93.06902	8.94358
33	17.18203	0.05820	0.00556	179.80032	0.09556	10.46444	94.93144	9.07181
34	18.72841	0.05339	0.00508	196.98234	0.09508	10.51784	96.69346	9.19329
35	20.41397	0.04899	0.00464	215.71075	0.09464	10.56682	98.35899	9.30829
36	22.25123	0.04494	0.00424	236.12472	0.09424	10.61176	99.93194	9.41709
40	31.40942	0.03184	0.00296	337.88245	0.09296	10.75736	105.37619	9.79573
45	48.32729	0.02069	0.00190	525.85873	0.09190	10.88120	110.55607	10.16029
50	74.35752	0.01345	0.00123	815.08356	0.09123	10.96168	114.32507	10.42952
55	114.40826	0.00874	0.00079	1260.09180	0.09079	11.01399	117.03621	10.62614
60	176.03129	0.00568	0.00051	1944.79213	0.09051	11.04799	118.96825	10.76832
65	270.84596	0.00369	0.00033	2998.28847	0.09033	11.07009	120.33443	10.87023
70	416.73009	0.00240	0.00022	4619.22318	0.09022	11.08445	121.29416	10.94273
75	641.19089	0.00156	0.00014	7113.23215	0.09014	11.09378	121.96458	10.99396
80	986.55167	0.00101	0.00009	10950.5741	0.09009	11.09985	122.43064	11.02994
85	1517.93203	0.00066	0.00006	16854.8003	0.09006	11.10379	122.75327	11.05508
90	2335.52658	0.00043	0.00004	25939.1842	0.09004	11.10635	122.97576	11.07256
96	3916.91189	0.00026	0.00002	43510.1321	0.09002	11.10827	123.15295	11.08660
98	4653.68302	0.00021	0.00002	51696.4780	0.09002	11.10872	123.19628	11.09005
100	5529.04079	0.00018	0.00002	61422.6755	0.09002	11.10910	123.23350	11.09302

Table A16 Compound Interest Factors 10%

n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.10000	0.90909	1.00000	1.00000	1.10000	0.90909		
2	1.21000	0.82645	0.47619	2.10000	0.57619	1.73554	0.82645	0.47619
3	1.33100	0.75131	0.30211	3.31000	0.40211	2.48685	2.32908	0.93656
4	1.46410	0.68301	0.21547	4.64100	0.31547	3.16987	4.37812	1.38117
5	1.61051	0.62092	0.16380	6.10510	0.26380	3.79079	6.86180	1.81013
6	1.77156	0.56447	0.12961	7.71561	0.22961	4.35526	9.68417	2.22356
7	1.94872	0.51316	0.10541	9.48717	0.20541	4.86842	12.76312	2.62162
8	2.14359	0.46651	0.08744	11.43589	0.18744	5.33493	16.02867	3.00448

(Continued)

Table A16 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
9	2.35795	0.42410	0.07364	13.57948	0.17364	5.75902	19.42145	3.37235
10	2.59374	0.38554	0.06275	15.93742	0.16275	6.14457	22.89134	3.72546
11	2.85312	0.35049	0.05396	18.53117	0.15396	6.49506	26.39628	4.06405
12	3.13843	0.31863	0.04676	21.38428	0.14676	6.81369	29.90122	4.38840
13	3.45227	0.28966	0.04078	24.52271	0.14078	7.10336	33.37719	4.69879
14	3.79750	0.26333	0.03575	27.97498	0.13575	7.36669	36.80050	4.99553
15	4.17725	0.23939	0.03147	31.77248	0.13147	7.60608	40.15199	5.27893
16	4.59497	0.21763	0.02782	35.94973	0.12782	7.82371	43.41642	5.54934
17	5.05447	0.19784	0.02466	40.54470	0.12466	8.02155	46.58194	5.80710
18	5.55992	0.17986	0.02193	45.59917	0.12193	8.20141	49.63954	6.05256
19	6.11591	0.16351	0.01955	51.15909	0.11955	8.36492	52.58268	6.28610
20	6.72750	0.14864	0.01746	57.27500	0.11746	8.51356	55.40691	6.50808
21	7.40025	0.13513	0.01562	64.00250	0.11562	8.64869	58.10952	6.71888
22	8.14027	0.12285	0.01401	71.40275	0.11401	8.77154	60.68929	6.91889
23	8.95430	0.11168	0.01257	79.54302	0.11257	8.88322	63.14621	7.10848
24	9.84973	0.10153	0.01130	88.49733	0.11130	8.98474	65.48130	7.28805
25	10.83471	0.09230	0.01017	98.34706	0.11017	9.07704	67.69640	7.45798
26	11.91818	0.08391	0.00916	109.18177	0.10916	9.16095	69.79404	7.61865
27	13.10999	0.07628	0.00826	121.09994	0.10826	9.23722	71.77726	7.77044
28	14.42099	0.06934	0.00745	134.20994	0.10745	9.30657	73.64953	7.91372
29	15.86309	0.06304	0.00673	148.63093	0.10673	9.36961	75.41463	8.04886
30	17.44940	0.05731	0.00608	164.49402	0.10608	9.42691	77.07658	8.17623
31	19.19434	0.05210	0.00550	181.94342	0.10550	9.47901	78.63954	8.29617
32	21.11378	0.04736	0.00497	201.13777	0.10497	9.52638	80.10777	8.40905
33	23.22515	0.04306	0.00450	222.25154	0.10450	9.56943	81.48559	8.51520
34	25.54767	0.03914	0.00407	245.47670	0.10407	9.60857	82.77729	8.61494
35	28.10244	0.03558	0.00369	271.02437	0.10369	9.64416	83.98715	8.70860
36	30.91268	0.03235	0.00334	299.12681	0.10334	9.67651	85.11938	8.79650
40	45.25926	0.02209	0.00226	442.59256	0.10226	9.77905	88.95254	9.09623
45	72.89048	0.01372	0.00139	718.90484	0.10139	9.86281	92.45443	9.37405

50	117.39085	0.00852	0.00086	1163.90853	0.10086	9.91481	94.88887	9.57041
55	189.05914	0.00529	0.00053	1880.59142	0.10053	9.94711	96.56192	9.70754
60	304.48164	0.00328	0.00033	3034.81640	0.10033	9.96716	97.70101	9.80229
65	490.37073	0.00204	0.00020	4893.70725	0.10020	9.97961	98.47054	9.86718
70	789.74696	0.00127	0.00013	7887.46957	0.10013	9.98734	98.98702	9.91125
75	1271.89537	0.00079	0.00008	12708.9537	0.10008	9.99214	99.33171	9.94099
80	2048.40021	0.00049	0.00005	20474.0021	0.10005	9.99512	99.56063	9.96093
85	3298.96903	0.00030	0.00003	32979.6903	0.10003	9.99697	99.71203	9.97423
90	5313.02261	0.00019	0.00002	53120.2261	0.10002	9.99812	99.81178	9.98306
96	9412.34365	0.00011	0.00001	94113.4365	0.10001	9.99894	99.88738	9.98980
98	11388.9358	0.00009	0.00001	113879.358	0.10001	9.99912	99.90517	9.99139
100	13780.6123	0.00007	0.00001	137796.123	0.10001	9.99927	99.92018	9.99274

Table A17 Compound Interest Factors 11%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.11000	0.90090	1.00000	1.00000	1.11000	0.90090		
2	1.23210	0.81162	0.47393	2.11000	0.58393	1.71252	0.81162	0.47393
3	1.36763	0.73119	0.29921	3.34210	0.40921	2.44371	2.27401	0.93055
4	1.51807	0.65873	0.21233	4.70973	0.32233	3.10245	4.25020	1.36995
5	1.68506	0.59345	0.16057	6.22780	0.27057	3.69590	6.62400	1.79226
6	1.87041	0.53464	0.12638	7.91286	0.23638	4.23054	9.29721	2.19764
7	2.07616	0.48166	0.10222	9.78327	0.21222	4.71220	12.18716	2.58630
8	2.30454	0.43393	0.08432	11.85943	0.19432	5.14612	15.22464	2.95847
9	2.55804	0.39092	0.07060	14.16397	0.18060	5.53705	18.35204	3.31441
10	2.83942	0.35218	0.05980	16.72201	0.16980	5.88923	21.52170	3.65442
11	3.15176	0.31728	0.05112	19.56143	0.16112	6.20652	24.69454	3.97881
12	3.49845	0.28584	0.04403	22.71319	0.15403	6.49236	27.83878	4.28793
13	3.88328	0.25751	0.03815	26.21164	0.14815	6.74987	30.92896	4.58216
14	4.31044	0.23199	0.03323	30.09492	0.14323	6.98187	33.94489	4.86187
15	4.78459	0.20900	0.02907	34.40536	0.13907	7.19087	36.87095	5.12747
16	5.31089	0.18829	0.02552	39.18995	0.13552	7.37916	39.69533	5.37938

(Continued)

Table A17 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
17	5.89509	0.16963	0.02247	44.50084	0.13247	7.54879	42.40945	5.61804
18	6.54355	0.15282	0.01984	50.39594	0.12984	7.70162	45.00743	5.84389
19	7.26334	0.13768	0.01756	56.93949	0.12756	7.83929	47.48563	6.05739
20	8.06231	0.12403	0.01558	64.20283	0.12558	7.96333	49.84227	6.25898
21	8.94917	0.11174	0.01384	72.26514	0.12384	8.07507	52.07712	6.44912
22	9.93357	0.10067	0.01231	81.21431	0.12231	8.17574	54.19116	6.62829
23	11.02627	0.09069	0.01097	91.14788	0.12097	8.26643	56.18640	6.79693
24	12.23916	0.08170	0.00979	102.17415	0.11979	8.34814	58.06561	6.95552
25	13.58546	0.07361	0.00874	114.41331	0.11874	8.42174	59.83220	7.10449
26	15.07986	0.06631	0.00781	127.99877	0.11781	8.48806	61.49004	7.24430
27	16.73865	0.05974	0.00699	143.07864	0.11699	8.54780	63.04334	7.37539
28	18.57990	0.05382	0.00626	159.81729	0.11626	8.60162	64.49652	7.49818
29	20.62369	0.04849	0.00561	178.39719	0.11561	8.65011	65.85418	7.61310
30	22.89230	0.04368	0.00502	199.02088	0.11502	8.69379	67.12098	7.72056
31	25.41045	0.03935	0.00451	221.91317	0.11451	8.73315	68.30160	7.82096
32	28.20560	0.03545	0.00404	247.32362	0.11404	8.76860	69.40067	7.91468
33	31.30821	0.03194	0.00363	275.52922	0.11363	8.80054	70.42277	8.00210
34	34.75212	0.02878	0.00326	306.83744	0.11326	8.82932	71.37235	8.08356
35	38.57485	0.02592	0.00293	341.58955	0.11293	8.85524	72.25375	8.15944
36	42.81808	0.02335	0.00263	380.16441	0.11263	8.87859	73.07116	8.23004
37	47.52807	0.02104	0.00236	422.98249	0.11236	8.89963	73.82861	8.29569
38	52.75616	0.01896	0.00213	470.51056	0.11213	8.91859	74.52995	8.35670
39	58.55934	0.01708	0.00191	523.26673	0.11191	8.93567	75.17887	8.41335
40	65.00087	0.01538	0.00172	581.82607	0.11172	8.95105	75.77886	8.46592
45	109.53024	0.00913	0.00101	986.63856	0.11101	9.00791	78.15513	8.67628
50	184.56483	0.00542	0.00060	1668.77115	0.11060	9.04165	79.73405	8.81853
55	311.00247	0.00322	0.00035	2818.20424	0.11035	9.06168	80.77119	8.91349
60	524.05724	0.00191	0.00021	4755.06584	0.11021	9.07356	81.44610	8.97620
65	883.06693	0.00113	0.00012	8018.79027	0.11012	9.08061	81.88188	9.01722
70	1488.01913	0.00067	0.00007	13518.3557	0.11007	9.08480	82.16143	9.04384

75	2507.39877	0.00040	0.00004	22785.4434	0.11004	9.08728	82.33975	9.06099
80	4225.11275	0.00024	0.00003	38401.0250	0.11003	9.08876	82.45294	9.07197
85	7119.56070	0.00014	0.00002	64714.1881	0.11002	9.08963	82.52448	9.07897
90	11996.8738	0.00008	0.00001	109053.398	0.11001	9.09015	82.56954	9.08341

Table A18 Compound Interest Factors 12%

n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.12000	0.89286	1.00000	1.00000	1.12000	0.89286		
2	1.25440	0.79719	0.47170	2.12000	0.59170	1.69005	0.79719	0.47170
3	1.40493	0.71178	0.29635	3.37440	0.41635	2.40183	2.22075	0.92461
4	1.57352	0.63552	0.20923	4.77933	0.32923	3.03735	4.12731	1.35885
5	1.76234	0.56743	0.15741	6.35285	0.27741	3.60478	6.39702	1.77459
6	1.97382	0.50663	0.12323	8.11519	0.24323	4.11141	8.93017	2.17205
7	2.21068	0.45235	0.09912	10.08901	0.21912	4.56376	11.64427	2.55147
8	2.47596	0.40388	0.08130	12.29969	0.20130	4.96764	14.47145	2.91314
9	2.77308	0.36061	0.06768	14.77566	0.18768	5.32825	17.35633	3.25742
10	3.10585	0.32197	0.05698	17.54874	0.17698	5.65022	20.25409	3.58465
11	3.47855	0.28748	0.04842	20.65458	0.16842	5.93770	23.12885	3.89525
12	3.89598	0.25668	0.04144	24.13313	0.16144	6.19437	25.95228	4.18965
13	4.36349	0.22917	0.03568	28.02911	0.15568	6.42355	28.70237	4.46830
14	4.88711	0.20462	0.03087	32.39260	0.15087	6.62817	31.36242	4.73169
15	5.47357	0.18270	0.02682	37.27971	0.14682	6.81086	33.92017	4.98030
16	6.13039	0.16312	0.02339	42.75328	0.14339	6.97399	36.36700	5.21466
17	6.86604	0.14564	0.02046	48.88367	0.14046	7.11963	38.69731	5.43530
18	7.68997	0.13004	0.01794	55.74971	0.13794	7.24967	40.90798	5.64274
19	8.61276	0.11611	0.01576	63.43968	0.13576	7.36578	42.99790	5.83752
20	9.64629	0.10367	0.01388	72.05244	0.13388	7.46944	44.96757	6.02020
21	10.80385	0.09256	0.01224	81.69874	0.13224	7.56200	46.81876	6.19132
22	12.10031	0.08264	0.01081	92.50258	0.13081	7.64465	48.55425	6.35141
23	13.55235	0.07379	0.00956	104.60289	0.12956	7.71843	50.17759	6.50101
24	15.17863	0.06588	0.00846	118.15524	0.12846	7.78432	51.69288	6.64064

(Continued)

Table A18 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
25	17.00006	0.05882	0.00750	133.33387	0.12750	7.84314	53.10464	6.77084
26	19.04007	0.05252	0.00665	150.33393	0.12665	7.89566	54.41766	6.89210
27	21.32488	0.04689	0.00590	169.37401	0.12590	7.94255	55.63689	7.00491
28	23.88387	0.04187	0.00524	190.69889	0.12524	7.98442	56.76736	7.10976
29	26.74993	0.03738	0.00466	214.58275	0.12466	8.02181	57.81409	7.20712
30	29.95992	0.03338	0.00414	241.33268	0.12414	8.05518	58.78205	7.29742
31	33.55511	0.02980	0.00369	271.29261	0.12369	8.08499	59.67610	7.38110
32	37.58173	0.02661	0.00328	304.84772	0.12328	8.11159	60.50097	7.45858
33	42.09153	0.02376	0.00292	342.42945	0.12292	8.13535	61.26122	7.53025
34	47.14252	0.02121	0.00260	384.52098	0.12260	8.15656	61.96123	7.59649
35	52.79962	0.01894	0.00232	431.66350	0.12232	8.17550	62.60517	7.65765
36	59.13557	0.01691	0.00206	484.46312	0.12206	8.19241	63.19703	7.71409
37	66.23184	0.01510	0.00184	543.59869	0.12184	8.20751	63.74058	7.76613
38	74.17966	0.01348	0.00164	609.83053	0.12164	8.22099	64.23936	7.81406
39	83.08122	0.01204	0.00146	684.01020	0.12146	8.23303	64.69675	7.85819
40	93.05097	0.01075	0.00130	767.09142	0.12130	8.24378	65.11587	7.89879
45	163.98760	0.00610	0.00074	1358.23003	0.12074	8.28252	66.73421	8.05724
50	289.00219	0.00346	0.00042	2400.01825	0.12042	8.30450	67.76241	8.15972
55	509.32061	0.00196	0.00024	4236.00505	0.12024	8.31697	68.40821	8.22513
60	897.59693	0.00111	0.00013	7471.64111	0.12013	8.32405	68.81003	8.26641
65	1581.87249	0.00063	0.00008	13173.9374	0.12008	8.32807	69.05812	8.29222
70	2787.79983	0.00036	0.00004	23223.3319	0.12004	8.33034	69.21029	8.30821
75	4913.05584	0.00020	0.00002	40933.7987	0.12002	8.33164	69.30310	8.31806
80	8658.48310	0.00012	0.00001	72145.6925	0.12001	8.33237	69.35943	8.32409
85	15259.2057	0.00007	0.00001	127151.714	0.12001	8.33279	69.39347	8.32776
90	26891.9342	0.00004	0.00000	224091.119	0.12000	8.33302	69.41397	8.32999

Table A19 Compound Interest Factors 13%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.13000	0.88496	1.00000	1.00000	1.13000	0.88496		
2	1.27690	0.78315	0.46948	2.13000	0.59948	1.66810	0.78315	0.46948
3	1.44290	0.69305	0.29352	3.40690	0.42352	2.36115	2.16925	0.91872
4	1.63047	0.61332	0.20619	4.84980	0.33619	2.97447	4.00920	1.34787
5	1.84244	0.54276	0.15431	6.48027	0.28431	3.51723	6.18024	1.75713
6	2.08195	0.48032	0.12015	8.32271	0.25015	3.99755	8.58184	2.14677
7	2.35261	0.42506	0.09611	10.40466	0.22611	4.42261	11.13220	2.51711
8	2.65844	0.37616	0.07839	12.75726	0.20839	4.79877	13.76532	2.86851
9	3.00404	0.33288	0.06487	15.41571	0.19487	5.13166	16.42840	3.20138
10	3.39457	0.29459	0.05429	18.41975	0.18429	5.42624	19.07969	3.51619
11	3.83586	0.26070	0.04584	21.81432	0.17584	5.68694	21.68667	3.81342
12	4.33452	0.23071	0.03899	25.65018	0.16899	5.91765	24.22443	4.09359
13	4.89801	0.20416	0.03335	29.98470	0.16335	6.12181	26.67441	4.35727
14	5.53475	0.18068	0.02867	34.88271	0.15867	6.30249	29.02320	4.60504
15	6.25427	0.15989	0.02474	40.41746	0.15474	6.46238	31.26167	4.83749
16	7.06733	0.14150	0.02143	46.67173	0.15143	6.60388	33.38412	5.05523
17	7.98608	0.12522	0.01861	53.73906	0.14861	6.72909	35.38760	5.25890
18	9.02427	0.11081	0.01620	61.72514	0.14620	6.83991	37.27141	5.44911
19	10.19742	0.09806	0.01413	70.74941	0.14413	6.93797	39.03656	5.62651
20	11.52309	0.08678	0.01235	80.94683	0.14235	7.02475	40.68543	5.79172
21	13.02109	0.07680	0.01081	92.46992	0.14081	7.10155	42.22140	5.94538
22	14.71383	0.06796	0.00948	105.49101	0.13948	7.16951	43.64863	6.08809
23	16.62663	0.06014	0.00832	120.20484	0.13832	7.22966	44.97181	6.22046
24	18.78809	0.05323	0.00731	136.83147	0.13731	7.28288	46.19599	6.34309
25	21.23054	0.04710	0.00643	155.61956	0.13643	7.32998	47.32643	6.45655
26	23.99051	0.04168	0.00565	176.85010	0.13565	7.37167	48.36851	6.56141
27	27.10928	0.03689	0.00498	200.84061	0.13498	7.40856	49.32759	6.65819
28	30.63349	0.03264	0.00439	227.94989	0.13439	7.44120	50.20898	6.74743
29	34.61584	0.02889	0.00387	258.58338	0.13387	7.47009	51.01786	6.82962
30	39.11590	0.02557	0.00341	293.19922	0.13341	7.49565	51.75925	6.90523

(Continued)

Table A19 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
31	44.20096	0.02262	0.00301	332.31511	0.13301	7.51828	52.43796	6.97473
32	49.94709	0.02002	0.00266	376.51608	0.13266	7.53830	53.05862	7.03854
33	56.44021	0.01772	0.00234	426.46317	0.13234	7.55602	53.62559	7.09707
34	63.77744	0.01568	0.00207	482.90338	0.13207	7.57170	54.14302	7.15071
35	72.06851	0.01388	0.00183	546.68082	0.13183	7.58557	54.61479	7.19983
36	81.43741	0.01228	0.00162	618.74933	0.13162	7.59785	55.04457	7.24475
37	92.02428	0.01087	0.00143	700.18674	0.13143	7.60872	55.43577	7.28582
38	103.98743	0.00962	0.00126	792.21101	0.13126	7.61833	55.79158	7.32333
39	117.50580	0.00851	0.00112	896.19845	0.13112	7.62684	56.11497	7.35756
40	132.78155	0.00753	0.00099	1013.70424	0.13099	7.63438	56.40868	7.38878
45	244.64140	0.00409	0.00053	1874.16463	0.13053	7.66086	57.51478	7.50761
50	450.73593	0.00222	0.00029	3459.50712	0.13029	7.67524	58.18701	7.58113
55	830.45173	0.00120	0.00016	6380.39789	0.13016	7.68304	58.59089	7.62600
60	1530.05347	0.00065	0.00009	11761.9498	0.13009	7.68728	58.83128	7.65307
65	2819.02434	0.00035	0.00005	21677.1103	0.13005	7.68958	58.97324	7.66924
70	5193.86962	0.00019	0.00003	39945.1510	0.13003	7.69083	59.05653	7.67883
75	9569.36811	0.00010	0.00001	73602.8316	0.13001	7.69150	59.10513	7.68447
80	17630.9405	0.00006	0.00001	135614.927	0.13001	7.69187	59.13334	7.68777
85	32483.8649	0.00003	0.00000	249868.192	0.13000	7.69207	59.14965	7.68969
90	59849.4155	0.00002	0.00000	460372.427	0.13000	7.69218	59.15904	7.69080

Table A20 Compound Interest Factors 14%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.14000	0.87719	1.00000	1.00000	1.14000	0.87719		
2	1.29960	0.76947	0.46729	2.14000	0.60729	1.64666	0.76947	0.46729
3	1.48154	0.67497	0.29073	3.43960	0.43073	2.32163	2.11941	0.91290
4	1.68896	0.59208	0.20320	4.92114	0.34320	2.91371	3.89565	1.33701
5	1.92541	0.51937	0.15128	6.61010	0.29128	3.43308	5.97313	1.73987

6	2.19497	0.45559	0.11716	8.53552	0.25716	3.88867	8.25106	2.12182
7	2.50227	0.39964	0.09319	10.73049	0.23319	4.28830	10.64888	2.48324
8	2.85259	0.35056	0.07557	13.23276	0.21557	4.63886	13.10280	2.82457
9	3.25195	0.30751	0.06217	16.08535	0.20217	4.94637	15.56286	3.14632
10	3.70722	0.26974	0.05171	19.33730	0.19171	5.21612	17.99055	3.44903
11	4.22623	0.23662	0.04339	23.04452	0.18339	5.45273	20.35673	3.73331
12	4.81790	0.20756	0.03667	27.27075	0.17667	5.66029	22.63988	3.99977
13	5.49241	0.18207	0.03116	32.08865	0.17116	5.84236	24.82471	4.24909
14	6.26135	0.15971	0.02661	37.58107	0.16661	6.00207	26.90094	4.48194
15	7.13794	0.14010	0.02281	43.84241	0.16281	6.14217	28.86229	4.69904
16	8.13725	0.12289	0.01962	50.98035	0.15962	6.26506	30.70567	4.90110
17	9.27646	0.10780	0.01692	59.11760	0.15692	6.37286	32.43046	5.08884
18	10.57517	0.09456	0.01462	68.39407	0.15462	6.46742	34.03800	5.26299
19	12.05569	0.08295	0.01266	78.96923	0.15266	6.55037	35.53107	5.42429
20	13.74349	0.07276	0.01099	91.02493	0.15099	6.62313	36.91354	5.57343
21	15.66758	0.06383	0.00954	104.76842	0.14954	6.68696	38.19006	5.71113
22	17.86104	0.05599	0.00830	120.43600	0.14830	6.74294	39.36581	5.83807
23	20.36158	0.04911	0.00723	138.29704	0.14723	6.79206	40.44627	5.95494
24	23.21221	0.04308	0.00630	158.65862	0.14630	6.83514	41.43713	6.06237
25	26.46192	0.03779	0.00550	181.87083	0.14550	6.87293	42.34410	6.16100
26	30.16658	0.03315	0.00480	208.33274	0.14480	6.90608	43.17283	6.25143
27	34.38991	0.02908	0.00419	238.49933	0.14419	6.93515	43.92886	6.33423
28	39.20449	0.02551	0.00366	272.88923	0.14366	6.96066	44.61756	6.40996
29	44.69312	0.02237	0.00320	312.09373	0.14320	6.98304	45.24405	6.47914
30	50.95016	0.01963	0.00280	356.78685	0.14280	7.00266	45.81324	6.54226
31	58.08318	0.01722	0.00245	407.73701	0.14245	7.01988	46.32974	6.59979
32	66.21483	0.01510	0.00215	465.82019	0.14215	7.03498	46.79791	6.65217
33	75.48490	0.01325	0.00188	532.03501	0.14188	7.04823	47.22184	6.69981
34	86.05279	0.01162	0.00165	607.51991	0.14165	7.05985	47.60532	6.74311
35	98.10018	0.01019	0.00144	693.57270	0.14144	7.07005	47.95191	6.78240
36	111.83420	0.00894	0.00126	791.67288	0.14126	7.07899	48.26487	6.81805
37	127.49099	0.00784	0.00111	903.50708	0.14111	7.08683	48.54724	6.85035
38	145.33973	0.00688	0.00097	1030.99808	0.14097	7.09371	48.80182	6.87959

(Continued)

Table A20 (Continued)

n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
39	165.68729	0.00604	0.00085	1176.33781	0.14085	7.09975	49.03117	6.90604
40	188.88351	0.00529	0.00075	1342.02510	0.14075	7.10504	49.23764	6.92996
45	363.67907	0.00275	0.00039	2590.56480	0.14039	7.12322	49.99629	7.01878
50	700.23299	0.00143	0.00020	4994.52135	0.14020	7.13266	50.43751	7.07135
55	1348.23881	0.00074	0.00010	9623.13434	0.14010	7.13756	50.69118	7.10203
60	2595.91866	0.00039	0.00005	18535.1333	0.14005	7.14011	50.83566	7.11974
65	4998.21964	0.00020	0.00003	35694.4260	0.14003	7.14143	50.91731	7.12985
70	9623.64498	0.00010	0.00001	68733.1785	0.14001	7.14211	50.96315	7.13558
75	18529.506	0.00005	0.00001	132346.474	0.14001	7.14247	50.98874	7.13881
80	35676.982	0.00003	0.00000	254828.441	0.14000	7.14266	51.00296	7.14061
85	68692.981	0.00001	0.00000	490657.007	0.14000	7.14275	51.01083	7.14162
90	132262.467	0.00001	0.00000	944724.767	0.14000	7.14280	51.01516	7.14218

Table A21 Compound Interest Factors 15%

n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.15000	0.86957	1.00000	1.00000	1.15000	0.86957		
2	1.32250	0.75614	0.46512	2.15000	0.61512	1.62571	0.75614	0.46512
3	1.52088	0.65752	0.28798	3.47250	0.43798	2.28323	2.07118	0.90713
4	1.74901	0.57175	0.20027	4.99338	0.35027	2.85498	3.78644	1.32626
5	2.01136	0.49718	0.14832	6.74238	0.29832	3.35216	5.77514	1.72281
6	2.31306	0.43233	0.11424	8.75374	0.26424	3.78448	7.93678	2.09719
7	2.66002	0.37594	0.09036	11.06680	0.24036	4.16042	10.19240	2.44985
8	3.05902	0.32690	0.07285	13.72682	0.22285	4.48732	12.48072	2.78133
9	3.51788	0.28426	0.05957	16.78584	0.20957	4.77158	14.75481	3.09223
10	4.04556	0.24718	0.04925	20.30372	0.19925	5.01877	16.97948	3.38320
11	4.65239	0.21494	0.04107	24.34928	0.19107	5.23371	19.12891	3.65494
12	5.35025	0.18691	0.03448	29.00167	0.18448	5.42062	21.18489	3.90820

13	6.15279	0.16253	0.02911	34.35192	0.17911	5.58315	23.13522	4.14376
14	7.07571	0.14133	0.02469	40.50471	0.17469	5.72448	24.97250	4.36241
15	8.13706	0.12289	0.02102	47.58041	0.17102	5.84737	26.69302	4.56496
16	9.35762	0.10686	0.01795	55.71747	0.16795	5.95423	28.29599	4.75225
17	10.76126	0.09293	0.01537	65.07509	0.16537	6.04716	29.78280	4.92509
18	12.37545	0.08081	0.01319	75.83636	0.16319	6.12797	31.15649	5.08431
19	14.23177	0.07027	0.01134	88.21181	0.16134	6.19823	32.42127	5.23073
20	16.36654	0.06110	0.00976	102.44358	0.15976	6.25933	33.58217	5.36514
21	18.82152	0.05313	0.00842	118.81012	0.15842	6.31246	34.64479	5.48832
22	21.64475	0.04620	0.00727	137.63164	0.15727	6.35866	35.61500	5.60102
23	24.89146	0.04017	0.00628	159.27638	0.15628	6.39884	36.49884	5.70398
24	28.62518	0.03493	0.00543	184.16784	0.15543	6.43377	37.30232	5.79789
25	32.91895	0.03038	0.00470	212.79302	0.15470	6.46415	38.03139	5.88343
26	37.85680	0.02642	0.00407	245.71197	0.15407	6.49056	38.69177	5.96123
27	43.53531	0.02297	0.00353	283.56877	0.15353	6.51353	39.28899	6.03190
28	50.06561	0.01997	0.00306	327.10408	0.15306	6.53351	39.82828	6.09600
29	57.57545	0.01737	0.00265	377.16969	0.15265	6.55088	40.31460	6.15408
30	66.21177	0.01510	0.00230	434.74515	0.15230	6.56598	40.75259	6.20663
31	76.14354	0.01313	0.00200	500.95692	0.15200	6.57911	41.14658	6.25412
32	87.56507	0.01142	0.00173	577.10046	0.15173	6.59053	41.50060	6.29700
33	100.69983	0.00993	0.00150	664.66552	0.15150	6.60046	41.81838	6.33567
34	115.80480	0.00864	0.00131	765.36535	0.15131	6.60910	42.10334	6.37051
35	133.17552	0.00751	0.00113	881.17016	0.15113	6.61661	42.35864	6.40187
36	153.15185	0.00653	0.00099	1014.34568	0.15099	6.62314	42.58717	6.43006
37	176.12463	0.00568	0.00086	1167.49753	0.15086	6.62881	42.79157	6.45539
38	202.54332	0.00494	0.00074	1343.62216	0.15074	6.63375	42.97425	6.47812
39	232.92482	0.00429	0.00065	1546.16549	0.15065	6.63805	43.13739	6.49851
40	267.86355	0.00373	0.00056	1779.09031	0.15056	6.64178	43.28299	6.51678
45	538.76927	0.00186	0.00028	3585.12846	0.15028	6.65429	43.80513	6.58299
48	819.40071	0.00122	0.00018	5456.00475	0.15018	6.65853	43.99967	6.60802
50	1083.65744	0.00092	0.00014	7217.71628	0.15014	6.66051	44.09583	6.62048
55	2179.62218	0.00046	0.00007	14524.1479	0.15007	6.66361	44.25583	6.64142
60	4383.99875	0.00023	0.00003	29219.9916	0.15003	6.66515	44.34307	6.65298

(Continued)

Table A21 (Continued)

n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
65	8817.78739	0.00011	0.00002	58778.5826	0.15002	6.66591	44.39026	6.65929
70	17735.720	0.00006	0.00001	118231.467	0.15001	6.66629	44.41563	6.66272
75	35672.868	0.00003	0.00000	237812.453	0.15000	6.66648	44.42918	6.66456
80	71750.879	0.00001	0.00000	478332.529	0.15000	6.66657	44.43639	6.66555
85	144316.647	0.00001	0.00000	962104.313	0.15000	6.66662	44.44021	6.66608

Table A22 Compound Interest Factors 16%

n	Single Payment		Uniform Payment Series				Arithmetic Gradient	
	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.16000	0.86207	1.00000	1.00000	1.16000	0.86207		
2	1.34560	0.74316	0.46296	2.16000	0.62296	1.60523	0.74316	0.46296
3	1.56090	0.64066	0.28526	3.50560	0.44526	2.24589	2.02448	0.90141
4	1.81064	0.55229	0.19738	5.06650	0.35738	2.79818	3.68135	1.31562
5	2.10034	0.47611	0.14541	6.87714	0.30541	3.27429	5.58580	1.70596
6	2.43640	0.41044	0.11139	8.97748	0.27139	3.68474	7.63801	2.07288
7	2.82622	0.35383	0.08761	11.41387	0.24761	4.03857	9.76099	2.41695
8	3.27841	0.30503	0.07022	14.24009	0.23022	4.34359	11.89617	2.73879
9	3.80296	0.26295	0.05708	17.51851	0.21708	4.60654	13.99979	3.03911
10	4.41144	0.22668	0.04690	21.32147	0.20690	4.83323	16.03995	3.31868
11	5.11726	0.19542	0.03886	25.73290	0.19886	5.02864	17.99412	3.57832
12	5.93603	0.16846	0.03241	30.85017	0.19241	5.19711	19.84721	3.81890
13	6.88579	0.14523	0.02718	36.78620	0.18718	5.34233	21.58993	4.04129
14	7.98752	0.12520	0.02290	43.67199	0.18290	5.46753	23.21747	4.24643
15	9.26552	0.10793	0.01936	51.65951	0.17936	5.57546	24.72844	4.43523
16	10.74800	0.09304	0.01641	60.92503	0.17641	5.66850	26.12405	4.60864
17	12.46768	0.08021	0.01395	71.67303	0.17395	5.74870	27.40737	4.76757
18	14.46251	0.06914	0.01188	84.14072	0.17188	5.81785	28.58282	4.91295
19	16.77652	0.05961	0.01014	98.60323	0.17014	5.87746	29.65575	5.04568

20	19.46076	0.05139	0.00867	115.37975	0.16867	5.92884	30.63207	5.16662
21	22.57448	0.04430	0.00742	134.84051	0.16742	5.97314	31.51803	5.27663
22	26.18640	0.03819	0.00635	157.41499	0.16635	6.01133	32.31997	5.37651
23	30.37622	0.03292	0.00545	183.60138	0.16545	6.04425	33.04422	5.46705
24	35.23642	0.02838	0.00467	213.97761	0.16467	6.07263	33.69696	5.54899
25	40.87424	0.02447	0.00401	249.21402	0.16401	6.09709	34.28412	5.62303
26	47.41412	0.02109	0.00345	290.08827	0.16345	6.11818	34.81139	5.68983
27	55.00038	0.01818	0.00296	337.50239	0.16296	6.13636	35.28412	5.75000
28	63.80044	0.01567	0.00255	392.50277	0.16255	6.15204	35.70731	5.80414
29	74.00851	0.01351	0.00219	456.30322	0.16219	6.16555	36.08565	5.85279
30	85.84988	0.01165	0.00189	530.31173	0.16189	6.17720	36.42345	5.89643
31	99.58586	0.01004	0.00162	616.16161	0.16162	6.18724	36.72469	5.93555
32	115.51959	0.00866	0.00140	715.74746	0.16140	6.19590	36.99305	5.97057
33	134.00273	0.00746	0.00120	831.26706	0.16120	6.20336	37.23185	6.00188
34	155.44317	0.00643	0.00104	965.26979	0.16104	6.20979	37.44414	6.02985
35	180.31407	0.00555	0.00089	1120.71295	0.16089	6.21534	37.63270	6.05481
36	209.16432	0.00478	0.00077	1301.02703	0.16077	6.22012	37.80004	6.07706
37	242.63062	0.00412	0.00066	1510.19135	0.16066	6.22424	37.94841	6.09687
38	281.45151	0.00355	0.00057	1752.82197	0.16057	6.22779	38.07987	6.11450
39	326.48376	0.00306	0.00049	2034.27348	0.16049	6.23086	38.19626	6.13018
40	378.72116	0.00264	0.00042	2360.75724	0.16042	6.23350	38.29924	6.14410
42	509.60719	0.00196	0.00031	3178.79494	0.16031	6.23774	38.47075	6.16742
44	685.72744	0.00146	0.00023	4279.54648	0.16023	6.24089	38.60450	6.18574
45	795.44383	0.00126	0.00020	4965.27391	0.16020	6.24214	38.65982	6.19336
46	922.71484	0.00108	0.00017	5760.71774	0.16017	6.24323	38.70859	6.20009
50	1670.70380	0.00060	0.00010	10435.6488	0.16010	6.24626	38.85207	6.22005
55	3509.04880	0.00028	0.00005	21925.3050	0.16005	6.24822	38.95341	6.23432
60	7370.20137	0.00014	0.00002	46057.5085	0.16002	6.24915	39.00632	6.24186
65	15479.9410	0.00006	0.00001	96743.3810	0.16001	6.24960	39.03373	6.24580
70	32513.1648	0.00003	0.00000	203201.030	0.16000	6.24981	39.04784	6.24785
75	68288.7545	0.00001	0.00000	426798.466	0.16000	6.24991	39.05506	6.24890

Table A23 Compound Interest Factors 17%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.17000	0.85470	1.00000	1.00000	1.17000	0.85470		
2	1.36890	0.73051	0.46083	2.17000	0.63083	1.58521	0.73051	0.46083
3	1.60161	0.62437	0.28257	3.53890	0.45257	2.20958	1.97925	0.89576
4	1.87389	0.53365	0.19453	5.14051	0.36453	2.74324	3.58020	1.30510
5	2.19245	0.45611	0.14256	7.01440	0.31256	3.19935	5.40465	1.68930
6	2.56516	0.38984	0.10861	9.20685	0.27861	3.58918	7.35384	2.04889
7	3.00124	0.33320	0.08495	11.77201	0.25495	3.92238	9.35301	2.38453
8	3.51145	0.28478	0.06769	14.77325	0.23769	4.20716	11.34649	2.69695
9	4.10840	0.24340	0.05469	18.28471	0.22469	4.45057	13.29372	2.98697
10	4.80683	0.20804	0.04466	22.39311	0.21466	4.65860	15.16606	3.25549
11	5.62399	0.17781	0.03676	27.19994	0.20676	4.83641	16.94415	3.50345
12	6.58007	0.15197	0.03047	32.82393	0.20047	4.98839	18.61587	3.73184
13	7.69868	0.12989	0.02538	39.40399	0.19538	5.11828	20.17458	3.94167
14	9.00745	0.11102	0.02123	47.10267	0.19123	5.22930	21.61783	4.13398
15	10.53872	0.09489	0.01782	56.11013	0.18782	5.32419	22.94626	4.30982
16	12.33030	0.08110	0.01500	66.64885	0.18500	5.40529	24.16278	4.47021
17	14.42646	0.06932	0.01266	78.97915	0.18266	5.47461	25.27185	4.61620
18	16.87895	0.05925	0.01071	93.40561	0.18071	5.53385	26.27902	4.74878
19	19.74838	0.05064	0.00907	110.28456	0.17907	5.58449	27.19049	4.86893
20	23.10560	0.04328	0.00769	130.03294	0.17769	5.62777	28.01280	4.97760
21	27.03355	0.03699	0.00653	153.13854	0.17653	5.66476	28.75262	5.07570
22	31.62925	0.03162	0.00555	180.17209	0.17555	5.69637	29.41657	5.16409
23	37.00623	0.02702	0.00472	211.80134	0.17472	5.72340	30.01106	5.24357
24	43.29729	0.02310	0.00402	248.80757	0.17402	5.74649	30.54227	5.31494
25	50.65783	0.01974	0.00342	292.10486	0.17342	5.76623	31.01604	5.37891
26	59.26966	0.01687	0.00292	342.76268	0.17292	5.78311	31.43784	5.43615
27	69.34550	0.01442	0.00249	402.03234	0.17249	5.79753	31.81277	5.48730
28	81.13423	0.01233	0.00212	471.37783	0.17212	5.80985	32.14555	5.53294
29	94.92705	0.01053	0.00181	552.51207	0.17181	5.82039	32.44052	5.57360
30	111.06465	0.00900	0.00154	647.43912	0.17154	5.82939	32.70163	5.60979

31	129.94564	0.00770	0.00132	758.50377	0.17132	5.83709	32.93249	5.64194
32	152.03640	0.00658	0.00113	888.44941	0.17113	5.84366	33.13639	5.67048
33	177.88259	0.00562	0.00096	1040.48581	0.17096	5.84928	33.31629	5.69579
34	208.12263	0.00480	0.00082	1218.36839	0.17082	5.85409	33.47485	5.71820
35	243.50347	0.00411	0.00070	1426.49102	0.17070	5.85820	33.61447	5.73803
36	284.89906	0.00351	0.00060	1669.99450	0.17060	5.86171	33.73733	5.75555
37	333.33191	0.00300	0.00051	1954.89356	0.17051	5.86471	33.84533	5.77102
38	389.99833	0.00256	0.00044	2288.22547	0.17044	5.86727	33.94020	5.78467
40	533.86871	0.00187	0.00032	3134.52184	0.17032	5.87133	34.09653	5.80729
45	1170.47941	0.00085	0.00015	6879.29065	0.17015	5.87733	34.34636	5.84387
50	2566.21528	0.00039	0.00007	15089.5017	0.17007	5.88006	34.47398	5.86286
55	5626.29366	0.00018	0.00003	33089.9627	0.17003	5.88131	34.53842	5.87258
60	12335.3565	0.00008	0.00001	72555.0381	0.17001	5.88188	34.57066	5.87749
65	27044.6281	0.00004	0.00001	159080.165	0.17001	5.88214	34.58666	5.87995
70	59293.9417	0.00002	0.00000	348782.010	0.17000	5.88225	34.59455	5.88117

Table A24 Compound Interest Factors 18%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.18000	0.84746	1.00000	1.00000	1.18000	0.84746		
2	1.39240	0.71818	0.45872	2.18000	0.63872	1.56564	0.71818	0.45872
3	1.64303	0.60863	0.27992	3.57240	0.45992	2.17427	1.93545	0.89016
4	1.93878	0.51579	0.19174	5.21543	0.37174	2.69006	3.48281	1.29470
5	2.28776	0.43711	0.13978	7.15421	0.31978	3.12717	5.23125	1.67284
6	2.69955	0.37043	0.10591	9.44197	0.28591	3.49760	7.08341	2.02522
7	3.18547	0.31393	0.08236	12.14152	0.26236	3.81153	8.96696	2.35259
8	3.75886	0.26604	0.06524	15.32700	0.24524	4.07757	10.82922	2.65581
9	4.43545	0.22546	0.05239	19.08585	0.23239	4.30302	12.63287	2.93581
10	5.23384	0.19106	0.04251	23.52131	0.22251	4.49409	14.35245	3.19363
11	6.17593	0.16192	0.03478	28.75514	0.21478	4.65601	15.97164	3.43033
12	7.28759	0.13722	0.02863	34.93107	0.20863	4.79322	17.48106	3.64703
13	8.59936	0.11629	0.02369	42.21866	0.20369	4.90951	18.87651	3.84489

(Continued)

Table A24 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
14	10.14724	0.09855	0.01968	50.81802	0.19968	5.00806	20.15765	4.02504
15	11.97375	0.08352	0.01640	60.96527	0.19640	5.09158	21.32687	4.18866
16	14.12902	0.07078	0.01371	72.93901	0.19371	5.16235	22.38852	4.33688
17	16.67225	0.05998	0.01149	87.06804	0.19149	5.22233	23.34820	4.47084
18	19.67325	0.05083	0.00964	103.74028	0.18964	5.27316	24.21231	4.59161
19	23.21444	0.04308	0.00810	123.41353	0.18810	5.31624	24.98769	4.70026
20	27.39303	0.03651	0.00682	146.62797	0.18682	5.35275	25.68130	4.79778
21	32.32378	0.03094	0.00575	174.02100	0.18575	5.38368	26.30004	4.88514
22	38.14206	0.02622	0.00485	206.34479	0.18485	5.40990	26.85061	4.96324
23	45.00763	0.02222	0.00409	244.48685	0.18409	5.43212	27.33942	5.03292
24	53.10901	0.01883	0.00345	289.49448	0.18345	5.45095	27.77249	5.09498
25	62.66863	0.01596	0.00292	342.60349	0.18292	5.46691	28.15546	5.15016
26	73.94898	0.01352	0.00247	405.27211	0.18247	5.48043	28.49353	5.19914
27	87.25980	0.01146	0.00209	479.22109	0.18209	5.49189	28.79149	5.24255
28	102.96656	0.00971	0.00177	566.48089	0.18177	5.50160	29.05371	5.28096
29	121.50054	0.00823	0.00149	669.44745	0.18149	5.50983	29.28416	5.31489
30	143.37064	0.00697	0.00126	790.94799	0.18126	5.51681	29.48643	5.34484
31	169.17735	0.00591	0.00107	934.31863	0.18107	5.52272	29.66376	5.37123
32	199.62928	0.00501	0.00091	1103.49598	0.18091	5.52773	29.81905	5.39445
33	235.56255	0.00425	0.00077	1303.12526	0.18077	5.53197	29.95490	5.41487
34	277.96381	0.00360	0.00065	1538.68781	0.18065	5.53557	30.07362	5.43280
35	327.99729	0.00305	0.00055	1816.65161	0.18055	5.53862	30.17728	5.44852
36	387.03680	0.00258	0.00047	2144.64890	0.18047	5.54120	30.26771	5.46230
37	456.70343	0.00219	0.00039	2531.68570	0.18039	5.54339	30.34653	5.47436
38	538.91004	0.00186	0.00033	2988.38913	0.18033	5.54525	30.41519	5.48491
40	750.37834	0.00133	0.00024	4163.21303	0.18024	5.54815	30.52692	5.50218
45	1716.68388	0.00058	0.00010	9531.57711	0.18010	5.55232	30.70059	5.52933
50	3927.35686	0.00025	0.00005	21813.0937	0.18005	5.55414	30.78561	5.54282
55	8984.84112	0.00011	0.00002	49910.2284	0.18002	5.55494	30.82675	5.54943
60	20555.140	0.00005	0.00001	114189.666	0.18001	5.55529	30.84648	5.55264
65	47025.181	0.00002	0.00000	261245.449	0.18000	5.55544	30.85586	5.55417
70	107582.222	0.00001	0.00000	597673.458	0.18000	5.55550	30.86030	5.55490

Table A25 Compound Interest Factors 19%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.19000	0.84034	1.00000	1.00000	1.19000	0.84034		
2	1.41610	0.70616	0.45662	2.19000	0.64662	1.54650	0.70616	0.45662
3	1.68516	0.59342	0.27731	3.60610	0.46731	2.13992	1.89300	0.88461
4	2.00534	0.49867	0.18899	5.29126	0.37899	2.63859	3.38900	1.28440
5	2.38635	0.41905	0.13705	7.29660	0.32705	3.05763	5.06520	1.65657
6	2.83976	0.35214	0.10327	9.68295	0.29327	3.40978	6.82591	2.00186
7	3.37932	0.29592	0.07985	12.52271	0.26985	3.70570	8.60142	2.32114
8	4.02139	0.24867	0.06289	15.90203	0.25289	3.95437	10.34211	2.61537
9	4.78545	0.20897	0.05019	19.92341	0.24019	4.16333	12.01385	2.88563
10	5.69468	0.17560	0.04047	24.70886	0.23047	4.33893	13.59427	3.13309
11	6.77667	0.14757	0.03289	30.40355	0.22289	4.48650	15.06992	3.35895
12	8.06424	0.12400	0.02690	37.18022	0.21690	4.61050	16.43397	3.56446
13	9.59645	0.10421	0.02210	45.24446	0.21210	4.71471	17.68443	3.75091
14	11.41977	0.08757	0.01823	54.84091	0.20823	4.80228	18.82280	3.91956
15	13.58953	0.07359	0.01509	66.26068	0.20509	4.87586	19.85301	4.07169
16	16.17154	0.06184	0.01252	79.85021	0.20252	4.93770	20.78057	4.20855
17	19.24413	0.05196	0.01041	96.02175	0.20041	4.98966	21.61199	4.33135
18	22.90052	0.04367	0.00868	115.26588	0.19868	5.03333	22.35433	4.44126
19	27.25162	0.03670	0.00724	138.16640	0.19724	5.07003	23.01484	4.53939
20	32.42942	0.03084	0.00605	165.41802	0.19605	5.10086	23.60073	4.62681
21	38.59101	0.02591	0.00505	197.84744	0.19505	5.12677	24.11898	4.70451
22	45.92331	0.02178	0.00423	236.43846	0.19423	5.14855	24.57627	4.77343
23	54.64873	0.01830	0.00354	282.36176	0.19354	5.16685	24.97884	4.83444
24	65.03199	0.01538	0.00297	337.01050	0.19297	5.18223	25.33251	4.88835
25	77.38807	0.01292	0.00249	402.04249	0.19249	5.19515	25.64264	4.93588
26	92.09181	0.01086	0.00209	479.43056	0.19209	5.20601	25.91410	4.97773
27	109.58925	0.00912	0.00175	571.52237	0.19175	5.21513	26.15135	5.01451
28	130.41121	0.00767	0.00147	681.11162	0.19147	5.22280	26.35839	5.04679
29	155.18934	0.00644	0.00123	811.52283	0.19123	5.22924	26.53882	5.07508
30	184.67531	0.00541	0.00103	966.71217	0.19103	5.23466	26.69585	5.09983

(Continued)

Table A25 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
32	261.51871	0.00382	0.00073	1371.15110	0.19073	5.24303	26.95090	5.14033
34	370.33664	0.00270	0.00051	1943.87708	0.19051	5.24895	27.14283	5.17110
35	440.70061	0.00227	0.00043	2314.21372	0.19043	5.25122	27.21998	5.18356
36	524.43372	0.00191	0.00036	2754.91433	0.19036	5.25312	27.28672	5.19438
38	742.65059	0.00135	0.00026	3903.42418	0.19026	5.25607	27.39423	5.21192
40	1051.66751	0.00095	0.00018	5529.82898	0.19018	5.25815	27.47431	5.22509
45	2509.65060	0.00040	0.00008	13203.424	0.19008	5.26106	27.59542	5.24522
50	5988.91390	0.00017	0.00003	31515.336	0.19003	5.26228	27.65226	5.25481
55	14291.6666	0.00007	0.00001	75214.035	0.19001	5.26279	27.67864	5.25931
60	34104.9709	0.00003	0.00001	179494.584	0.19001	5.26300	27.69076	5.26140

Table A26 Compound Interest Factors 20%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.20000	0.83333	1.00000	1.00000	1.20000	0.83333		
2	1.44000	0.69444	0.45455	2.20000	0.65455	1.52778	0.69444	0.45455
3	1.72800	0.57870	0.27473	3.64000	0.47473	2.10648	1.85185	0.87912
4	2.07360	0.48225	0.18629	5.36800	0.38629	2.58873	3.29861	1.27422
5	2.48832	0.40188	0.13438	7.44160	0.33438	2.99061	4.90612	1.64051
6	2.98598	0.33490	0.10071	9.92992	0.30071	3.32551	6.58061	1.97883
7	3.58318	0.27908	0.07742	12.91590	0.27742	3.60459	8.25510	2.29016
8	4.29982	0.23257	0.06061	16.49908	0.26061	3.83716	9.88308	2.57562
9	5.15978	0.19381	0.04808	20.79890	0.24808	4.03097	11.43353	2.83642
10	6.19174	0.16151	0.03852	25.95868	0.23852	4.19247	12.88708	3.07386
11	7.43008	0.13459	0.03110	32.15042	0.23110	4.32706	14.23296	3.28929
12	8.91610	0.11216	0.02526	39.58050	0.22526	4.43922	15.46668	3.48410
13	10.69932	0.09346	0.02062	48.49660	0.22062	4.53268	16.58825	3.65970
14	12.83918	0.07789	0.01689	59.19592	0.21689	4.61057	17.60078	3.81749
15	15.40702	0.06491	0.01388	72.03511	0.21388	4.67547	18.50945	3.95884

16	18.48843	0.05409	0.01144	87.44213	0.21144	4.72956	19.32077	4.08511
17	22.18611	0.04507	0.00944	105.93056	0.20944	4.77463	20.04194	4.19759
18	26.62333	0.03756	0.00781	128.11667	0.20781	4.81219	20.68048	4.29752
19	31.94800	0.03130	0.00646	154.74000	0.20646	4.84350	21.24390	4.38607
20	38.33760	0.02608	0.00536	186.68800	0.20536	4.86958	21.73949	4.46435
21	46.00512	0.02174	0.00444	225.02560	0.20444	4.89132	22.17423	4.53339
22	55.20614	0.01811	0.00369	271.03072	0.20369	4.90943	22.55462	4.59414
23	66.24737	0.01509	0.00307	326.23686	0.20307	4.92453	22.88671	4.64750
24	79.49685	0.01258	0.00255	392.48424	0.20255	4.93710	23.17603	4.69426
25	95.39622	0.01048	0.00212	471.98108	0.20212	4.94759	23.42761	4.73516
26	114.47546	0.00874	0.00176	567.37730	0.20176	4.95632	23.64600	4.77088
27	137.37055	0.00728	0.00147	681.85276	0.20147	4.96360	23.83527	4.80201
28	164.84466	0.00607	0.00122	819.22331	0.20122	4.96967	23.99906	4.82911
29	197.81359	0.00506	0.00102	984.06797	0.20102	4.97472	24.14061	4.85265
30	237.37631	0.00421	0.00085	1181.88157	0.20085	4.97894	24.26277	4.87308
32	341.82189	0.00293	0.00059	1704.10946	0.20059	4.98537	24.45878	4.90611
34	492.22352	0.00203	0.00041	2456.11762	0.20041	4.98984	24.60384	4.93079
35	590.66823	0.00169	0.00034	2948.34115	0.20034	4.99154	24.66140	4.94064
36	708.80187	0.00141	0.00028	3539.00937	0.20028	4.99295	24.71078	4.94914
38	1020.67470	0.00098	0.00020	5098.37350	0.20020	4.99510	24.78936	4.96273
40	1469.77157	0.00068	0.00014	7343.85784	0.20014	4.99660	24.84691	4.97277
45	3657.26199	0.00027	0.00005	18281.3099	0.20005	4.99863	24.93164	4.98769
50	9100.43815	0.00011	0.00002	45497.1908	0.20002	4.99945	24.96978	4.99451
55	22644.8023	0.00004	0.00001	113219.011	0.20001	4.99978	24.98675	4.99757
60	56347.5144	0.00002	0.00000	281732.572	0.20000	4.99991	24.99423	4.99894

Table A27 Compound Interest Factors 21%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.21000	0.82645	1.00000	1.00000	1.21000	0.82645		
2	1.46410	0.68301	0.45249	2.21000	0.66249	1.50946	0.68301	0.45249
3	1.77156	0.56447	0.27218	3.67410	0.48218	2.07393	1.81196	0.87368

(Continued)

Table A27 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
4	2.14359	0.46651	0.18363	5.44566	0.39363	2.54044	3.21148	1.26414
5	2.59374	0.38554	0.13177	7.58925	0.34177	2.92598	4.75366	1.62464
6	3.13843	0.31863	0.09820	10.18299	0.30820	3.24462	6.34681	1.95611
7	3.79750	0.26333	0.07507	13.32142	0.28507	3.50795	7.92680	2.25967
8	4.59497	0.21763	0.05841	17.11892	0.26841	3.72558	9.45020	2.53658
9	5.55992	0.17986	0.04605	21.71389	0.25605	3.90543	10.88907	2.78818
10	6.72750	0.14864	0.03667	27.27381	0.24667	4.05408	12.22687	3.01594
11	8.14027	0.12285	0.02941	34.00131	0.23941	4.17692	13.45532	3.22135
12	9.84973	0.10153	0.02373	42.14158	0.23373	4.27845	14.57211	3.40593
13	11.91818	0.08391	0.01923	51.99132	0.22923	4.36235	15.57897	3.57123
14	14.42099	0.06934	0.01565	63.90949	0.22565	4.43170	16.48044	3.71876
15	17.44940	0.05731	0.01277	78.33049	0.22277	4.48901	17.28276	3.85002
16	21.11378	0.04736	0.01044	95.77989	0.22044	4.53637	17.99319	3.96643
17	25.54767	0.03914	0.00855	116.89367	0.21855	4.57551	18.61947	4.06937
18	30.91268	0.03235	0.00702	142.44134	0.21702	4.60786	19.16941	4.16015
19	37.40434	0.02673	0.00577	173.35402	0.21577	4.63460	19.65064	4.23999
20	45.25926	0.02209	0.00474	210.75836	0.21474	4.65669	20.07044	4.31002
21	54.76370	0.01826	0.00391	256.01762	0.21391	4.67495	20.43564	4.37131
22	66.26408	0.01509	0.00322	310.78131	0.21322	4.69004	20.75256	4.42481
23	80.17953	0.01247	0.00265	377.04539	0.21265	4.70251	21.02694	4.47143
24	97.01723	0.01031	0.00219	457.22492	0.21219	4.71282	21.26401	4.51195
25	117.39085	0.00852	0.00180	554.24216	0.21180	4.72134	21.46846	4.54711
26	142.04293	0.00704	0.00149	671.63301	0.21149	4.72838	21.64446	4.57756
27	171.87195	0.00582	0.00123	813.67594	0.21123	4.73420	21.79574	4.60389
28	207.96506	0.00481	0.00101	985.54789	0.21101	4.73901	21.92557	4.62662
29	251.63772	0.00397	0.00084	1193.51295	0.21084	4.74298	22.03684	4.64620
30	304.48164	0.00328	0.00069	1445.15066	0.21069	4.74627	22.13208	4.66305
32	445.79157	0.00224	0.00047	2118.05509	0.21047	4.75122	22.28305	4.68996
34	652.68344	0.00153	0.00032	3103.25445	0.21032	4.75461	22.39293	4.70973
35	789.74696	0.00127	0.00027	3755.93789	0.21027	4.75588	22.43599	4.71753
36	955.59382	0.00105	0.00022	4545.68485	0.21022	4.75692	22.47261	4.72419

38	1399.08491	0.00071	0.00015	6657.54718	0.21015	4.75850	22.53019	4.73472
40	2048.40021	0.00049	0.00010	9749.52483	0.21010	4.75958	22.57168	4.74237
45	5313.02261	0.00019	0.00004	25295.3458	0.21004	4.76101	22.63114	4.75343
50	13780.6123	0.00007	0.00002	65617.2016	0.21002	4.76156	22.65681	4.75828
55	35743.3594	0.00003	0.00001	170201.711	0.21001	4.76177	22.66778	4.76037
60	92709.0688	0.00001	0.00000	441466.994	0.21000	4.76185	22.67241	4.76126

Table A28 Compound Interest Factors 22%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.22000	0.81967	1.00000	1.00000	1.22000	0.81967		
2	1.48840	0.67186	0.45045	2.22000	0.67045	1.49153	0.67186	0.45045
3	1.81585	0.55071	0.26966	3.70840	0.48966	2.04224	1.77328	0.86830
4	2.21533	0.45140	0.18102	5.52425	0.40102	2.49364	3.12747	1.25418
5	2.70271	0.37000	0.12921	7.73958	0.34921	2.86364	4.60747	1.60896
6	3.29730	0.30328	0.09576	10.44229	0.31576	3.16692	6.12386	1.93370
7	4.02271	0.24859	0.07278	13.73959	0.29278	3.41551	7.61539	2.22965
8	4.90771	0.20376	0.05630	17.76231	0.27630	3.61927	9.04172	2.49822
9	5.98740	0.16702	0.04411	22.67001	0.26411	3.78628	10.37786	2.74091
10	7.30463	0.13690	0.03489	28.65742	0.25489	3.92318	11.60995	2.95932
11	8.91165	0.11221	0.02781	35.96205	0.24781	4.03540	12.73208	3.15510
12	10.87221	0.09198	0.02228	44.87370	0.24228	4.12737	13.74383	3.32992
13	13.26410	0.07539	0.01794	55.74591	0.23794	4.20277	14.64853	3.48545
14	16.18220	0.06180	0.01449	69.01001	0.23449	4.26456	15.45188	3.62332
15	19.74229	0.05065	0.01174	85.19221	0.23174	4.31522	16.16102	3.74513
16	24.08559	0.04152	0.00953	104.93450	0.22953	4.35673	16.78380	3.85238
17	29.38442	0.03403	0.00775	129.02009	0.22775	4.39077	17.32831	3.94653
18	35.84899	0.02789	0.00631	158.40451	0.22631	4.41866	17.80252	4.02894
19	43.73577	0.02286	0.00515	194.25350	0.22515	4.44152	18.21408	4.10086
20	53.35764	0.01874	0.00420	237.98927	0.22420	4.46027	18.57017	4.16347
21	65.09632	0.01536	0.00343	291.34691	0.22343	4.47563	18.87740	4.21782
22	79.41751	0.01259	0.00281	356.44323	0.22281	4.48822	19.14183	4.26490
23	96.88936	0.01032	0.00229	435.86075	0.22229	4.49854	19.36889	4.30559

(Continued)

Table A28 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
24	118.20502	0.00846	0.00188	532.75011	0.22188	4.50700	19.56347	4.34069
25	144.21013	0.00693	0.00154	650.95513	0.22154	4.51393	19.72989	4.37089
26	175.93636	0.00568	0.00126	795.16526	0.22126	4.51962	19.87199	4.39683
27	214.64236	0.00466	0.00103	971.10162	0.22103	4.52428	19.99312	4.41908
28	261.86368	0.00382	0.00084	1185.74398	0.22084	4.52810	20.09623	4.43812
29	319.47368	0.00313	0.00069	1447.60765	0.22069	4.53123	20.18387	4.45440
30	389.75789	0.00257	0.00057	1767.08134	0.22057	4.53379	20.25828	4.46829
32	580.11565	0.00172	0.00038	2632.34386	0.22038	4.53762	20.37481	4.49020
34	863.44413	0.00116	0.00026	3920.20061	0.22026	4.54019	20.45824	4.50603
35	1053.40184	0.00095	0.00021	4783.64474	0.22021	4.54114	20.49052	4.51220
36	1285.15025	0.00078	0.00017	5837.04658	0.22017	4.54192	20.51775	4.51742
38	1912.81763	0.00052	0.00012	8690.08013	0.22012	4.54308	20.56006	4.52558
40	2847.03776	0.00035	0.00008	12936.5353	0.22008	4.54386	20.59004	4.53140
45	7694.71219	0.00013	0.00003	34971.4191	0.22003	4.54486	20.63189	4.53961
50	20796.5615	0.00005	0.00001	94525.2793	0.22001	4.54524	20.64924	4.54305
55	56207.0364	0.00002	0.00000	255481.984	0.22000	4.54537	20.65634	4.54448
60	151911.216	0.00001	0.00000	690500.982	0.22000	4.54542	20.65923	4.54506

Table A29 Compound Interest Factors 23%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.23000	0.81301	1.00000	1.00000	1.23000	0.81301		
2	1.51290	0.66098	0.44843	2.23000	0.67843	1.47399	0.66098	0.44843
3	1.86087	0.53738	0.26717	3.74290	0.49717	2.01137	1.73575	0.86297
4	2.28887	0.43690	0.17845	5.60377	0.40845	2.44827	3.04644	1.24432
5	2.81531	0.35520	0.12670	7.89263	0.35670	2.80347	4.46725	1.59347
6	3.46283	0.28878	0.09339	10.70794	0.32339	3.09225	5.91115	1.91160
7	4.25928	0.23478	0.07057	14.17077	0.30057	3.32704	7.31984	2.20011
8	5.23891	0.19088	0.05426	18.43004	0.28426	3.51792	8.65600	2.46055

9	6.44386	0.15519	0.04225	23.66895	0.27225	3.67310	9.89749	2.69459
10	7.92595	0.12617	0.03321	30.11281	0.26321	3.79927	11.03300	2.90398
11	9.74891	0.10258	0.02629	38.03876	0.25629	3.90185	12.05876	3.09053
12	11.99116	0.08339	0.02093	47.78767	0.25093	3.98524	12.97610	3.25604
13	14.74913	0.06780	0.01673	59.77883	0.24673	4.05304	13.78971	3.40231
14	18.14143	0.05512	0.01342	74.52796	0.24342	4.10816	14.50630	3.53109
15	22.31396	0.04481	0.01079	92.66940	0.24079	4.15298	15.13371	3.64406
16	27.44617	0.03643	0.00870	114.98336	0.23870	4.18941	15.68023	3.74282
17	33.75879	0.02962	0.00702	142.42953	0.23702	4.21904	16.15419	3.82888
18	41.52331	0.02408	0.00568	176.18832	0.23568	4.24312	16.56359	3.90364
19	51.07368	0.01958	0.00459	217.71163	0.23459	4.26270	16.91603	3.96839
20	62.82062	0.01592	0.00372	268.78531	0.23372	4.27862	17.21847	4.02431
21	77.26936	0.01294	0.00302	331.60593	0.23302	4.29156	17.47731	4.07249
22	95.04132	0.01052	0.00245	408.87530	0.23245	4.30208	17.69827	4.11389
23	116.90082	0.00855	0.00198	503.91662	0.23198	4.31063	17.88646	4.14938
24	143.78801	0.00695	0.00161	620.81744	0.23161	4.31759	18.04642	4.17974
25	176.85925	0.00565	0.00131	764.60545	0.23131	4.32324	18.18212	4.20567
26	217.53688	0.00460	0.00106	941.46470	0.23106	4.32784	18.29704	4.22775
27	267.57036	0.00374	0.00086	1159.00158	0.23086	4.33158	18.39421	4.24654
28	329.11155	0.00304	0.00070	1426.57194	0.23070	4.33462	18.47625	4.26249
29	404.80720	0.00247	0.00057	1755.68349	0.23057	4.33709	18.54542	4.27601
30	497.91286	0.00201	0.00046	2160.49070	0.23046	4.33909	18.60366	4.28745
32	753.29237	0.00133	0.00031	3270.83637	0.23031	4.34205	18.69380	4.30529
34	1139.65602	0.00088	0.00020	4950.67835	0.23020	4.34401	18.75729	4.31797
35	1401.77690	0.00071	0.00016	6090.33437	0.23016	4.34472	18.78155	4.32284
36	1724.18559	0.00058	0.00013	7492.11127	0.23013	4.34530	18.80185	4.32693
38	2608.52038	0.00038	0.00009	11337.0451	0.23009	4.34616	18.83301	4.33325
40	3946.43049	0.00025	0.00006	17154.0456	0.23006	4.34672	18.85473	4.33769
45	11110.4082	0.00009	0.00002	48301.7747	0.23002	4.34743	18.88428	4.34378
50	31279.1953	0.00003	0.00001	135992.154	0.23001	4.34769	18.89604	4.34623
55	88060.4964	0.00001	0.00000	382867.376	0.23000	4.34778	18.90066	4.34720
60	247917.216	0.00000	0.00000	1077896.59	0.23000	4.34781	18.90246	4.34758

Table A30 Compound Interest Factors 24%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.24000	0.80645	1.00000	1.00000	1.24000	0.80645		
2	1.53760	0.65036	0.44643	2.24000	0.68643	1.45682	0.65036	0.44643
3	1.90662	0.52449	0.26472	3.77760	0.50472	1.98130	1.69934	0.85769
4	2.36421	0.42297	0.17593	5.68422	0.41593	2.40428	2.96826	1.23457
5	2.93163	0.34111	0.12425	8.04844	0.36425	2.74538	4.33269	1.57817
6	3.63522	0.27509	0.09107	10.98006	0.33107	3.02047	5.70812	1.88981
7	4.50767	0.22184	0.06842	14.61528	0.30842	3.24232	7.03919	2.17104
8	5.58951	0.17891	0.05229	19.12294	0.29229	3.42122	8.29154	2.42356
9	6.93099	0.14428	0.04047	24.71245	0.28047	3.56550	9.44577	2.64921
10	8.59443	0.11635	0.03160	31.64344	0.27160	3.68186	10.49296	2.84991
11	10.65709	0.09383	0.02485	40.23787	0.26485	3.77569	11.43131	3.02761
12	13.21479	0.07567	0.01965	50.89495	0.25965	3.85136	12.26371	3.18425
13	16.38634	0.06103	0.01560	64.10974	0.25560	3.91239	12.99602	3.32176
14	20.31906	0.04921	0.01242	80.49608	0.25242	3.96160	13.63582	3.44199
15	25.19563	0.03969	0.00992	100.81514	0.24992	4.00129	14.19147	3.54672
16	31.24259	0.03201	0.00794	126.01077	0.24794	4.03330	14.67158	3.63761
17	38.74081	0.02581	0.00636	157.25336	0.24636	4.05911	15.08459	3.71623
18	48.03860	0.02082	0.00510	195.99416	0.24510	4.07993	15.43847	3.78400
19	59.56786	0.01679	0.00410	244.03276	0.24410	4.09672	15.74064	3.84226
20	73.86415	0.01354	0.00329	303.60062	0.24329	4.11026	15.99787	3.89218
21	91.59155	0.01092	0.00265	377.46477	0.24265	4.12117	16.21623	3.93486
22	113.57352	0.00880	0.00213	469.05632	0.24213	4.12998	16.40114	3.97124
23	140.83116	0.00710	0.00172	582.62984	0.24172	4.13708	16.55735	4.00218
24	174.63064	0.00573	0.00138	723.46100	0.24138	4.14281	16.68906	4.02844
25	216.54199	0.00462	0.00111	898.09164	0.24111	4.14742	16.79989	4.05068
26	268.51207	0.00372	0.00090	1114.63363	0.24090	4.15115	16.89300	4.06947
27	332.95497	0.00300	0.00072	1383.14570	0.24072	4.15415	16.97109	4.08533
28	412.86416	0.00242	0.00058	1716.10067	0.24058	4.15657	17.03648	4.09868
29	511.95156	0.00195	0.00047	2128.96483	0.24047	4.15853	17.09117	4.10991
30	634.81993	0.00158	0.00038	2640.91639	0.24038	4.16010	17.13686	4.11933

32	976.09913	0.00102	0.00025	4062.91304	0.24025	4.16240	17.20673	4.13385
34	1500.85002	0.00067	0.00016	6249.37509	0.24016	4.16389	17.25515	4.14400
35	1861.05403	0.00054	0.00013	7750.22511	0.24013	4.16443	17.27342	4.14785
36	2307.70699	0.00043	0.00010	9611.27913	0.24010	4.16486	17.28859	4.15106
38	3548.33027	0.00028	0.00007	14780.5428	0.24007	4.16549	17.31160	4.15595
40	5455.91262	0.00018	0.00004	22728.8026	0.24004	4.16590	17.32738	4.15933
45	15994.6902	0.00006	0.00002	66640.3758	0.24002	4.16641	17.34830	4.16385
50	46890.4346	0.00002	0.00001	195372.644	0.24001	4.16658	17.35630	4.16560
55	137465.173	0.00001	0.00000	572767.389	0.24000	4.16664	17.35932	4.16627
60	402996.347	0.00000	0.00000	1679147.28	0.24000	4.16666	17.36045	4.16652

Table A31 Compound Interest Factors 25%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.25000	0.80000	1.00000	1.00000	1.25000	0.80000		
2	1.56250	0.64000	0.44444	2.25000	0.69444	1.44000	0.64000	0.44444
3	1.95313	0.51200	0.26230	3.81250	0.51230	1.95200	1.66400	0.85246
4	2.44141	0.40960	0.17344	5.76563	0.42344	2.36160	2.89280	1.22493
5	3.05176	0.32768	0.12185	8.20703	0.37185	2.68928	4.20352	1.56307
6	3.81470	0.26214	0.08882	11.25879	0.33882	2.95142	5.51424	1.86833
7	4.76837	0.20972	0.06634	15.07349	0.31634	3.16114	6.77253	2.14243
8	5.96046	0.16777	0.05040	19.84186	0.30040	3.32891	7.94694	2.38725
9	7.45058	0.13422	0.03876	25.80232	0.28876	3.46313	9.02068	2.60478
10	9.31323	0.10737	0.03007	33.25290	0.28007	3.57050	9.98705	2.79710
11	11.64153	0.08590	0.02349	42.56613	0.27349	3.65640	10.84604	2.96631
12	14.55192	0.06872	0.01845	54.20766	0.26845	3.72512	11.60195	3.11452
13	18.18989	0.05498	0.01454	68.75958	0.26454	3.78010	12.26166	3.24374
14	22.73737	0.04398	0.01150	86.94947	0.26150	3.82408	12.83341	3.35595
15	28.42171	0.03518	0.00912	109.68684	0.25912	3.85926	13.32599	3.45299
16	35.52714	0.02815	0.00724	138.10855	0.25724	3.88741	13.74820	3.53660
17	44.40892	0.02252	0.00576	173.63568	0.25576	3.90993	14.10849	3.60838
18	55.51115	0.01801	0.00459	218.04460	0.25459	3.92794	14.41473	3.66979

(Continued)

Table A31 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
19	69.38894	0.01441	0.00366	273.55576	0.25366	3.94235	14.67414	3.72218
20	86.73617	0.01153	0.00292	342.94470	0.25292	3.95388	14.89320	3.76673
21	108.42022	0.00922	0.00233	429.68087	0.25233	3.96311	15.07766	3.80451
22	135.52527	0.00738	0.00186	538.10109	0.25186	3.97049	15.23262	3.83646
23	169.40659	0.00590	0.00148	673.62636	0.25148	3.97639	15.36248	3.86343
24	211.75824	0.00472	0.00119	843.03295	0.25119	3.98111	15.47109	3.88613
25	264.69780	0.00378	0.00095	1054.79118	0.25095	3.98489	15.56176	3.90519
26	330.87225	0.00302	0.00076	1319.48898	0.25076	3.98791	15.63732	3.92118
27	413.59031	0.00242	0.00061	1650.36123	0.25061	3.99033	15.70019	3.93456
28	516.98788	0.00193	0.00048	2063.95153	0.25048	3.99226	15.75241	3.94574
29	646.23485	0.00155	0.00039	2580.93941	0.25039	3.99381	15.79574	3.95506
30	807.79357	0.00124	0.00031	3227.17427	0.25031	3.99505	15.83164	3.96282
32	1262.17745	0.00079	0.00020	5044.70979	0.25020	3.99683	15.88591	3.97463
34	1972.15226	0.00051	0.00013	7884.60905	0.25013	3.99797	15.92293	3.98275
35	2465.19033	0.00041	0.00010	9856.76132	0.25010	3.99838	15.93672	3.98580
36	3081.48791	0.00032	0.00008	12321.9516	0.25008	3.99870	15.94808	3.98831
38	4814.82486	0.00021	0.00005	19255.2994	0.25005	3.99917	15.96511	3.99211
40	7523.16385	0.00013	0.00003	30088.6554	0.25003	3.99947	15.97661	3.99468
45	22958.8740	0.00004	0.00001	91831.4962	0.25001	3.99983	15.99146	3.99804
50	70064.9232	0.00001	0.00000	280255.693	0.25000	3.99994	15.99692	3.99929
55	213821.177	0.00000	0.00000	855280.707	0.25000	3.99998	15.99890	3.99974
60	652530.447	0.00000	0.00000	2610117.79	0.25000	3.99999	15.99961	3.99991

Table A32 Compound Interest Factors 30%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.30000	0.76923	1.00000	1.00000	1.30000	0.76923		
2	1.69000	0.59172	0.43478	2.30000	0.73478	1.36095	0.59172	0.43478
3	2.19700	0.45517	0.25063	3.99000	0.55063	1.81611	1.50205	0.82707

4	2.85610	0.35013	0.16163	6.18700	0.46163	2.16624	2.55243	1.17828
5	3.71293	0.26933	0.11058	9.04310	0.41058	2.43557	3.62975	1.49031
6	4.82681	0.20718	0.07839	12.75603	0.37839	2.64275	4.66563	1.76545
7	6.27485	0.15937	0.05687	17.58284	0.35687	2.80211	5.62183	2.00628
8	8.15731	0.12259	0.04192	23.85769	0.34192	2.92470	6.47995	2.21559
9	10.60450	0.09430	0.03124	32.01500	0.33124	3.01900	7.23435	2.39627
10	13.78585	0.07254	0.02346	42.61950	0.32346	3.09154	7.88719	2.55122
11	17.92160	0.05580	0.01773	56.40535	0.31773	3.14734	8.44518	2.68328
12	23.29809	0.04292	0.01345	74.32695	0.31345	3.19026	8.91732	2.79517
13	30.28751	0.03302	0.01024	97.62504	0.31024	3.22328	9.31352	2.88946
14	39.37376	0.02540	0.00782	127.91255	0.30782	3.24867	9.64369	2.96850
15	51.18589	0.01954	0.00598	167.28631	0.30598	3.26821	9.91721	3.03444
16	66.54166	0.01503	0.00458	218.47220	0.30458	3.28324	10.14263	3.08921
17	86.50416	0.01156	0.00351	285.01386	0.30351	3.29480	10.32759	3.13451
18	112.45541	0.00889	0.00269	371.51802	0.30269	3.30369	10.47876	3.17183
19	146.19203	0.00684	0.00207	483.97343	0.30207	3.31053	10.60189	3.20247
20	190.04964	0.00526	0.00159	630.16546	0.30159	3.31579	10.70186	3.22754
21	247.06453	0.00405	0.00122	820.21510	0.30122	3.31984	10.78281	3.24799
22	321.18389	0.00311	0.00094	1067.27963	0.30094	3.32296	10.84819	3.26462
23	417.53905	0.00239	0.00072	1388.46351	0.30072	3.32535	10.90088	3.27812
24	542.80077	0.00184	0.00055	1806.00257	0.30055	3.32719	10.94326	3.28904
25	705.64100	0.00142	0.00043	2348.80334	0.30043	3.32861	10.97727	3.29785
26	917.33330	0.00109	0.00033	3054.44434	0.30033	3.32970	11.00452	3.30496
27	1192.53329	0.00084	0.00025	3971.77764	0.30025	3.33054	11.02632	3.31067
28	1550.29328	0.00065	0.00019	5164.31093	0.30019	3.33118	11.04374	3.31526
30	2619.99564	0.00038	0.00011	8729.98548	0.30011	3.33206	11.06870	3.32188
32	4427.79264	0.00023	0.00007	14755.9755	0.30007	3.33258	11.08451	3.32610
34	7482.96956	0.00013	0.00004	24939.8985	0.30004	3.33289	11.09448	3.32879
35	9727.86043	0.00010	0.00003	32422.8681	0.30003	3.33299	11.09798	3.32974
36	12646.2186	0.00008	0.00002	42150.7285	0.30002	3.33307	11.10074	3.33049
38	21372.1094	0.00005	0.00001	71237.0312	0.30001	3.33318	11.10466	3.33156
40	36118.8648	0.00003	0.00001	120392.883	0.30001	3.33324	11.10711	3.33223

Table A33 Compound Interest Factors 35%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.35000	0.74074	1.00000	1.00000	1.35000	0.74074		
2	1.82250	0.54870	0.42553	2.35000	0.77553	1.28944	0.54870	0.42553
3	2.46038	0.40644	0.23966	4.17250	0.58966	1.69588	1.36158	0.80288
4	3.32151	0.30107	0.15076	6.63288	0.50076	1.99695	2.26479	1.13412
5	4.48403	0.22301	0.10046	9.95438	0.45046	2.21996	3.15684	1.42202
6	6.05345	0.16520	0.06926	14.43841	0.41926	2.38516	3.98282	1.66983
7	8.17215	0.12237	0.04880	20.49186	0.39880	2.50752	4.71702	1.88115
8	11.03240	0.09064	0.03489	28.66401	0.38489	2.59817	5.35151	2.05973
9	14.89375	0.06714	0.02519	39.69641	0.37519	2.66531	5.88865	2.20937
10	20.10656	0.04974	0.01832	54.59016	0.36832	2.71504	6.33626	2.33376
11	27.14385	0.03684	0.01339	74.69672	0.36339	2.75188	6.70467	2.43639
12	36.64420	0.02729	0.00982	101.84057	0.35982	2.77917	7.00486	2.52048
13	49.46967	0.02021	0.00722	138.48476	0.35722	2.79939	7.24743	2.58893
14	66.78405	0.01497	0.00532	187.95443	0.35532	2.81436	7.44209	2.64433
15	90.15847	0.01109	0.00393	254.73848	0.35393	2.82545	7.59737	2.68890
16	121.71393	0.00822	0.00290	344.89695	0.35290	2.83367	7.72061	2.72460
17	164.31381	0.00609	0.00214	466.61088	0.35214	2.83975	7.81798	2.75305
18	221.82364	0.00451	0.00158	630.92469	0.35158	2.84426	7.89462	2.77563
19	299.46192	0.00334	0.00117	852.74834	0.35117	2.84760	7.95473	2.79348
20	404.27359	0.00247	0.00087	1152.21025	0.35087	2.85008	8.00173	2.80755
21	545.76935	0.00183	0.00064	1556.48384	0.35064	2.85191	8.03837	2.81859
22	736.78862	0.00136	0.00048	2102.25319	0.35048	2.85327	8.06687	2.82724
23	994.66463	0.00101	0.00035	2839.04180	0.35035	2.85427	8.08899	2.83400
24	1342.79725	0.00074	0.00026	3833.70643	0.35026	2.85502	8.10612	2.83926
25	1812.77629	0.00055	0.00019	5176.50369	0.35019	2.85557	8.11936	2.84334
26	2447.24799	0.00041	0.00014	6989.27998	0.35014	2.85598	8.12957	2.84651
27	3303.78479	0.00030	0.00011	9436.52797	0.35011	2.85628	8.13744	2.84897
28	4460.10947	0.00022	0.00008	12740.3128	0.35008	2.85650	8.14350	2.85086
30	8128.54950	0.00012	0.00004	23221.5700	0.35004	2.85679	8.15172	2.85345
32	14814.2815	0.00007	0.00002	42323.6613	0.35002	2.85695	8.15654	2.85498

34	26999.0280	0.00004	0.00001	77137.2228	0.35001	2.85704	8.15936	2.85588
35	36448.6878	0.00003	0.00001	104136.251	0.35001	2.85706	8.16030	2.85618
36	49205.7285	0.00002	0.00001	140584.939	0.35001	2.85708	8.16101	2.85641
38	89677.4402	0.00001	0.00000	256218.400	0.35000	2.85711	8.16196	2.85672
40	163437.135	0.00001	0.00000	466960.385	0.35000	2.85713	8.16252	2.85690

Table A34 Compound Interest Factors 40%

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
1	1.40000	0.71429	1.00000	1.00000	1.40000	0.71429		
2	1.96000	0.51020	0.41667	2.40000	0.81667	1.22449	0.51020	0.41667
3	2.74400	0.36443	0.22936	4.36000	0.62936	1.58892	1.23907	0.77982
4	3.84160	0.26031	0.14077	7.10400	0.54077	1.84923	2.01999	1.09234
5	5.37824	0.18593	0.09136	10.94560	0.49136	2.03516	2.76373	1.35799
6	7.52954	0.13281	0.06126	16.32384	0.46126	2.16797	3.42778	1.58110
7	10.54135	0.09486	0.04192	23.85338	0.44192	2.26284	3.99697	1.76635
8	14.75789	0.06776	0.02907	34.39473	0.42907	2.33060	4.47129	1.91852
9	20.66105	0.04840	0.02034	49.15262	0.42034	2.37900	4.85849	2.04224
10	28.92547	0.03457	0.01432	69.81366	0.41432	2.41357	5.16964	2.14190
11	40.49565	0.02469	0.01013	98.73913	0.41013	2.43826	5.41658	2.22149
12	56.69391	0.01764	0.00718	139.23478	0.40718	2.45590	5.61060	2.28454
13	79.37148	0.01260	0.00510	195.92869	0.40510	2.46850	5.76179	2.33412
14	111.12007	0.00900	0.00363	275.30017	0.40363	2.47750	5.87878	2.37287
15	155.56810	0.00643	0.00259	386.42024	0.40259	2.48393	5.96877	2.40296
16	217.79533	0.00459	0.00185	541.98833	0.40185	2.48852	6.03764	2.42620
17	304.91347	0.00328	0.00132	759.78367	0.40132	2.49180	6.09012	2.44406
18	426.87885	0.00234	0.00094	1064.69714	0.40094	2.49414	6.12994	2.45773
19	597.63040	0.00167	0.00067	1491.57599	0.40067	2.49582	6.16006	2.46815
20	836.68255	0.00120	0.00048	2089.20639	0.40048	2.49701	6.18277	2.47607
21	1171.35558	0.00085	0.00034	2925.88894	0.40034	2.49787	6.19984	2.48206
22	1639.89781	0.00061	0.00024	4097.24452	0.40024	2.49848	6.21265	2.48658
23	2295.85693	0.00044	0.00017	5737.14232	0.40017	2.49891	6.22223	2.48998

(Continued)

Table A34 (Continued)

	Single Payment		Uniform Payment Series				Arithmetic Gradient	
n	F/P	P/F	A/F	F/A	A/P	P/A	P/G	A/G
24	3214.19970	0.00031	0.00012	8032.99925	0.40012	2.49922	6.22939	2.49253
25	4499.87958	0.00022	0.00009	11247.1990	0.40009	2.49944	6.23472	2.49444
26	6299.83141	0.00016	0.00006	15747.0785	0.40006	2.49960	6.23869	2.49587
27	8819.76398	0.00011	0.00005	22046.9099	0.40005	2.49972	6.24164	2.49694
28	12347.6696	0.00008	0.00003	30866.6739	0.40003	2.49980	6.24382	2.49773
30	24201.4324	0.00004	0.00002	60501.0809	0.40002	2.49990	6.24664	2.49876
32	47434.8074	0.00002	0.00001	118584.519	0.40001	2.49995	6.24818	2.49933
34	92972.2225	0.00001	0.00000	232428.056	0.40000	2.49997	6.24902	2.49963
35	130161.112	0.00001	0.00000	325400.279	0.40000	2.49998	6.24928	2.49973
36	182225.556	0.00001	0.00000	455561.390	0.40000	2.49999	6.24947	2.49980
38	357162.090	0.00000	0.00000	892902.725	0.40000	2.49999	6.24972	2.49989
40	700037.697	0.00000	0.00000	1750091.74	0.40000	2.50000	6.24985	2.49994

Appendix B: Airlines Vital Statistics

Rank	Airline	Passenger (in Millions)	Fleet	Yield	Load Factor
1	Delta Air Lines	326,465	796	17.24	84.7%
2	United Airlines	288,036	707	18.86	83.6%
3	Emirates Airline	235,498	234	13.02	79.6%
4	American Airlines	208,108	964	16.17	82.1%
5	Southwest Airlines	173,829	670	16.35	82.5%
6	Lufthansa	156,826	276	18.50	79.4%
7	British Airways	138,431	262	19.99	81.0%
8	China Southern Airlines	135,637	491	15.21	80.5%
9	Air France	134,000	231	30.98	82.5%
10	China Eastern Airlines	127,637	289	15.39	79.5%
11	Ryanair	120,000	323	7.13	88.2%
12	Air China	112,495	333	21.47	77.1%
13	Turkish Airlines	106,787	247	14.85	78.9%
14	Cathay Pacific	100,048	143	15.71	84.4%
15	Air Canada	99,140	167	17.30	83.4%
16	Qatar Airways	95,977	161	12.10	75.7%
17	Singapore Airlines	94,209	105	13.92	78.7%
18	KLM	91,477	112	15.88	86.5%
19	ANA – All Nippon Airways	74,221	190	22.55	67.7%
20	EasyJet	72,933	216	16.35	91.7%
21	Qantas	71,270	118	27.31	78.1%
22	Etihad Airways	68,600	112		79.2%
23	Korean Air	67,948	162	15.71	74.7%
24	Aeroflot	67,122	160	16.30	78.2%
25	JetBlue Airways	60,842	208	14.13	84.0%
26	TAM Linhas Aereas	60,247	162		82.9%

(Continued)

Rank	Airline	Passenger (in Millions)	Fleet	Yield	Load Factor
27	Thai Airways International	56,377	80	13.58	69.0%
28	Japan Airlines	52,890	157	25.88	72.5%
29	Saudia	51,782	161		76.3%
30	Air Berlin	49,270	98	16.40	83.5%
31	LAN Airlines	48,250	82		83.9%
32	Transaero Airlines	47,066	98		83.5%
33	Malaysia Airlines	46,375	97		74.1%
34	Hainan Airlines	45,072	139		87.1%
35	Alaska Airlines	44,695	139	14.08	85.7%
36	Iberia	42,686	75	15.79	78.6%
37	Shenzhen Airlines	38,668	155		80.8%
38	GOL	38,085	129	16.19	76.9%
39	Virgin Atlantic Airways	37,664	41		77.2%
40	Norwegian	37,615	86	10.91	80.9%
41	China Airlines	35,886	79	13.42	77.7%
42	Swiss	35,717	76		83.7%
43	Asiana Airlines	35,223	84		75.3%
44	Air India	35,101	100		73.6%
45	SAS	34,594	146	19.84	76.6%
46	Alitalia	34,500	106		75.0%
47	Jet Airways	34,423	104		82.4%
48	WestJet	33,514	107	15.59	81.4%
49	Virgin Australia	33,067	95		78.3%

Appendix C: Top 50 Commercial Airports

Rank	Airport	Country	Code	Passengers	Operations per Week
1	Atlanta Hartsfield Int'l	USA	ATL	96,179	18,378
2	Capital	China	PEK	86,128	11,796
3	Heathrow	UK	LHR	73,408	9,726
4	Haneda International	Japan	HND	72,827	9,198
5	International	USA	LAX	70,663	13,022
6	International	UAE	DXB	70,476	7,560
7	O'Hare International	USA	ORD	69,999	17,986
8	Charles de Gaulle	France	CDG	63,814	8,898
9	International	USA	DFW	63,554	13,444
10	International	China	HKG	63,122	6,852
11	International	Germany	FRA	59,566	9,100
12	Soekarno Hatta	Indonesia	CGK	57,221	8,958
13	Ataturk International	Turkey	IST	56,767	8,740
14	Schiphol	Netherlands	AMS	54,978	8,634
15	Baiyun International	China	CAN	54,780	8,374
16	Changi	Singapore	SIN	54,093	6,742
17	International	USA	DEN	53,473	12,702
18	JFK	USA	JFK	53,255	8,582
19	Pudong	China	PVG	51,688	8,612
20	International	Malaysia	KUL	48,930	6,970
21	International	USA	SFO	47,115	8,870
22	Suvarnabhumi	Thailand	BKK	46,423	6,112
23	Incheon International	South Korea	ICN	45,662	5,280
24	Douglas	USA	CLT	44,280	10,482
25	McCarran	USA	LAS	42,870	8,658
26	Sky Harbor	USA	PHX	42,125	8,908

(Continued)

Rank	Airport	Country	Code	Passengers	Operations per Week
27	Barajas	Spain	MAD	41,823	6,962
28	George Bush	USA	IAH	41,240	10,494
29	International	USA	MIA	40,942	6,626
30	Guarulhos Int'l	Brazil	GRU	39,766	5,552
31	Indira Gandhi Int'l	India	DEL	39,753	7,188
32	International	Germany	MUC	39,701	7,030
33	International	Australia	SYD	38,863	6,070
34	Pearson International	Canada	YYZ	38,569	8,420
35	Fiumicino	Italy	FCO	38,506	6,664
36	Gatwick	UK	LGW	38,106	5,312
37	Hongqiao Int'l	China	SHA	37,971	5,902
38	Shuangliu Int'l	China	CTU	37,712	6,294
39	El Prat	Spain	BCN	37,540	5,724
40	International	USA	SEA	37,498	7,912
41	Baoan International	China	SZX	36,273	6,530
42	Taoyuan Int'l	Taiwan	TPE	35,804	3,756
43	International	USA	MCO	35,714	7,290
44	Liberty International	USA	EWR	35,611	8,494
45	Narita International	Japan	NRT	35,535	4,094
46	International	USA	MSP	35,147	7,856
47	International	India	BOM	34,994	6,226
48	Benito Juarez Int'l	Mexico	MEX	34,256	7,658
49	Ninoy Aquino Int'l	Philippines	MNL	34,015	5,016
50	Domodedovo Int'l	Russia	DME	33,108	4,680

Source: Compiled by the authors from several databases, 2016.

Appendix D:

Aircraft Leasing Companies

Rank	Leasing Firm	Total Fleet	Fleet Value
1	AerCap	1,256	31,613
2	GECAS	1,567	30,894
3	BBAM	297	12,803
4	SMBC Aviation Capital	395	10,869
5	Air Lease Corporation	276	10,479
6	BOC Aviation	267	10,159
7	CIT Aerospace	326	9,528
8	ICBC Leasing	204	7,692
9	AWAS	260	6,998
10	Avolon Aerospace Leasing	158	6,420
11	Aviation Capital Group	263	6,087
12	Aircastle Advisor	164	5,027
13	CDB Leasing Company	134	4,949
14	Macquarie AirFinance	196	4,880
15	Jackson Square Aviation	110	4,295
16	Standard Chartered Aviation Finance	100	3,625
17	BoCom Leasing	93	3,528
18	Doric	37	3,427
19	ORIX Aviation	157	3,411
20	Nordic Aviation Capital	249	3,074
21	DAE Capital	70	3,074
22	MC Aviation Partners	88	2,946
23	FLY Leasing	116	2,942
24	Hong Kong Aviation Capital	76	2,661
25	Intrepid Aviation Group	28	2,344
26	VEB-Leasing JSC	70	2,288
27	Aircraft Leasing & Management	70	2,271

(Continued)

Rank	Leasing Firm	Total Fleet	Fleet Value
28	China Aircraft Leasing	64	2,258
29	Boeing Capital	243	2,233
30	Novus Aviation	24	1,997
31	Changjiang Leasing	70	1,594
32	CCB Financial Leasing	34	1,560
33	Amedeo	8	1,558
34	AVIC International Leasing	80	1,496
35	Amentum Capital	44	1,484
36	ALAFCO	50	1,473
37	CMB Financial Leasing	30	1,225
38	Castlelake	125	1,194
39	Guggenheim Aviation Partners	36	1,136
40	Jackson Square Aviation	23	1,052
41	VTB-Leasing	58	1,052
42	Banc of America Leasing Ireland	29	1,013
43	Comsys Aviation Leasing	20	993
44	Apollo Aviation Group	66	988
45	Dragon Aviation Leasing	30	985
46	Transportation Partners Pte	33	921
47	SPDB Financial Leasing	19	887
48	GTLK – State Transport Leasing	31	862
49	Showa Leasing	32	821
50	Jetscape	46	808

Appendix E:

Major Excel Project For Airport

Computer Project Microsoft Excel Assignment

Greenville Airport Construction by Redford Airport Authority

Redford Airport Authority is considering construction of a reliever airport in Greenville County, which is about 40 miles away from Redford International Airport, to reduce current congestion at Redford International Airport. This facility will be built to accommodate up to an additional 30 million enplanements at ultimate build out in five years. The following are costs associated with the construction of Greenville Airport. All costs and revenues are incurred at the end of the year unless they are specifically stated to be at the beginning of the year.

- ◆ Purchase of 15,000 acres of land at \$100 million in Greenville County to be paid today (time 0).
- ◆ Cost of environmental assessment of \$30 million due in year 1.
- ◆ Airfield improvement costs including taxiway, ramps, access roads, airfield storm drainage, and fencing of \$150 million due in year 2.
- ◆ Construction of two terminals, each with 25 gates, and supporting spaces with the cost of \$400 million in year 3, \$500 million in year 4, and \$600 million in year 5.
- ◆ Construction of parking garage and rental car facilities for \$80 million in year 4.
- ◆ Construction of concession areas in both terminals for \$100 million in year 4 and \$125 million in year 5.
- ◆ Construction cost of the following two parallel runways:

Runway A (150 ft × 15,000 ft)	\$400 million in year 4
Runway B (150 ft × 12,000 ft)	\$300 million in year 5
- ◆ Other capital equipment costing \$50 million at the end of each year during the five years of construction.
- ◆ Maintenance costs and recurring expenses: \$100 million in the first year of airport operation (year 6) with increase of 5 percent per year thereafter.

Enplanement (# of passengers boarding the planes) is expected to be 15 million in the first year of operation (year 6) with 5 percent increase per year.

The expected airport revenue and fees are as follows:

- ◆ Passenger facility charges will be \$4.50 per enplaned passenger.
- ◆ Landing fees are estimated to be \$50 million in the first year of operation with 15 percent increase per year.

- ◆ Net revenue from parking structure is estimated to be \$50 million in the first year of operation with 10 percent increase per year up to maximum of \$100 million.
- ◆ Advertising revenue is estimated to be \$400 million per year for years 6–8, \$450 million per year for years 9–12, and \$500 million per year for years 13–15, with the airport receiving 85%.
- ◆ Food revenue is estimated to be \$200 million per year for years 6–8, \$250 million per year for years 9–12, and \$300 million per year for years 13–15 in which airport concession is 15%.
- ◆ Revenue for gate usage is estimated to be \$40 million in the first year of airport operation with increase of 10 percent per year through year 15.
- ◆ Revenue from alcoholic beverages is estimated to be \$180 million per year for years 6–8, \$280 million per year for years 9–12, and \$380 million per year for years 13–15 with airport concession of 18%.
- ◆ Revenue from gift and general merchandise is estimated to be \$300 million per year for years 6–8, \$400 million per year for years 9–12, and \$500 million per year for years 13–15 with airport concession of 12%.
- ◆ Airport revenue from car rental concession is \$6 per rental day. The annual car rental days are estimated to be 10 million per year for years 6–8, 20 million per year for years 9–12, and 30 million per year for years 13–15.
- ◆ Redford Authority also agreed to sell the Greenville Airport to Greenville County at the end of 10 years of airport operation at an expected price of \$1,500 million.

Assuming Redford's MARR is 8%, using **Excel Functions**, complete the following:

- ◆ Calculate the total annual investment (cost) per year through year 5.
- ◆ Calculate the PW of annual investments (costs) occurring in the first five years. This is considered the first cost.
- ◆ Calculate the net annual cash flow per year for the first 10 years of operations (Years 6–15).

Use the above information and the following techniques to evaluate the project.

- ◆ Calculate the PW (NPV) of all cash flows.
- ◆ Calculate the ROR (IRR) of all cash flows.
- ◆ Calculate the modified B/C ratio.
- ◆ Calculate the no-return payback period.

Add a brief statement at the bottom of the page indicating whether or not you recommend that Redford construct the Greenville Airport.

Appendix F:

Solutions to Selected Problems

Chapter 2

2.2

$$I = (\$10,000) \times \left(\frac{5}{100} \right) \times (4)$$

$$I = \$2,000$$

2.4

$$F_n = P + P \times (i) \times (n)$$

$$\$14,000,000 = P + P \times (0.05) \times (8)$$

$$\$14,000,000 = 1.4 \times P$$

$$P = \$10,000,000$$

2.6

$$F_n = P \times (1 + i)^n$$

$$F_6 = \$100 \times (1 + 0.08)^6$$

$$F_6 = \$100 \times (1.5869)$$

$$F_6 = \$158.69 \text{ million}$$

2.10

$$P = F \times \frac{1}{(1 + i)^n}$$

$$P = \$2,000,000 \times \frac{1}{(1 + 0.08)^5}$$

$$P = \$2,000,000 \times [0.6806]$$

$$P = \$1,361,166$$

Alternatively:

$$P = F (P/F, i, n)$$

$$P = \$2,000,000 (P/F, 8\%, 5)$$

From Appendix A, Table A14, for 8% interest rate, the value of the interest factor (P/F , 8%, 5) is 0.6806. We plug in 0.6806 in the above relationship and compute the present value.

$$P = \$2,000,000 \times (0.6806)$$

$$P = \$1,361,166$$

2.12

Since payments are not in uniform amounts, first compute the future value of each payment at the end of year 4 and simply add them together.

$$F_n = P \times (1 + i)^n$$

Future value of the first payment at the end of year 4 = $\$50,000 \times (1 + 0.10)^3$

$$\begin{aligned} &= \$50,000 \times (1.331) \\ &= \$66,550 \end{aligned}$$

Future value of the second payment at the end of year 4 = $\$65,000 \times (1 + 0.10)^2$

$$\begin{aligned} &= \$65,000 \times (1.21) \\ &= \$78,650 \end{aligned}$$

Future value of the third payment at the end of year 4 = $\$80,000 \times (1 + 0.10)^1$

$$\begin{aligned} &= \$80,000 \times (1.10) \\ &= \$88,000 \end{aligned}$$

Future value of the fourth payment at the end of year 4 = $\$100,000$

$$\begin{aligned} \text{Total Future value} &= \$66,550 + \$78,650 + \$88,000 + \$100,000 \\ \text{Total Future value} &= \$333,200 \end{aligned}$$

2.14

Using rule 72 for compound interest,

$$n = \frac{72}{12} = 6 \text{ years}$$

Chapter 3**3.2**

$$A = F \times \left[\frac{i}{(1 + i)^n - 1} \right]$$

$$A = \$400,000 \times \left[\frac{0.08}{(1 + 0.08)^{18} - 1} \right]$$

$$A = \$400,000 \times \left[\frac{0.08}{3.9960 - 1} \right]$$

$$A = \frac{\$32,000}{2.9960}$$

$$A = \$10,681$$

Alternatively:

$$A = F (A/F, i, n)$$

$$A = \$400,000 \times (A/F, 8\%, 18)$$

From Table A14 in Appendix A, for 8% interest rate, the value of the interest factor ($A/F, 8\%, 18$) is 0.0267.

$$A = \$400,000 \times (0.0267)$$

$$A = \$10,680$$

3.5

We compare the two alternatives on a present value bases.

First, we compute the present value of the leasing cost (P_L) for 10 years.

$$P_L = \$100,000 (P/A, 6\%, 10)$$

$$P_L = \$100,000 \times (7.3601)$$

$$P_L = \$736,010$$

Second, we compute the present value of the cost of building the hangar and the maintenance cost (P_B).

$$P_B = \$960,000 + \$5,000 (P/A, 6\%, 10) - \$500,000 (P/F, 6\%, 10)$$

$$P_B = \$960,000 + \$5,000 \times (7.3601) - \$500,000 \times (0.5584)$$

$$P_B = \$960,000 + \$36,801 - \$279,200$$

$$P_B = \$717,601$$

Building is cost efficient.

3.8

$$P_G = \$100,000 (P/A, 8\%, 5) + \$20,000 (P/G, 8\%, 5)$$

$$P_G = \$100,000 \times (3.9927) + \$20,000 \times (7.3724)$$

$$P_G = \$399,270 + \$147,448$$

$$P_G = \$546,718$$

3.11

$$F_G = A (F/A, i, n) + G/i [F/A, i, n] - n]$$

$$F_G = \$12,000 (F/A, 10\%, 25) + \$1,200/0.10 [F/A, 10\%, 25) - 25]$$

$$F_G = \$12,000 \times (98.3471) + \frac{\$1,200}{0.10} \times [98.3471 - 25]$$

$$F_G = \$1,180,165 + \$880,165$$

$$F_G = \$2,060,330$$

Alternatively, one may calculate the present value of investments and then compute the future value of that amount at the end of 20 years.

$$P_G = \$12,000 (P/A, 10\%, 25) + \$1,200 (P/G, 10\%, 25)$$

$$P_G = \$12,000 \times (9.0770) + \$1,200 \times (67.6964)$$

$$P_G = \$108,924 + \$81,236$$

$$P_G = \$190,160$$

$$F_{20} = \$190,160 \times (1 + 0.10)^{25}$$

$$F_{20} = \$190,160 \times (10.8347)$$

$$F_{20} = \$2,060,327$$

3.14

Total contributions to retirement account in the first year = $\$120,000 \times (0.04 + 0.04) = \$9,600$.

$$P_s = \$9,600 \times \left[\frac{1 - \left\{ \frac{1 + 0.03}{1 + 0.10} \right\}^{30}}{0.10 - 0.03} \right]$$

$$P_s = \$9,600 \times \left[\frac{1 - 0.1391}{0.07} \right]$$

$$P_s = \$9,600 \times (12.2985)$$

$$P_s = \$118,066$$

The accumulation at the end of 30 years is

$$F_{30} = \$118,066 \times (1 + 0.10)^{30}$$

$$F_{30} = \$118,066 \times (17.4494)$$

$$F_{30} = \$2,060,181$$

3.17

The payments for the first five years are the same amounts, an annuity of \$40,000 per year for five years. The payments for the next five years are on an arithmetic gradient with the base amount of \$60,000 in year 6 and increasing by gradient of \$20,000 per year through year 10.

$$P_T = P_A + P_G$$

$$P_T = \$40,000 (P/A, 8\%, 5) + \{[\$60,000 (P/A, 8\%, 5) + \$20,000 (P/G, 8\%, 5)] \times (P/F, 8\%, 5)\}$$

$$P_T = \$40,000 (3.9927) + \{[(\$60,000 \times 3.9927) + (\$20,000 \times 7.3724)] \times 0.6806\}$$

$$P_T = \$159,708 + [(\$239,562 + \$147,448) \times 0.6806]$$

$$P_T = \$159,708 + (\$387,010 \times 0.6806)$$

$$P_T = \$159,708 + \$263,399$$

$$P_T = \$423,107$$

Chapter 4

4.2

a.

Nominal rate per month = $18\%/12 = 1.5\%$

$$i_{\text{effective}} = \left(1 + \frac{0.015}{30}\right)^{30} - 1$$

$$i_{\text{effective}} = 1.0151 - 1$$

$$i_{\text{effective}} = 0.0151$$

$$i_{\text{effective}} = 1.51\%$$

b.

Nominal rate per quarter = 4.5%

$$i_{\text{effective}} = \left(1 + \frac{0.045}{90}\right)^{90} - 1$$

$$i_{\text{effective}} = 1.0460 - 1$$

$$i_{\text{effective}} = 0.0460$$

$$i_{\text{effective}} = 4.60\%$$

c.

$$i_{\text{effective}} = \left(1 + \frac{0.18}{360}\right)^{360} - 1$$

$$i_{\text{effective}} = 1.1972 - 1$$

$$i_{\text{effective}} = 0.1972$$

$$i_{\text{effective}} = 19.72\%$$

4.5

$$0.1956 = \left(1 + \frac{r}{12}\right)^{12} - 1$$

$$1.1956 = \left(1 + \frac{r}{12}\right)^{12}$$

$$(1.1956)^{1/12} = \left(1 + \frac{r}{12}\right)$$

$$1.015 = 1 + \frac{r}{12}$$

$$0.015 = \frac{r}{12}$$

$$r = 0.17998$$

$$r = 18\%$$

4.8

Calculating effective rate

$$\text{Effective rate} = \left(1 + \frac{0.08}{4}\right)^4$$

$$\text{Effective rate} = 8.2432\%$$

$$P = \frac{\$5,000,000}{(1 + 0.082432)^6}$$

$$P = \frac{\$5,000,000}{1.60844}$$

$$P = \$3,108,610$$

4.11

In this example $PP > CP$, and we need to calculate quarterly payments to match the compounding period.

$$\text{Quarterly Payment} = \$200,000 \times 3 = \$600,000$$

Now, we can re-write the problem as Kazak Airline is depositing \$600,000 per quarter for four years. At interest rate of 8% per year compounded quarterly, what is the future value of all payments at the end of four years? Now $PP = CP$ and we compute the accumulated amount.

$$F = A \left(F/A, \frac{i\%}{4}, n \times 4 \right)$$

$$F = \$600,000 \left(F/A, \frac{8\%}{4}, 4 \times 4 \right)$$

$$F = \$600,000 (F/A, 2\%, 16)$$

$$F = \$600,000 \times (18.6393)$$

$$F = \$11,183,580$$

4.14

- a. $\text{Nominal rate per month} = \frac{12\%}{12} = 1\%$
- b. $\text{Effective rate per month} = e^{0.01} - 1 = 1.005\%$
- c. $\text{Nominal rate per quarter} = \frac{12\%}{4} = 3\%$
- d. $\text{Effective rate per quarter} = e^{0.03} - 1 = 3.045\%$
- e. $\text{Nominal rate per semi-annual} = \frac{12\%}{2} = 6\%$
- f. $\text{Effective rate per semi-annual} = e^{0.06} - 1 = 6.184\%$
- g. $\text{Effective rate per year} = e^{0.12} - 1 = 12.75\%$

Chapter 5

5.3

$$\text{Bond price} = \$50 \times \frac{1}{0.04} \left[1 - \left[\frac{1}{(1 + 0.04)^4} \right] \right] + \frac{\$1,000}{(1 + 0.04)^4}$$

$$\text{Bond Price} = \$1,036.3$$

5.12

$$P = \$100 \times PVA F(14\%, 5) + PVF(14\%, 5)$$

$$P = \$862.71$$

5.17

$$P = \$300,000(P/A, 12\%, 6) - \$50,000(P/G, 12\%, 6) = \$786,800$$

5.18

$$A = \$100,000 \text{ and } i = 0.05.$$

$$CC = \$100,000/0.05 = \$2,000,000$$

Chapter 6

6.2

$$AW = -\$5,000,000 (A/P, 10\%, 10) - \$400,000 + \$1,500,000$$

$$AW = -\$5,000,000 (0.16275) - \$400,000 + \$1,500,000$$

$$AW = -\$813,750 - \$400,000 + \$1,500,000$$

$$AW = \$286,250$$

Since $AW > 0$, this project is economically justified.

6.4

Since revenues are constant per year, to calculate AW of the project, we simply compute the AW of all costs and add it to the annual revenue.

$$AW_{\text{System}} = -AW_{\text{Cost}} + AW_{\text{Revenue}}$$

$$AW_{\text{Cost}} = -\$4,000,000 (A/P, 8\%, 10) - [\$500,000 (P/F, 10\%, 4) + \$600,000 (P/F, 10\%, 8)] \times [(A/P, 8\%, 10)] + \$200,000 (A/F, 8\%, 10)$$

$$AW_{\text{Cost}} = -\$4,000,000 (0.14903) - [\$500,000 (0.7350) + \$600,000 (0.5403)] \times [(0.14903)] + \$200,000 (0.06903)$$

$$AW_{\text{Cost}} = -\$596,120 - [\$367,500 + \$324,180] \times [0.14903] + \$13,806$$

$$AW_{\text{Cost}} = -\$596,120 - \$103,081 + \$13,806$$

$$AW_{\text{Cost}} = -\$685,395$$

To compute the annual worth of the project, we simply add the AW of costs to AW of revenue.

$$AW_{System} = -\$685,395 + \$1,200,000$$

$$AW_{System} = +\$514,605$$

Since AW value of the project is greater than zero, then this project is economically justified.

6.6

$$CR = [-\$4,000,000 - \$500,000 (P/F, 12\%, 5)] [(A/P, 12\%, 10)] + \$200,000 (A/F, 12\%, 10)$$

$$CR = [-\$4,000,000 - \$500,000 (0.5674)] [(0.17698)] + \$200,000 (0.05698)$$

$$CR = [-\$4,000,000 - \$283,700] [(0.17698)] + 11,396$$

$$CR = -\$758,129.23 + 11,396$$

$$CR = -\$746,733.23$$

6.8

$$CR = -\$2,000,000 (0.17401) + \$2,000,000 (0.10) (0.09401)$$

$$CR = -\$348,020 + \$18,802$$

$$CR = -\$329,218$$

Since this is a cost-based project and there is no annual revenue, we use annual operating cost (AOC) instead of NACF.

$$A \text{ of } AOC = -\$100,000 - \$50,000 (A/G, 9\%, 8)$$

$$A \text{ of } AOC = -\$100,000 - \$50,000 (3.0985)$$

$$A \text{ of } AOC = -\$100,000 - \$154,925$$

$$A \text{ of } AOC = -\$254,925$$

$$AW = -\$329,218 - \$254,925$$

$$AW = -\$584,143$$

6.10

$$AW = CR + A \text{ of } NACF$$

$$CR = -\$1,100,000 (A/P, 10\%, 10) + \$80,000 (A/F, 10\%, 10)$$

$$CR = -\$1,100,000 (0.16275) + \$80,000 (0.06275)$$

$$CR = -\$179,025 + \$5,020$$

$$CR = -\$174,005$$

$$A \text{ of } NACF = (\$300,000 - \$100,000) + (\$75,000 - \$50,000) (A/G, 10\%, 10)$$

$$A \text{ of } NACF = (\$200,000) + (\$25,000) (3.7255)$$

$$A \text{ of } NACF = \$200,000 + \$93,137.50$$

$$A \text{ of } NACF = \$293,137.50$$

$$AW = -\$174,005 + 293,137.50$$

$$AW = \$119,132.50$$

Since AW value is greater than zero, it is a good project or economically justified.

6.14

$$AW_P = -\$180,000,000 (0.10) = -\$18,000,000$$

$$AW_{AOC} = -\$2,000,000 - \$2,000,000 (P/F, 10\%, 10)$$

$$AW_{AOC} = -\$2,000,000 - \$2,000,000 (0.3855)$$

$$AW_{AOC} = -\$2,000,000 - \$771,000$$

$$AW_{AOC} = -\$2,771,000$$

$$AW_{Resurfacing} = -\$5,000,000 (A/F, 10\%, 5)$$

$$AW_{Resurfacing} = -\$5,000,000 \times (0.1638)$$

$$AW_{Resurfacing} = -\$819,000$$

$$AW_{Revenue} = \$15,000,000 + \$5,000,000 (P/F, 10\%, 4) + \$10,000,000 (P/F, 10\%, 10)$$

$$AW_{Revenue} = \$15,000,000 + \$5,000,000 (0.6830) + \$10,000,000 (0.3855)$$

$$AW_{Revenue} = \$15,000,000 + \$3,415,000 + \$3,855,000$$

$$AW_{Revenue} = \$22,270,000$$

To calculate the total annual worth of the project, we add the AW_P , AW_{AOC} , $AW_{Resurfacing}$, and $AW_{Revenue}$ together as follow:

$$AW_{Total} = -\$18,000,000 - \$2,771,000 - \$819,000 + \$22,270,000$$

$$AW_{Total} = \$680,000 > 0$$

Since AW value of the parking garage project is positive, it is therefore economically justified.

Chapter 7**7.7**

$$PW = -\$10,000,000 - \$400,000 (P/A, i^*, 10) - \$50,000 (P/G, i^*, 10) + \$2,500,000 (P/A, i^*, 10) = 0$$

$$PW = -\$10,000,000 + (\$2,500,000 - \$400,000) (P/A, i^*, 10) - \$50,000 (P/G, i^*, 10) = 0$$

$$PW = -\$10,000,000 + \$2,100,000 (P/A, i^*, 10) - \$50,000 (P/G, i^*, 10) = 0$$

$$\text{Try } i^* = 10\%$$

$$PW = -\$10,000,000 + \$2,100,000 (6.1446) - \$50,000 (22.8913)$$

$$PW = -\$10,000,000 + \$12,903,660 - \$1,144,565$$

$$PW = \$1,759,095 > 0$$

$$\text{Try } i^* = 12\%$$

$$PW = -\$10,000,000 + \$2,100,000 \times (5.6502) - \$50,000 \times (20.2541)$$

$$PW = -\$10,000,000 + \$11,865,420 - \$1,012,705$$

$$PW = \$852,715 > 0$$

Try $i^* = 14\%$

$$PW = -\$10,000,000 + \$2,100,000 \times (5.2161) - \$50,000 \times (17.9906)$$

$$PW = -\$10,000,000 + \$10,953,810 - \$899,530$$

$$PW = \$54,280 > 0$$

Try $i^* = 15\%$

$$PW = -\$10,000,000 + \$2,100,000 \times (5.0188) - \$50,000 \times (16.9795)$$

$$PW = -\$10,000,000 + \$10,539,480 - \$848,975$$

$$PW = -\$309,495 < 0$$

i^*	PW
14%	\$54,280
14% + X%	0
15%	-\$309,495

$$\frac{X}{1} = \frac{\$54,280}{\$363,775}$$

$$X\% = \frac{\$54,280}{\$363,775} = 0.15\%$$

Therefore, $ROR (i^*) = 14.15\%$. Since $ROR > MARR$, this project is economically justified.

7.10

	Hangar A	Hangar B	$\Delta B-A$
First cost	-\$2,000,000	-\$3,000,000	-\$1,000,000
Annual operating cost (AOC)	-\$500,000	-\$200,000	+ \$300,000
Deposit return	\$400,000	\$800,000	+\$400,000
Life, years	10	10	10

$$PW\Delta = -\$1,000,000 + \$300,000 (P/A, i^*, 10) + \$400,000 (P/F, i^*, 10) = 0$$

Try $i^* = MARR = 12\%$

$$PW\Delta = -\$1,000,000 + \$300,000 (P/A, i^*, 10) + \$400,000 (P/F, i^*, 10)$$

$$PW\Delta = -\$1,000,000 + \$300,000 \times (5.6502) + \$400,000 \times (0.3220)$$

$$PW\Delta = -\$1,000,000 + \$1,695,060 + \$128,800$$

$$PW\Delta = \$823,860 > 0$$

Since at i^* equal to MARR, the PW value of the Δ project is positive, it indicates that $i^* > MARR$, and that the Δ project is a profitable project and economically justified, and therefore, the hangar with larger first cost, hangar B, should be selected.

7.14

We compute the ROR for each project by setting the PW of all cash flows to zero and solve for i^* .

New Parking Garage

$$\begin{aligned}
 PW &= -\$120 - 5 (P/A, i^*, 20) + \$25 (P/A, i^*, 20) = 0 \\
 \$120 &= (\$25 - \$5) (P/A, i^*, 20) \\
 \$120 &= \$20 (P/A, i^*, 20) \\
 (P/A, i^*, 20) &= 6
 \end{aligned}$$

i^*	$(P/A, i^*, 20)$
15%	6.2593
15% + X%	6.0000
16%	5.9288

$$\begin{array}{rcl}
 \frac{X\%}{1\%} & = & \frac{0.2593}{0.3305}
 \end{array}$$

$$X\% = \frac{0.2593}{0.3305} = 0.78\%$$

Therefore, $ROR = i^* = 15.78\% > \text{MARR}$

New Parking Garage is economically justified.

Expanding Existing Terminal

$$\begin{aligned}
 PW &= -\$540 - 25 (P/A, i^*, 25) + \$85 (P/A, i^*, 25) = 0 \\
 \$540 &= (\$85 - \$25) (P/A, i^*, 25) \\
 \$540 &= \$60 (P/A, i^*, 25) \\
 (P/A, i^*, 25) &= 9
 \end{aligned}$$

i^*	$(P/A, i^*, 25)$
10%	9.0770
10% + X%	9.0000
11%	8.4217

$$\begin{array}{rcl}
 \frac{X\%}{1\%} & = & \frac{0.0770}{0.6553}
 \end{array}$$

$$X\% = \frac{0.0770}{0.6553} = 0.12\%$$

$ROR = i^* = 10.12\% < \text{MARR}$

Expanding Existing Terminal is not economically justified and should be eliminated.

Building New Terminal

$$\begin{aligned}
 PW &= -\$780 - \$30 (P/A, i^*, 25) + \$150 (P/A, i^*, 25) = 0 \\
 \$780 &= (\$150 - \$30) (P/A, i^*, 25) \\
 \$780 &= \$120 (P/A, i^*, 25) \\
 (P/A, i^*, 25) &= 6.5
 \end{aligned}$$

i^*	$(P/A, i^*, 25)$
14%	6.8729
14% + X%	6.5000
15%	6.4641

$$\begin{aligned}
 X &= \frac{0.3729}{0.4088} \\
 X &= 0.91\%
 \end{aligned}$$

Therefore, $ROR = i^* = 14.91\% > MARR$

Building New Terminal is economically justified.

Out of the three projects only the new parking garage and building the new terminal are economically justified and Legacy Regional Airport should only undertake those two projects.

Chapter 8

8.8

$$\begin{aligned}
 AW \text{ of Benefits} &= \$150 \\
 AW \text{ of Disbenefits} &= \$4 \\
 AW \text{ of AOC} &= \$25 \\
 AW \text{ of } P &= \$800 (0.10) + \$100 (A/F, 10\%, 10) = \$80 + \$100 (0.06275) \\
 AW \text{ of } P &= \$80 + \$6.275 = \$86.275
 \end{aligned}$$

$$B/C = \frac{\$150 - \$4}{\$86.275 + \$25} = \frac{\$146}{\$111.275}$$

$$B/C = 1.31 > 1$$

Because the B/C ratio of the new terminal is greater than 1, it is economically justified.

8.10

$$\Delta \text{ Project} = \text{Bridge B} - \text{Bridge A}$$

The following table presents the cash flows for the Δ project. Note the differential user travel costs are the implied benefits.

(in millions)	Bridge A	Bridge B	Δ Project B-A
Initial investment (first cost)	\$50	\$80	\$30
Annual operating cost (AOC)	\$4	\$5	1\$
Repair cost (RC) every 10 years	\$2	\$2.5	\$0.5
User travel costs per year	\$20	\$10	\$10 (Benefits)
Estimated life, years	∞	∞	∞

$$AW_{AP} = \$30 (0.08) = \$2.4$$

$$AW_{AAOC} = \$1$$

$$AW_{ARC} = \$10 (A/F, 8\%, 10) = \$10 (0.06903) = \$0.6903$$

$$AW_{AB} = \$10$$

$$\Delta B/C = \frac{\Delta B - \Delta D}{\Delta TC - \Delta S}$$

$$\Delta B/C = \frac{\$10 - \$0}{\$2.4 + \$1 + \$0.6903} = \frac{\$10}{\$4.0903} = 2.44 > 1$$

Because the $\Delta B/C$ ratio is greater than 1, the Δ project is economically justified and therefore Busan Global should select Bridge B.

8.12

For simplicity, all values are converted to AW.

$$AW \text{ of } P \text{ for } CAE = \$300 (0.08) = \$24$$

$$AW \text{ of } P \text{ for } CBE = \$400 (0.08) = \$32$$

$$AW \text{ of } P \text{ for } NRW = \$500 (0.08) = \$40$$

$$AW \text{ of } D \text{ for } CAE = \$15 (0.08) = \$1.2$$

$$AW \text{ of } D \text{ for } CBE = \$20 (0.08) = \$1.6$$

$$AW \text{ of } D \text{ for } NRW = \$18 (0.08) = \$1.44$$

(in millions)	Concourse A Expansion (CAE)	Concourse B Expansion (CBE)	New Runway (NRW)
AW of first cost, P	\$24	\$32	\$40
AW of annual operating cost (AOC)	\$15	\$20	\$25
AW of benefits, B	\$85	\$100	\$120
AW of disbenefits, D	\$1.2	\$1.6	\$1.44
Life, years	∞	∞	∞

Because these projects are benefit-based alternatives, Do-nothing becomes an option.

$$\Delta \text{ Project} = \text{CAE} - \text{Do-nothing}$$

The following table presents the cash flows for Do-nothing, for the Concourse A Expansion, and for the Δ project.

(in millions)	Do-nothing (DN)	Concourse A Expansion (CAE)	Δ Project CAE – DN
AW of P	\$0	\$24	\$24
AW of AOC	\$0	\$15	\$15
AW of benefits	\$0	\$85	\$85
AW of disbenefits	\$0	\$1.2	\$1.2
Life, years	∞	∞	∞

$$\Delta B/C = \frac{\$85 - \$1.2}{\$24 + \$15} = \frac{\$83.8}{\$39}$$

$$\Delta B/C = 2.14 > 1$$

Because $\Delta B/C > 1$, the Δ project is economically justified. The Do-nothing alternative is eliminated, and the CAE survives. Now, we compare the CAE with the CBE.

$$\Delta \text{ Project} = \text{CAE} - \text{CBE}$$

The following table presents the cash flows for Concourse A Expansion and Concourse B Expansion, and for the Δ project.

(in millions)	Concourse A Expansion (CAE)	Concourse B Expansion (CBE)	Δ Project CBE – CAE
AW of P	\$24	\$32	\$8
AW of AOC	\$15	\$20	\$5
AW of benefits	\$85	\$100	\$15
AW of disbenefits	\$1.2	\$1.6	\$0.4
Life, years	∞	∞	∞

$$\Delta B/C = \frac{\$15 - \$0.4}{\$8 + 5} = \frac{\$14.6}{\$13}$$

$$\Delta B/C = 1.12 > 1$$

Because $\Delta B/C > 1$, the Δ project is economically justified. The CAE is eliminated, and the CBE survives. Now, we compare the CBE with the NRW.

$$\Delta \text{ Project} = \text{CAE} - \text{CBE}$$

The following table presents the cash flows for Concourse B Expansion, the New Runway and for the Δ Project.

(in millions)	Concourse B Expansion (CBE)	New Runway (NRW)	Δ Project NRW – CBE
AW of P	\$32	\$40	\$8
AW of AOC	\$20	\$25	\$5
AW of benefits	\$100	\$120	\$20
AW of disbenefits	\$1.6	\$1.44	–\$0.16
Life, years	∞	∞	∞

$$\Delta B/C = \frac{\$20 - (-\$0.16)}{\$8 + \$5} = \frac{\$20.16}{\$13}$$

$$\Delta B/C = 1.55 > 1$$

Because $\Delta B/C > 1$, the Δ project is economically justified. Therefore, the CBE is eliminated, and the New Runway survives. Legacy International Airport should select the New Runway.

Chapter 9

9.1

a.

$$Q_{BE} = \frac{\$1,500}{\$0.9 - \$0.5}$$

$$Q_{BE} = 3,750 \text{ units}$$

b.

$$\text{Profit} = 4,000 \times (\$900,000 - \$500,000) - \$1,500,000$$

$$\text{Profit} = \$1,000,000$$

9.3

$$\text{Load Factor} = \frac{1,957 \text{ million}}{2,284 \text{ million}} = 0.85 \text{ or } 85\%$$

$$\text{CASM} = \frac{\text{Total Costs (TC)}}{\text{Available Seat Miles (ASM)}}$$

$$\text{CASM} = \frac{\$365 \text{ million}}{2,284 \text{ million}} = \$0.1598$$

$$\text{RRPM} = \frac{\text{Total Revenue (TR)}}{\text{Revenue per Passenger Mile (RPM)}}$$

$$\text{RRPM} = \frac{\$388 \text{ million}}{1,957 \text{ million}} = \$0.1983$$

$$\text{Break-even Load Factor} = \frac{\$0.1598}{\$0.1983} = 0.81 \text{ or } 81\%$$

Since load factor > the break-even load factor, the route is economically justified.

9.7

$$\text{Variable cost per unit (V)} = \frac{6 \times 8 \times \$50}{8} = \$300$$

$$AW = 0$$

$$[-\$12,000,000 - \$1,000,000 (P/F, 8\%, 4)] \times [A/P, 8\%, 10] + \$2,000,000$$

$$(A/F, 8\%, 10) - \$500,000 - \$300 Q + \$15,000 Q = 0$$

$$[-\$12,000,000 - \$1,000,000 (0.7350)] \times [0.14903] + \$2,000,000 (0.06903) - \$500,000$$

$$+ (\$15,000 - \$300) Q = 0$$

$$-\$1,897,897 + \$138,060 - \$500,000 + \$14,700 Q = 0$$

$$\$2,259,837 = \$14,700 Q$$

$$Q_{BE} = 153.73$$

9.9

$$-\$24,000 Q = -\$12,000,000 (A/P, 10\%, 10) + \$12,000,000 \times (0.08) \times (A/F, 10\%, 10)$$

$$-\$300,000 - \$12,000 Q$$

$$-\$12,000 Q = -\$12,000,000 (0.16275) + \$12,000,000 \times (0.08) \times (0.06275) - \$300,000$$

$$-\$12,000 Q = -\$1,953,000 + \$60,240 - \$300,000$$

$$-\$12,000 Q = -\$2,192,760$$

$$Q_{BE} = 182.73$$

9.11

We first calculate the cumulative NACFs as shown in the table below.

Year	NACF	Cumulative NACF
1	\$2,000,000	\$2,000,000
2	\$1,750,000	\$3,750,000
3	\$1,500,000	\$5,250,000
4	\$1,250,000	\$6,500,000
5	\$1,000,000	\$7,500,000
6	\$750,000	\$8,250,000
7	\$500,000	
8	\$500,000	
9	\$500,000	
10	\$500,000	

$$N_p = (6 - 1) + \frac{\$8,100,000 - 7,500,000}{\$750,000}$$

$$N_p = (5) + 0.8$$

$$N_p = 5.8 \text{ years}$$

9.14

$$Q_{BE} = \frac{FC}{P - V} = \frac{16,000 \text{ million}}{P - 170 \text{ million}} = 200 \text{ units}$$

$$200 (P - 170) = 16,000$$

$$200 P - 34,000 = 16,000$$

$$200 P = 16,000 + 34,000 = 50,000$$

$$P = \frac{50,000}{200} = \$250 \text{ million}$$

9.15

$$\text{Load Factor (LF)} = \frac{RPM}{ASM} = \frac{RPM}{56,000 \text{ million}} = 80\%$$

$$RPM = 56,000 * 80\% = 44,800 \text{ million}$$

$$RRPM = \frac{\text{Total Revenue}}{RPM} = \frac{5,376 \text{ million}}{44,800 \text{ million}} = 0.12$$

$$LF_{BE} = \frac{CASM}{RRPM} = 70\%$$

$$CASM = LF_{BE} \times RRPM$$

$$CASM = 70\% \times 0.12$$

$$CASM = 0.084$$

9.16

22	First Cost	\$10,000,000				22	First Cost	\$10,000,000
23	AOC	\$800,000				23	AOC	\$800,000
24	Life/years	10				24	Life/years	10
25	MARR	12%				25	MARR	12%
26	Variable cost per unit	\$240				26	Variable cost per unit	\$240
27	Price per unit	\$12,000				27	Price per unit	\$12,000
28						28		
29	Break-even quantity	=(PMT(B25,B24,-B22,0)+B23)/(B27-B26)				29	Break-even quantity	218.52
30	MARR = 12%					30	MARR = 12%	

*Variable Cost per unit = $(5 \times 60 \times 8)/10 = \$240$ per unit

9.17

34	First Cost	\$15,000,000				34	First Cost	\$15,000,000
35	AOC	\$400,000				35	AOC	\$400,000
36	Life/years	10				36	Life/years	10
37	MARR	10%				37	MARR	10%
38	Variable cost per unit	\$60				38	Variable cost per unit	\$60
39	Price per unit	\$24,000				39	Price per unit	\$24,000
40						40		
41						41		
42	Break-even quantity	=(PMT(B37,B36,-B34,0)+B35)/(B39-B38)				42	Break-even quantity	118.68
43	MARR = 9%					43	MARR = 9%	

9.18

26	First Cost	\$2,500,000	Year 1	50,000	26	First Cost	\$2,500,000	
27	AOC	\$1,400,000	Year 2	70,000	27	AOC	\$1,400,000	
28	Life/years	10	Year 3	90,000	28	Life/years	10	
29	MARR	9%	Year 4	110,000	29	MARR	9%	
30	Salvage Value	\$0	Year 5	130,000	30	Salvage Value	\$0	
31			Year 6	150,000	31			
32	Conventional Payback	=B26/B27	Year 7	170,000	32	Conventional Payback	1.79	
33	MARR = 0		Year 8	190,000	33	MARR = 0		
34			Year 9	210,000	34			
35	Discounted Payback	2.03	Year 10	230,000	35	Discounted Payback	=NPER(B29,B27,-B26,B30)	
36	MARR = 8%		AOC	1,400,000	36	MARR = 8%		

$$12\% - 10.5\% = R_{RF} + 0.75 R_{RF}$$

$$1.5\% = 1.75 R_{RF}$$

$$R_{RF} = 0.86\%$$

Chapter 10

10.6

$$R_{RE} = 6\% + 0.75 \times (12\% - 6\%)$$

$$R_{RE} = 6\% + 9\% - 4.5\%$$

$$R_{RE} = 10.5\%$$

10.9

Investment	Amount	Weight of Projects
Project A	\$20,000.0	0.2
Project B	\$25,000.0	0.25
Project C	\$40,000.0	0.4
Project D	\$15,000.0	0.15

$$\text{Portfolio Expected Return} = (0.2 \times 8\%) + (0.25 \times 12\%) + (0.4 \times 15\%) + (0.15 \times 18\%)$$

$$\text{Portfolio Expected Return} = 1.6\% + 3\% + 6\% + 9\%$$

$$\text{Portfolio Expected Return} = 19.6\%$$

$$\beta_{\text{Portfolio}} = .20 (0.7) + .25 (0.8) + .40 (1.1) + .15 (1.6)$$

$$\beta_{\text{Portfolio}} = 0.14 + 0.20 + 0.44 + 0.24$$

$$\beta_{\text{Portfolio}} = 1.02$$

10.13

a. $P(x = 101) = 0.95^{101} = 0.0056$

b. $P(x = 102) = 0.95^{102} = 0.0053$

c. $P(0 \leq x \leq 100) = \text{binomcdf}(102, 0.95, 100) = 0.9660$

10.14

$$\mu = n \times q$$

$$\mu = 225 \times 0.80 = 180$$

$$\sigma = \sqrt{n \times p \times q}$$

$$\sigma = \sqrt{225 \times 0.8 \times 0.2}$$

$$\sigma = 6$$

10.16

$$\text{Correlation coefficient} = 0.0026 / (0.04 \times 0.07) = 0.9286$$

10.18

$$\sigma^2 = (0.10) \times (0.2 - 0.14)^2 + (0.40) \times (0.1 - 0.14)^2 + (0.40) \times (0.14 - 0.14)^2 + (0.10) \times (0.24 - 0.14)^2$$

$$\sigma^2 = 0.002$$

Given that variance, the standard deviation of JLC's stock is simply the square root of the variance:

$$\sigma = 0.045$$

Chapter 11

11.2

The salvage value is an estimate of an amount potentially achievable when the asset is retired from service.

11.5

The initial cost, estimated salvage value, estimated useful life, and depreciation method.

11.6

Annual depreciation expense:

$$\text{Annual Depreciation} = \frac{\$1,000,000 - \$100,000}{3} = \$300,000$$

Year	Year-beginning Book Value	Depreciation Amount	Accumulated Depreciation	Year-end Value
2017	\$1,000,000	\$300,000	\$300,000	\$700,000
2018	\$700,000	\$300,000	\$600,000	\$400,000
2019	\$400,000	\$300,000	\$900,000	\$100,000

11.7

a. Solutions:

$$1/5 \text{ years} = 20\% \text{ Straight Line rate} \times 2 = 40\% \text{ DDB rate}$$

Year	Year-beginning Book Value	Depreciation Expense	Accumulated Depreciation	Year-end Book Value
2017	\$1,000,000	\$400,000	\$400,000	\$600,000
2018	600,000	\$240,000	640,000	\$360,000
2019	360,000	\$144,000	784,000	\$216,000
2020	216,000	\$86,400	870,400	\$129,600
2021	129,600	\$51,840	922,240	\$77,760
2022	77,760	\$31,104	953,344	\$46,656

2017: \$1,000,000 beginning book value × 40% DDB rate = \$400,000

2018: \$600,000 beginning book value × 40% DDB rate = \$240,000

2019: \$360,000 beginning book value × 40% DDB rate = \$144,000

2020: \$216,000 beginning book value × 40% DDB rate = \$86,400

2021: \$129,600 beginning book value $\times$ 40% DDB rate = \$51,840

2022: \$77,760 beginning book value $\times$ \$15,000 residual value = \$31,104

b. Depreciation Expense \$400,000
 Accumulated Depreciation \$400,000

11.8

Year	Year-beginning Book Value	SYD	Depreciation Expense	Accumulated Depreciation	Year-end Book Value
2017	\$900,000	1/3	\$300,000	\$300,000	\$700,000
2018	\$900,000	4/15	\$240,000	\$540,000	\$460,000
2019	\$900,000	1/5	\$180,000	\$720,000	\$280,000
2020	\$900,000	2/15	\$120,000	\$840,000	\$160,000
2021	\$900,000	1/15	\$60,000	\$900,000	\$100,000

Sum-of-the-Years' denominator $[(n(n+1))/2] = (5 \times 6)/2 = 15$

\$900,000 $\times$ 5/15 = \$300,000 first 12 months' depreciation

\$900,000 $\times$ 4/15 = \$240,000 second 12 months' depreciation

\$900,000 $\times$ 3/15 = \$180,000 third 12 months' depreciation

\$900,000 $\times$ 2/15 = \$120,000 fourth 12 months' depreciation

\$900,000 $\times$ 1/15 = \$60,000 last 12 months' depreciation

Depreciation expense:

2017: (\$900,000 $\times$ 5/15) = \$300,000

2018: (\$900,000 $\times$ 4/15) = \$240,000

2019: (\$900,000 $\times$ 3/15) = \$180,000

2020: (\$900,000 $\times$ 2/15) = \$120,000

2021: (\$900,000 $\times$ 1/15) = \$60,000

11.13

Component Cost of Repair and Overhaul Approach	Amount
Aircraft historical cost	\$50,000,000
Accumulated depreciation	−\$30,000,000
Carrying value of impaired asset	\$20,000,000
Restoration cost (\$10,000,000)/Replacement cost (\$75,000,000)	13.33%
Impairment loss = carrying value (\$20,000,000) multiplied by restoration cost ratio (13.33%)	\$2,666,000

Chapter 12

12.6

Marginal cost is the first derivative of the total cost function, so take the derivative to find the MC:

$$\begin{aligned}
 MC &= 1 - 20Q + 6Q^2 \\
 \text{a. } MC(Q = 4) &= 1 - 2(4) + 6(4)^2 \\
 MC(Q = 4) &= 89 \\
 \text{b. } MC(Q = 10) &= 1 - 2(10) + 6(10)^2 \\
 MC(Q = 10) &= 581
 \end{aligned}$$

12.10

Component	Quantity Needed	Unit Cost	
Paint Material	500 gallons	\$95	\$47,500
Sheet Metal	2000 tons	\$62	\$124,000
Labor, Mechanical/Electrical	1200 hours	\$85 per hour	\$102,000
Labor, Casting	1500 hours	\$60 per hour	\$90,000
Labor, Finishing	1000 hours	\$50 per hour	\$50,000
Labor, Painting	200 hours	\$40 per hour	\$240
Estimated Total Cost			\$413,740

12.13

$$\begin{aligned}
 C_2 &= \$200,000 \left(\frac{512}{342} \right) \left(\frac{500}{2000} \right)^{0.62} \\
 C_2 &= \$200,000 (1.4971)(0.4234) \\
 C_2 &= \$200,000 (0.6339) \\
 C_2 &= \$126,780
 \end{aligned}$$

12.14

$$\begin{aligned}
 h &= 1 + 0.85 + 0.46 + 1.20 + 0.55 + 0.42 = 4.48 \\
 C_T &= h \times C_E \\
 C_T &= 4.48 \times \$400,000 \\
 C_T &= \$1,792,000
 \end{aligned}$$

12.16

Route	Total RPM	Average Load Factor	Total ASM
Chicago	450,000	75%	600,000
Dallas	574,000	82%	700,000
Los Angeles	630,000	70%	900,000
Los Vegas	595,000	85%	700,000
Phoenix	480,000	80%	600,000
Total ASM			3,500,000

$$\text{Allocation rate per ASM} = \left(\frac{\$4,200,000}{3,500,000} \right) = \$1.20$$

Route	Total ASM	Allocation Rate	Indirect Cost Allocation Per Route
Chicago	600,000	\$1.20	\$720,000
Dallas	700,000	\$1.20	\$840,000
Los Angeles	900,000	\$1.20	\$1,080,000
Los Vegas	700,000	\$1.20	\$840,000
Phoenix	600,000	\$1.20	\$720,000
Total indirect cost			\$4,200,000

Chapter 13

13.2

Airport Services reported \$2,500,000 inventory at the beginning of the year. During the same year the company acquired more inventory (May, \$1,200,000 and October, \$1,000,000). Find the cost of goods sold, if the Airport Services holding \$1,750,000 inventory at the end of the year.

13.4

Total Revenue	\$800 million
Less: Cost of goods sold	\$700 million
Gross profit	\$100 million
Less: Depreciation	\$10 million
Less: Interest expenses	\$2.40 million
Income before tax and after interest	\$87.60 million

13.8

Year	Depreciation Rate	Base \$20,000,000	Book Value \$25,000,000
1	7.50%	\$1,500,000	\$23,500,000
2	18.50%	\$3,700,000	\$19,800,000
3	14.80%	\$2,960,000	\$16,840,000
4	11.84%	\$2,368,000	\$14,472,000
5	9.47%	\$1,894,000	\$12,578,000
6	7.58%	\$1,516,000	\$11,062,000
7	6.55%	\$1,310,000	\$9,752,000
8	6.55%	\$1,310,000	\$8,442,000
9	6.55%	\$1,310,000	\$7,132,000
10	6.55%	\$1,310,000	\$5,822,000
11	4.10%	\$820,000	\$5,000,000

13.9

Total Cash flow = Operating + Investment + Financing

Total Cash flow = \$20,000,000 – 8,000,000 + 18,000,000

Total Cash Flow = \$30,000,000

13.11

Total Aviation Services (TAS) is the leading Fixed Base Operator throughout the Eastern Europe. TAS has reported the following account activities:

EBIT = \$500 million

Depreciation Expenses = \$200 million

Change in NWC = -\$25 million

Capital Expenditure -\$50 million

What is the corporation's FCF?

Chapter 14**14.8**

$$CR = \frac{CA}{CL} = 1.2$$

$$CR = 1.2 \times CL$$

$$QR = \frac{CA - \text{Inventory}}{CL}$$

$$\frac{1.2 \times CL - 100,000,000}{CL} = 1.1$$

$$1.2 \times CL - 1.1 \times CL = 100,000,000$$

$$CL = \$1,000,000,000$$

$$CA = \$1,200,000,000$$

$$NWC = \$200,000,000$$

14.9

$$\text{Annual cost of goods sold} = \$3,000,000 \times \frac{365}{30}$$

$$\text{Annual cost of goods sold} = \$36,500,000$$

$$\text{Inventory turnover} = \frac{\$36,500,000}{\$3,000,000} = 12 \text{ times per year}$$

14.10

$$\text{Current Ratio} = \frac{\$500,000 + \$1,000,000 + \$2,000,000}{\$400,000 + \$800,000 + \$600,000}$$

$$\text{Current Ratio} = 1.94$$

$$\text{Quick Ratio} = \frac{\$500,000 + \$1,000,000 + \$2,000,000 - \$2,000,000}{\$400,000 + \$800,000 + \$600,000}$$

$$\text{Quick Ratio} = 0.83$$

$$\text{Cash Ratio} = \frac{\$500,000}{\$400,000 + \$800,000 + \$600,000}$$

$$\text{Cash Ratio} = 0.28$$

14.11

$$\text{Profit Margin} = \frac{\$3\text{m} - \$1\text{m}}{\$12\text{m}}$$

$$\text{Profit Margin} = 16.67\%$$

$$ROA = \frac{\$3.0}{\$50\text{m}}$$

$$ROA = 6.00\%$$

$$ROE = \frac{\$3.0}{\$20\text{m}}$$

$$ROE = 15.00\%$$

$$\text{Dividend} - \text{Payout} = \frac{\text{Dividends Paid}}{\text{Net Income}}$$

$$\text{Payout} = \frac{\$1.0 \text{ m}}{\$3.0 \text{ m}} = 33\%$$

14.12

$$\frac{\text{Debt}}{\text{Equity}} = 1.75$$

$$\frac{\$250 \text{ million}}{\text{Equity}} = 1.75$$

$$\text{Equity} = \$142.87 \text{ million}$$

14.13

$$\text{Current Ratio} = \frac{\text{Current Assets}}{\text{Current Liabilities}}$$

$$2.0 = \frac{\text{Current Assets}}{\$15}$$

$$\text{Current Assets} = \$30 \text{ million}$$

$$\text{Cash} = 10\% \times 30 = \$3 \text{ million}$$

$$\text{Accounts Receivable} = 40\% \times 30 = \$12 \text{ million}$$

$$\text{Inventory} = \$30 - (\$3 + \$12) = \$15 \text{ million}$$

Chapter 15**15.1**

Certainty:

- ♦ The decision maker has complete knowledge about the problem and knows about the results.
- ♦ Examples are investing in government bonds or an insured savings account.

Uncertainty:

- ♦ The decision maker must make a decision for which the outcome and the probabilities of the possible outcomes are not known.

Risk:

- ♦ The decision maker must make a decision for which the outcome is not known with certainty, but can assign probabilities to the various outcomes.
- ♦ Examples are aircraft accidents or the number of passenger no-shows on a given flight.

15.4

$$\text{EMV (Build a large plant)} = 0.5 \times \$200,000 + 0.5 \times -\$180,000 = \$10,000$$

$$\text{EMV (Build a small plant)} = 0.5 \times \$100 + 0.5 \times -\$20,000 = -\$9,950$$

$$\text{EMV (Do nothing)} = 0$$

15.6

EVPI quantifies how much better a decision maker could do on this decision if he could always know when each state of nature would occur.

15.9

a. Maximax Approach—Build new plant.

Build new plant	\$122,500,000
Subcontract	\$63,000,000
Overtime	\$38,500,000

b. Maximin Approach—overtime.

Build new plant	−\$105,000,000
Subcontract	−\$7,000,000
Overtime	−\$3,500,000

c. Equally Likely Approach—Build new plant.

Build new plant	\$33,833,333
Subcontract	\$29,166,667
Overtime	\$18,666,667

15.10

$$\begin{aligned}
 \text{Increase in Price (EMV)} &= 0.3 (\$5,500,000) + 0.5 (\$3,000,000) + 0.2 (-\$500,000) \\
 &= \$1,650,000 + \$1,500,000 - \$100,000 \\
 &= \$3,050,000
 \end{aligned}$$

$$\begin{aligned}
 \text{Keep the Price Unchanged (EMV)} &= 0.3 (\$3,500,000) + 0.5 (\$2,500,000) + 0.2 (\$1,500,000) \\
 &= \$1,050,000 + \$1,250,000 + \$300,000 \\
 &= \$2,600,000
 \end{aligned}$$

$$\begin{aligned}
 \text{Reduce the Price (EMV)} &= 0.3 (\$2,250,000) + 0.5 (\$2,000,000) + 0.2 (\$1,000,000) \\
 &= \$675,000 + \$1,000,000 + \$200,000 \\
 &= \$1,875,000
 \end{aligned}$$

MyJet should choose the increase the price alternative based on the EMV.

15.12

$$\begin{aligned}
 \text{Making the Investment (EMV)} &= 0.10(\$600\text{m}) + 0.5 (\$700\text{m}) + 0.4 (800\text{m}) - \$250\text{m} \\
 &= \$60\text{m} + \$350\text{m} + \$320\text{m} - \$250\text{m} \\
 &= \$480\text{m}
 \end{aligned}$$

$$\text{Not making investment (EMV)} = 0$$

GAA should invest in the regional airport.

Glossary

A

Accounts Payable (AP): Money the company owes to another party.

Accounts Receivable (AR): Accounts receivable consists of monies due from customers as a result of a credit sale. On a balance sheet, accounts receivable is often recorded as an asset because this represents a legal obligation for the customer to pay cash for its short-term debts.

Air Carrier: Any airline that undertakes directly, by lease or other arrangement, to engage in air transportation.

Aircraft Crews Maintenance Insurance (ACMI): A lease between two parties where the first party is a lessor with an AOC responsible for the aircraft crews, maintenance, and insurance and the second party is the lessee, usually with an AOC, who is responsible for schedules, flight charges, cargo handling, crew support, flight operations, ramp handling, and aircraft servicing and fueling.

Aircraft Utilization: Aircraft utilization is calculated by dividing aircraft block hours by the number of aircraft days assigned to service on air carrier routes and presented in block hours per day.

Airport and Airway Trust Fund: Fund established by Congress to pay for improvements to the nation's airports and air traffic control system. Money in the fund comes solely from users of the system, principally via taxation of domestic airline tickets.

Airport Improvement Program (AIP): Provides grants for public-use airports that are included in the National Plan of Integrated Airport Systems (NPIAS). These grants are given to public agencies, private owners, and entities for planning and development.

Air Traffic Control (ATC): A service operated by appropriate authority to promote the safe, orderly, and expeditious flow of air traffic.

Amortization: Repayment of loan principal and interest.

Ancillary Revenue: Revenue from non-ticket sources, such as commissions gained from hotel bookings, baggage fees and on-board food, premium seat assignments, early boarding, and other services.

Annuity: A series of consecutive payments of equal amounts.

Asset: An item of economic benefit to an institution.

Available Seat Kilometer (ASK): A measure of a passenger airline's carrying capacity that is calculated as follows:

$$\text{ASK} = \text{number of seats} \times \text{number of kilometers flown.}$$

Available Seat Mile (ASM): × A measure of a passenger airline's carrying capacity that is calculated as follows:

$$\text{ASM} = \text{Number of seats} \times \text{number of miles flown.}$$

Available Ton Mile (ATM): A measure of a cargo airline's carrying capacity that is calculated as follows:

ATM = weight in non-metric tons x number of miles flown.

Average Collection Period: The average time accounts receivable have been on the book.

$$ACP = \frac{ACR \times 365 \text{ days}}{\text{Credit Sales}}$$

Average Stage Length (ASL): The ASL is the average distance flown per aircraft departure.

$$ASL = \frac{\text{Plane Miles}}{\text{Departures}}$$

B

Balance Sheet: A financial that summarizes a company's assets, liabilities, and shareholders' equity at a specific point in time.

Bankruptcy: The inability of an airline to pay its creditor is called bankruptcy. There are two basic types of Bankruptcy proceedings. A filing under Chapter 7 is called liquidation. Chapter 11 bankruptcy allows an airline to reorganize and refinance to be able to prevent final insolvency.

Block Hour: Block hours are the airline industry's basic measure of aircraft utilization. Block hour is the time from the minute the aircraft door closes at departure of a revenue flight until the moment the aircraft door opens at the arrival gate.

Book Value: The book value of an asset is its original cost minus accumulated depreciation.

Break-even Load Factor: The load factor that covers the necessary operating costs for scheduled traffic revenue.

$$\text{Break-Even}_{LF} = \frac{CASM}{R / RPM} = \frac{CASM}{Yield}$$

Break-even Point: A point where total costs and total revenue are equal.

$$Q_{B-E} = \frac{FC}{\text{Price} - VC}$$

C

Cabotage Law: The term Cabotage which has a Spanish origin, denotes the transport of passengers and cargo within the same country.

Call Option: Is a contract that gives the option holder the right, but not the obligation, to buy an agreed quantity of a particular financial asset from the seller at a particular time (the expiration date) for a predetermined price (the strike price).

Capital: Any financial income, property, or physical goods used to generate income by investing in a business.

Capital Lease: A capital lease, or financial lease, generally lasts for the life of the asset, with the present value of lease payments covering the price of the asset; and the lease generally cannot be cancelled.

Carrying Costs: Carrying costs or holding costs are the costs associated with carrying inventories. For most inventories carrying costs include the warehouse charges, damage, obsolescence, shrinkage, and relocation cost.

Certificated Air Carrier: An air carrier that is certified by the DOT to conduct scheduled or non-scheduled services interstate. The certificate issued to the air carrier by the DOT is the Certificate of Public Convenience and Necessity.

City Pair (Segment): The operation between board point and any subsequent off point within the same flight number.

Code-sharing: An arrangement where an airline may place its own code to another carrier's flight. The airline that is actually operating the flight is called the operating carrier, and the airline that is marketing the flight is called the marketing carrier. Both carriers may sell tickets for the flight.

Collar: A collar is a hedging strategy of simultaneously buying a put option and selling (writing) a call option on the same asset.

Commercial Service Airport: Airport receiving scheduled passenger service and having 2,500 or more enplaned passengers per year.

Commuter Air Carrier: A passenger air carrier operating aircraft with 30 seats or less and performing at least five scheduled roundtrips per week. It operates for hire or compensation under FAR Part 135.

Computer Reservation System (CRS): CRS systems provide travel agencies with airline schedules, fares, and seat availability. They also enable travel agents to book seats and issue tickets.

Cost of Capital: The overall financial cost to an airline, expressed as a percentage taking into account its capital structure, the interest rate it pays on debt, and the cost of its equity.

Cost per Available Seat Mile (CASM): CASM is a measure of airline efficiency. It is represented in cents and is calculated as follows:

$$\text{CASM} = \frac{\text{Operating Costs}}{\text{ASM}}$$

Current Assets: Assets that can be converted into cash without significant loss of value in a very short period of time.

D

D-check: This is also known as a Heavy Maintenance Visit (HMV), which, more or less, takes the entire airplane apart for inspection and overhaul.

Degree of Financial Leverage (DFL): Financial leverage involves using debt to finance the project, and will include higher expenses before interest and taxes; DFL is defined as the percentage change in EPS for a given percentage change in EBIT.

$$DFL = \frac{Q(P - V)}{Q(P - V) - FC - I}$$

Degree of Operating Leverage (DOL): Operating leverage involves using a large proportion of fixed costs in the operations of the firm. DOL measures the degree to which a project incurs a combination of fixed and variable costs.

$$DOL = \frac{Q(P - V)}{Q(P - V) - FC}$$

Depreciation: Reduction in the price of an asset during its estimated useful life due to wear and tear, age, deterioration, and obsolescence.

Deregulation: The term refers to the Airline Deregulation Act of 1978, which ended US government regulation of airline routes and charges.

Discount: For a debt security, discount is the difference between the promised future payment and the present value.

Dry Lease: A leasing arrangement between two airlines where the first airline (lessee) leases an aircraft from the second airline (lessor) to operate. Under dry-lease the lessee pays for the crew, fuel, and maintenance.

E

Economy of Scale: The increase in efficiency of production as the number of goods being produced increases.

Economy of Scope: Average cost reduction as a result of increasing the number of different goods produced.

Eighth Freedom of the Air: Carry passengers and cargo wholly within a foreign country.

Enplanement: The boarding of scheduled and non-scheduled service aircraft by domestic, territorial, and international revenue passengers for intrastate, interstate, and foreign commerce and that includes in-transit passengers.

Equipment Trust Certificate (ETC): A trust of investors purchases the aircraft and then “leases” it to the operator, on condition that the airline will receive title upon full performance of the lease.

Essential Air Service (EAS): The program intended to guarantee small communities a minimal level of scheduled air service. The EAS program subsidizes commuter airlines to serve the rural communities across the country that otherwise would not receive any scheduled air service.

F

Federal Air Regulation (FAR): Title 14 of the US government’s Code of Federal Regulations. The FAR covers all the rules regarding aviation in the United States.

Federal Aviation Administration (FAA): A United States government agency responsible for air safety and operation of the air traffic control system.

Fifth Freedom of the Air: The privilege for an airline that is en route to or from the state it is registered in to take on revenue passengers, mail, and freight in a second state and to transport them to a third state.

Financial Lease: A financial lease, or capital lease, generally lasts for the life of the asset, with the present value of lease payments covering the price of the asset.

Financial Leverage: A measure of the amount of debt used in the capital structure of the airlines. An airline with high leverage is more vulnerable to downturns in the business cycle because the airline must continue to service its debt regardless of how bad business is.

Financial Risk: The risk that an airline may not have sufficient cash to meet current financial obligations.

First Freedom of the Air: The privilege to fly across another state (country) without landing.

Fixed Base Operator (FBO): A commercial business granted the right by an airport to operate on the airport and provide aeronautical services such as fueling, hangaring, parking, aircraft rental, aircraft maintenance, and flight instruction.

Fixed Cost: Costs are identified as fixed if they do not change as volume changes. Fixed costs cannot be eliminated in the short run.

Form 41 Data: Information derived from airline filings with the Bureau of Transportation Statistics. Airline financial data is filed with the BTS quarterly; traffic and employment numbers are filed monthly.

Fourth Freedom of the Air: The privilege for an airline to transport revenue passengers, mail, and freight from one state to the state where it is registered.

Freight: Any commodity other than mail and passenger baggage transported by air.

Freight Ton mile: A ton mile is defined as one ton of freight shipped one mile.

Frequent Flyer Programs: A service in which airline customers accrue points corresponding to the distance flown on an airline. These points can be used for free air travel, increased benefits such as airport lounge access, or priority bookings, and other products or services.

Future Value: The value that a current investment grows to at a given interest rate over a specified period of time.

Future Value of an Annuity: The future value of a series of consecutive equal payments.

G

Goodwill: Goodwill is a business advantage of customer loyalty and patronage developed with continuous business under the same name over a period of time.

Gross Domestic Product (GDP): The total output of goods and services produced by all the different productive resources within a given country, valued at market prices.

H

Half-life: A component, such as a jet engine, is in half-life condition when it is exactly midway between scheduled overhaul or replacement.

Hedging: Hedging is similar to buying insurance policies intended to minimize and transfer risk.

Hub and Spoke: Many airlines designate an airport as a hub through which they transit passengers from spoke (origin) to spoke (destination).

I

Income Statement: A financial statement that summarizes the various transactions of an airline during a specified period, showing the net profit or loss. Income statement is based on a fundamental accounting equation:

$$\text{Income} = \text{Revenue} - \text{Expenses}$$

Internal Rate of Return (IRR): The IRR is the discount rate that makes the net present value of all cash flows from a project equal to zero.

In transit Passengers: Revenue passengers on board international flights that transit an airport for non-traffic purposes domestically.

J

Joint Aviation Authorities (JAA): The JAA was an associated body of the European Civil Aviation Conference (ECAC) representing the civil aviation regulatory authorities of a number of European States who had agreed to cooperate in developing and implementing common safety regulatory standards and procedures.

Just in Time (JIT): JIT (also known as lean production or stockless production) is an inventory strategy that stresses taking possession of inventory just before the time it is needed for production.

L

Launch Customer: The carrier who places the first order for an aircraft.

Lease: A lease is a contract granting use or occupation of property during a specified period in exchange for specified lease payments.

Lessee: A person who leases a property from its owner (lessor).

Lessor: The owner of an asset who grants another party a lease on the asset.

Leverage: The use of debt to supplement investment.

Liquidity: The ability of an asset to be converted into cash quickly and without any loss of value.

Load Factor: Load factor is the ratio of revenue passenger miles over available seat miles, representing the proportion of aircraft seating capacity that is actually sold and utilized.

M

Major Airlines: Airlines earning revenues of \$1 billion or more annually in scheduled service.

Maximum Certificated Take-Off Weight (MCTOW): The maximum weight at which the pilot of the aircraft is allowed to attempt to take off, because of the aircraft's structural limitation.

Maximum Zero-Fuel: The total weight of the airplane minus the total weight of usable fuel on board.

Modified Accelerated Cost Recovery System (MACRS): MACRS allows for more depreciation toward the beginning of the life of the capital asset, allowing the tax deductible depreciation expense to be taken sooner.

N

National Plan of Integrated Airport Systems (NPIAS): A program that identifies more than 3,300 airports that are eligible to receive Federal grants under the Airport Improvement Program (AIP) based on their significance to national air transportation.

Net Present Value (NPV): The Net Present Value (NPV) equals the total present value of all cash inflows and outflows.

Net Profit Margin: Net profit (or loss) after interest and taxes as a percent of operating revenues.

Non-scheduled Service: Revenue flights not operated as regular scheduled service, such as charter flights, and all non-revenue flights incident to such flight.

O

Operating Expenses: Expenses incurred in the performance of air transportation, based on overall operating revenues and expenses.

Operating Lease: A short-term lease. For example, an aircraft which has an economic life of 30 years may be leased to an airline for four years on an operating lease.

Operating Leverage: A measure of the extent to which fixed assets are utilized in the business firm.

Operating Profit Margin: Operating profit (i.e. operating revenues minus operating expenses) as a percent of operating revenues.

Operating Revenues: Revenues from air transportation and related incidental services.

Options: An option is a contract that gives the holder the right but not the obligation to buy or sell a security at a predestined price for a given period of time.

Overbooking: When more seats have been booked on a flight than seats allowable for sale.

Oversale: When a passenger who has a valid completed ticket or of whom the airline has a positive reservation record is not accommodated at flight departure.

P

Par Value: Par value is the same as the face value, the principal value or maturity value of the bond. The par value is the amount of money the investor will receive once the bond matures.

Passenger Load Factor: Load factor is the ratio of revenue passenger miles over available seat miles, representing the proportion of aircraft seating capacity that is actually sold and utilized.

$$LF = \frac{RPM}{ASM} \times 100$$

Passenger Name Record (PNR): A record of each passenger's travel requirements which contains all information necessary to enable reservations to be processed and controlled by the booking and participating airlines.

Passenger Revenue per Available Seat Mile (PRASM): The average revenue received by the airline per unit of capacity available for sale.

Payback Period: The Payback Period represents the time required to recover an initial investment.

Perpetuity: A cash flow without a maturity date.

Preferred Stocks: A hybrid security combining some of the characteristics of bonds and common stocks but usually carrying no voting rights.

Primary Market: The market for raising of new capital for the first time.

Prime Rate: The rate that a commercial bank charges its most creditworthy customers.

Profitability Ratios: A group of ratios that are used to assess the return on assets, sales, and invested capital.

Proration: Proration is the division of passenger ticket revenue between two or more different flight legs which a passenger flies.

Put Option: A put option is a contract that gives the holder the right but not the obligation to sell a security at a predestined price for a given period of time.

R

Return on Investment (ROI): The percentage amount that is earned on a company's total capital, calculated by dividing the total capital by the earnings before interest, taxes, or dividends.

Revenue Management: A management approach to optimizing revenue that is based on managing revenues for different market segments or from different sources of funding around capacity and timing (yield management).

Revenue Passenger Enplanement: The total number of revenue passengers boarding aircraft including origination, stopover or connecting passengers.

Revenue Passenger Load Factor: (see Passenger Load Factor) Revenue passenger load factor is the ratio of revenue passenger miles over available seat miles, representing the proportion of aircraft seating capacity that is actually sold and utilized.

Revenue Passenger Mile (RPM): RPM is computed by the summation of the products of the revenue aircraft miles flown by the number of revenue passengers carried on that route. RPM is a principal measure of an airline's turnover.

Revenue per Available Seat Mile (RASM): The revenue in cents received for each seat mile offered. This is computed by dividing operating income by Available Seat Miles and is not limited to ticket sales revenue.

Revenue Ton Mile (RTM): One non-metric ton of revenue traffic transported one mile.

Risk Aversion: The propensity of investors or individuals to avoid risk unless it is properly compensated.

Risk Free Assets: Assets free of risk of default.

Risk Premium: The minimum difference an investor requires investing in a risky investment.

S

Safety Stocks: The amount of extra inventory needed to counter the variation in demand for an item during the replenishment lead time.

Sale and Leaseback: Arrangement in which an airline sells an aircraft to a buyer and the buyer immediately leases the aircraft back to the airline.

Seat Pitch: The distance between one seat in the aircraft and the same point on another seat directly in front or behind.

Secondary Market: The markets for securities that have already been issued and traded among investors with no proceeds going to the company.

Second Freedom of the Air: The privilege to land in another state for non-traffic purposes such as refueling or mechanical issues.

Securities and Exchange Commission (SEC): US Securities and Exchange Commission is a governmental agency responsible for enforcing federal securities laws, proposing securities rules, and regulating the securities industry, the nation's stock and options exchanges, and other activities and organizations, including the electronic securities markets in the United States.

Seventh Freedom of the Air: The privilege for an airline to pick up revenue passengers and freight from one state and take them to another without originating, stopping, or terminating in the state of registration.

Sixth Freedom of the Air: The privilege for an airline to transport revenue passengers, mail, and freight from one state to another, while passing through its state of registry.

Small Certificated Air Carrier: An air carrier holding a certificate issued under section 401 of the Federal Aviation Act of 1958, as amended, that operates aircraft designed to have a maximum seating capacity of 60 seats or fewer or a maximum payload of 18,000 pounds or less.

Stage 2: Term used to describe jets meeting certain noise parameters on takeoff and landing.

Stage 3: Term used to describe the quietest jets in service today.

Straight Line Depreciation: A method of depreciation that is calculated by taking the purchase price of an asset less the salvage value and dividing by the total productive years the asset can be reasonably expected to benefit the company.

Sunk Costs: A sunk cost is a cost that has already been incurred and consequently cannot be recuperated.

Supplemental Air Carrier: An air carrier authorized to perform passenger and cargo charter services.

Swap: Transactions involving an exchange of cash flows between two parties, such as interest rate swap (fixed rate and floating rate).

T

Third Freedom of the Air: The privilege to put down revenue passengers, mail, and freight taken from the state where the airline is registered to another state.

Treasury Bills (or T-bills): Treasury bills are short-term obligations of the federal government. They mature in one year or less and, like zero coupon bonds, they do not pay interest prior to the maturity.

Treasury Notes (T-notes): Treasury notes are short-term obligations of the federal government and they mature in two to 10 years.

U

US Flag Carrier: One of a class of air carriers holding a Certificate of Public Convenience and Necessity issued by the US Department of Transportation (DOT) and approved by the President, authorizing scheduled operations over specified routes between the United States and one or more foreign countries.

V

Variable Costs: A variable cost is a cost that changes in proportion to a change in a company's activity or business.

W

Walk-away Lease: A lease that allows the airline to return the aircraft at specified times without penalty.

Warrant: A security issued by a corporation along with a bond or preferred stock, giving the holder the right to buy a specific amount of securities at a specific price.

Weighted Average Cost of Capital (WACC): WACC is determined by simply multiplying the cost of each item of capital by its weighted representation in the overall capital structure.

$$WACC = \frac{D}{A} \times k_d \times (1 - T) + \frac{E}{A} \times k_e$$

Where:

A = Market value of the company

E = Market value of the firm's equity

Wet Lease: A leasing arrangement between a lessor and a lessee to lease an aircraft and crew from the second party which operates services on behalf of the first party.

White Tail: An unsold aircraft that has been built by an aircraft manufacturer.

Wide-body Aircraft: A commercial aircraft with two aisles.

Working Capital: The amount of liquidity asset an airline has on hand to meet its short-term financial obligations.

Y

Yield: A unit of airplane prices, defined as average revenue per revenue passenger mile or revenue ton mile.

Z

Zero Coupon Bonds: Zero coupon bonds are sold at discount and don't make regular interest payments like other bonds do. The bond holder receives all the interest in one lump sum when the bond matures.

Index

Please note that page references to non-textual content such as Figures will be in italics, followed by the letter 'f', whereas those for Tables will be in italics followed by the letter 't'.

- Abacus (GDS), Southeast Asian Airlines 20–1
- abandonment 308–11
- Aberdeen Airport (ABZ) 18
- accept and reject criteria: annual worth (AW) analysis *148t*; benefit to cost analysis *201t*, *203t*; present worth (PW) analysis *117t*; rate of return (ROR) analysis *170t*, *177t*, *189t*
- accounts payable (AP) *379t*, 383, 564
- accounts receivable (AR) 380–1, 564
- accounts receivable turnover ratio (ART) 405–6
- accrued expenses 383
- ACI *see* Airport Council International (ACI)
- acid-test ratio (quick ratio) 404–5
- AerCap 16
- air cargo 65, 375
- air carriers 2, 20, 564; *see also* low-cost carriers (LCCs)
- Air Lease Corporation (ALC) 17
- air navigation service providers (ANSP) 19
- Air New Zealand 297
- air traffic control (ATC) 19, 564
- Airbus (commercial airline manufacturer) 10, 11; A350 Airbus 320; A380 Airbus 12, 22, 468–9; and Boeing 12, 13, 22, 154; income statement 365; order book 458; portfolios 265
- aircraft crews maintenance insurance (ACMI) 564
- aircraft leasing companies 16–17, 553–4; annuities 60
- aircraft manufacturers 10–14; duopoly (Airbus and Boeing), 12, 13, 22; regional, 13–14; *see also* Airbus (commercial airline manufacturer); Boeing Company (aircraft manufacturer); *other specific airlines and manufacturers*
- aircraft utilization 2, 20, 564, 565
- airline break-even load factor 229–31, 433–4, 565
- airline cost structure 354–8
- Airline Deregulation Act 1978, US 3, 5, 27, 567
- airline mergers 4,5, 23–5, 27, 126
- airline reservation system (ARS) 19
- airline/air transport industry 1–30; prior to deregulation 2–3; after deregulation 3–6; abandonment issues 309; aircraft leasing companies 553–4; cost of aircraft 93–4; discussion questions and problems 27–8; first flight (1903) 24; global 8; indirect costs 338; key financial ratios for 428–33; profitability issues 6–8, 377; top 50 commercial airports 531–2; vital statistics 529–30
- Airlines for America (A4A) 355
- Airport and Airway Trust Fund 564
- Airport Council International (ACI) 564
- airport improvement program (AIP) 196, 564, 570
- airports 18, *19f*, 23; on-airport activities 198; benefits of an international airport 198–9; capital improvement projects (CIP) 198; commercial 198–9; community impacts 199; development 198; disbenefits 199; global air transportation networks 198; and industrial development 198; infrastructure 38; “stacking” 159; and supply chains 199; top 50 commercial airports 531–2; visitor categories 198; *see also specific airports, such as* London Heathrow (LHR)
- Airports Council International (ACI) World Airport Traffic Report 18
- Alaska Air Group 25, 429
- Alchian, A. 339
- Altman, Edward 427
- Altman insolvency index 427–8
- Altman Z-Score 427
- Amadeus (European GDS system) 20, 21
- America West 5
- American Airlines (AA) 4, 8, 20, 25, 304, 403; ratio analysis 417, 418
- American International Group (AIG) 16
- amortization 69–71, 564
- amortization schedule 70
- ancillary revenue 564
- annual equivalent rate (AER) *see* effective interest rates
- annual worth (AW) analysis 146–66; accept and reject criteria *148t*; annuity of net annual cash flows 153–4; and break-even

- analysis 227, 228; capital recovery 152–3, 162; congestion reduction 154; description 146, 161; DirectJet Airline, examples concerning 133, 134, 135, 136*t*, 137*t*; discussion questions and problems 162–5; of equal-life alternatives 155; evaluation of single projects 148–54; independent projects (IP), evaluating 157–8; least common multiple (LCM) approach 147, 148; motivation for using 147–8; mutually exclusive projects, evaluating 154–7; net annual cash flows (NACF) 149, 153–4, 160, 162; of non-equal alternatives 155–7; permanent projects 159–60; rate of return analysis compared 169; spreadsheet calculations 160–1; *see also* present worth (PW) analysis
- annuities 58–91; aircraft leasing companies 60; amortization 69–71, 88; annuities due 60, 71; capitalized cost (CC) 71–3; definitions/description 59, 87, 564; DirectJet Airline, examples concerning 65, 71; discussion questions and problems 88–91; equivalent annual annuity approach *see* capitalized cost (CC); formulas 60; future value 61–4, 568; geometric gradient series *see* geometric gradient series; net annual cash flows (NACF) 153–4; ordinary 60, 87; present value 64–8, 88; shifted cash flow series 82–4, 88; time intervals for annuity payments 58, 59–60; types 60; *see also* arithmetic gradient series
- Apollo (United Airlines' in-house CRS) 20
- arithmetic gradient series 73–9; annuity of 76–7; future value 78–9; present value 73–5; *see also* geometric gradient series
- ARJ-21 (turbofan short-medium range regional aircraft), Chinese Comac 14
- asset management ratios 405–9, 410*f*;
 accounts receivable turnover ratio (ART) 405–6; average collection period (ACP) 406; fixed asset turnover ratio (FAT) 408; inventory turnover (ITO) 407–8; total asset turnover ratio (TAT) 408–9, 410*f*
- asset replacement 303–6
- assets 564
- Atlanta (ATL) airport 18
- Australian and International Pilots Association (AIPA) 296
- available seat kilometer (ASK) 564
- available seat mile (ASM) 230, 353, 429, 564
- available ton kilometer (ATK) 231
- available ton mile (ATM) 565
- average collection period (ACP) 406, 565
- average daily block hour utilization 432–3
- average fixed cost (AFC) 325, 328–9
- average load factor 353
- average stage length (ASL) 432, 565
- average total cost (ATC) 325–8
- average variable cost (AVC) 325, 329–30
- aviation industry *see* airline/air transport industry
- aviation value chain: downstream sector 10; upstream sector 9–10
- AW *see* annual worth (AW) analysis
- B787 (Dreamliner) aircraft, Boeing 118–19, 154, 220–1, 343
- balance sheet 377–8, 379*t*, 380–5, 565;
 common size 392; current assets 377–82, 566; current liabilities 383–4; fixed assets 382–3; non-current liabilities 384; stakeholders' equity 384–5; total liabilities 383
- balking 462
- bankruptcies 4, 23–5, 380, 565
- banks: central banks 39; European Central Bank (ECB) 42; Federal Reserve Bank, US 39, 40; London Interbank Offered Rate (LIBOR) 40–1
- bargaining power 22
- Beijing Airport (PEK) 18
- benefit to cost analysis: accept and reject criteria 201*t*, 203*t*; computation, spreadsheet use 212–13; conventional ratio 200; definitions/description 200; Do-nothing (DN) alternative 203, 204, 206, 207, 214; minimum acceptable rate of return (MARR) 200, 202, 203, 208; modified B/C ratio 200, 210–12, 214; public sector projects 200–2, 210–12; sensitivity analysis 272; *see also* cost-benefit analysis (CBA); public sector projects and benefit to cost analysis
- Bethune, Gordon 354
- biodiversity impacts, airports 199
- Black-Scholes option pricing model 244
- Blank, L. 168
- block hours 565
- Boeing, William 11
- Boeing Company (aircraft manufacturer) 10, 11–12; and Airbus 12, 13, 22, 154; aircraft on order 173–4; B737 303–4, 319; B747, 317, 469; B787 (Dreamliner) aircraft 118–19, 154, 220–1, 343; cash and cash equivalents 380; cost of aircraft 93–4; evaluation of mutually exclusive projects 154; fixed assets 382, 383; income statement 365, 366*t*; inventories 381; Next-Generation aircraft (Boeing 737–800) 283, 287, 307; non-current liabilities 384; operating expenses 366*t*, 370; order book 458; portfolios 265; retained earnings 385

- Bombardier (regional aircraft manufacturer) 13, 114, 173
- bonds: annuity of future value 63; corporate 45–6; grading of 45; junk 252; maturity date 63, 120; municipal 44; rate of return (ROR) analysis 173–5; rates and maturity 46*f*; revenue 44; State and local government 43; Treasury Bonds (T-Bonds) 43, 55; valuation of 120–6; zero coupon bonds 574
- branches, decision trees 443–5
- break-even analysis 219–31; airline break-even load factor 229–31, 433–4, 566; break-even point 220–1; comparisons of alternatives 219, 225–6, 228–9; definitions/description 219, 237; discussion questions and problems 238–41; elements 221–9; fixed costs (FC) 221; mutually exclusive alternatives with interest rate greater than zero 228–9; non-linear variable costs 222, 223*f*; payback period analysis *see* payback period analysis; and sensitivity analysis 272–5; single projects, interest equal to zero 223–5; single projects, interest rate greater than zero 226–8; two alternatives with interest rate of zero 225–6
- break-even analysis and payback period, DirectJet Airline, examples concerning 230
- break-even load factor (LF_{be}) 229–31, 433–4, 566
- break-even point 566
- Bregier, F. 438
- British Airlines, privatization 6
- British Airports Authority (BAA) 6, 18
- Buffet, Warren 23–4
- Bureau of Air Commerce, US 2
- Bureau of Transportation Statistics 28n12, 29n39, 437n15, 568
- business decisions and probability 244–9
- C919 (short-medium range commercial aircraft), Chinese Comac 14
- Cabotage Law 566
- call options 566
- capital 566; additional paid-in 385
- capital (financial) leases 60, 566, 567
- capital asset pricing model (CAPM) 267–72, 334; security market line (SML) 269–72, 279
- capital improvement projects (CIP) 198
- capital recovery (CR) 152–3, 162
- capitalized cost (CC): annual worth (AW) analysis 159; annuities 71–3; present worth (PW) analysis 133–5
- CAPM *see* capital asset pricing model (CAPM)
- car loans 60, 87
- car rentals 34
- cargo, air 65, 375
- carrying costs 566
- cash advance (payday loans) 36, 93
- cash flows: constant *see* annuities/gradient series; diagram 50; from financing activities 387, 388*t*, 389*t*; free cash flow (FCF), measuring 390, 392–5; incremental 176–8; from investing activities 387; net annual cash flows (NACF) 114, 153–4, 232–3; non-uniform net annual cash flows 233–4; from operating activities 386–7; shifted cash flow series 82–4; statement 386–9; terminal cash flow (TCF) 114–15; time line 50; unequal series, future value 52; uniform net annual cash flows 232–3; *see also* project cash flows, measuring
- catering 17, 18
- CC *see* capitalized cost (CC)
- Cederholm, T. 167
- central banks 39
- certainty, decisions under 244, 249, 447–56; and uncertainty 447–54
- certificated air carriers 566
- certificates of deposit (CD) 41–2, 380, 447
- CFM International (CFMI) 15
- chance nodes 443
- Chapter 11 bankruptcy, US 24
- Charlotte Douglas International Airport 44
- China: civil aviation industry 18; short-medium range aircraft 14; TravelSky (Chinese GDS system) 21; *see also* Commercial Aircraft Corporation of China (Comac)
- Cirrus SR22 aircraft 139
- city pair (segment) 566
- Civil Aeronautics Act 1938, US 2
- Civil Aeronautics Administration, US 2
- Civil Aeronautics Authority, US 2
- Civil Aeronautics Board (CAB), US 2, 3, 25–6
- Clean Air Act/Clean Water Act, US 197
- climate changes, airports 199
- Cobb–Douglas production function 342
- code-sharing 566
- collar (hedging strategy) 566
- collateral, corporate bonds 45
- Commercial Aircraft Corporation of China (Comac) 13, 14, 458
- commercial aircraft manufacturer mergers, US 11*f*
- commercial loans 88, 93, 94
- commercial service airports 566
- commercialization, ANSPs 19
- common stock 385
- commuter air carriers 566
- compound interest 36–8, 54; amortization 69; arithmetic gradient series 75*t*, 77*t*; calculus

- 97; continuous compounding 95, 97; description 33, 36; frequency of compounding 95; matching compounding period to payment period 99–100; rule of 72 53; tables 48, 51, 75*t*, 77*t*, 473–527*t*; time frames 92; *see also* compounding period (CP)
- compounding period (CP): comparing with payment period 99–105, 109; matching to payment period 99–101; payment period equalling 101–2; payment period exceeding 102–4; payment period less than 104–5
- computer reservation systems (CRS) 20, 21, 566
- Concorde 3, 332, 438, 469
- Condit, Phil 22
- congestion reduction 154
- conservative approach (maximin), decision making 449–50, 452
- consortiums 15
- Consumer Price Index (CPI) 346
- Continental Airlines 5, 380
- corporate bonds 45–6
- cost capacity equation 349–50
- cost estimation, engineering 317–62; airline cost structure 354–8; average fixed cost (AFC) 325; average total cost (ATC) 325–8; average variable cost (AVC) 325, 329–30; bottom-up approach, unit cost method 344–5; cost capacity equation 349–50; cost divisions and categories 337–8; cost factor method 351–2; cost index method 346, 347*t*, 348–9; cubic cost function 324; debt, cost of 335; definitions/description 319–33; development of cost estimates 341–3; direct costs 337–8; discussion questions and problems 359–61; economic and accounting cost, relationship between 334; equity, cost of 335; financial costs 370–2; fixed costs (FC) 319–20, 568; indirect cost estimation, engineering and allocation 352–3, 354*t*; indirect costs 338; learning curves 339–41, 358; marginal costs 324–5; non-operating costs (NOC) 358; non-pecuniary/implied cost 334; operating costs (OC) *see* operating costs (OC) 358–9; opportunity costs 332–3; return on invested capital vs weighted average cost of capital 335–6, 337*t*; short-run cost schedule 322*t*; sunk costs 331–2, 572; techniques 343–58; top-down approach, unit cost method 344; unit cost method 344–6; variable costs (VC) 320–1, 573; weighted average cost of capital (WACC) 335–6, 337*f*, 573; *see also* expenses; total cost (TC)
- cost factor method 351–2
- cost index method 346, 347*t*, 348–9
- cost of capital 566
- cost of goods sold (COGS) 367–8
- cost per available seat mile (CASM) 230, 328, 429–30, 566; stage length adjusted 432
- cost-based project alternatives: present worth (PW) analysis 127; public sector projects 202–4; rate of return (ROR) analysis 178–9
- cost-benefit analysis (CBA) 272; *see also* benefit to cost analysis
- cost-capacity trade-off 467–8
- coupon rate 43; *see also* zero coupon bonds
- CP *see* compounding period (CP)
- credit cards 40, 93; companies 108–9
- credit rating agencies 45, 334
- CSeries airline family (Bombardier) 114
- cubic cost function 324
- cumulative distribution function (CDF) 246
- cumulative probability distribution 246
- current assets 377–82, 378, 566; accounts receivable (AR) 380–1, 564; cash and cash equivalents 378, 380; inventories 381; marketable securities (short-term investments) 380; prepaid expenses 381–2
- current liabilities 383–4
- current ratio (CR) 402–3, 435
- customer arrivals 459–62; arrival characteristics 462; arrival patterns 462; arrival rate 459; degree of patience 462; distribution of 459–61; size of arrival units 462; waiting line characteristics 462–4
- Dash 80 aircraft (707s) 11–12
- DC-6 (military aircraft) 10
- D-Check 566
- debt: cost of 335; financing 63; long-term, and interest rates 42–6; long-term, current portions of 384; *see also* interest/interest rates
- debt ratio (D/A) 410–12
- debt to equity ratio (D/E) 412–13, 435
- decision branches 443
- decision-making 243–4; asset replacement 303–6; business decisions and probability 244–9; conservative approach (maximin) 449–50, 452; cost-capacity trade-off 467–8; decision environments 445–7; decision theory 438–72; decision trees 441–5; decisions under certainty 447–56; DirectJet Airline, examples concerning 442, 448, 449*t*, 450, 451*t*; discussion questions and problems 469–71; economic theory of information 456–7; equally likely approach 450–1, 453; minimax regret approach 450, 453; optimistic approach (maximax) 448–9, 452; probability and business decisions

- 244–9; process 440–1; queuing theory 458–68; under risk 454–6; service facility characteristics 464–6; theory 438–72
- decision nodes 442
- decision trees 441–5; branches 443–5; nodes 442–3
- declining balance (DB) depreciation 289, 311
- default, risk of 45
- degree of financial leverage (DFL) 566
- degree of operating leverage (DOL) 567
- Delta Air Lines 5, 24, 33, 373, 377, 403, 411;
 - accounts payable (AP) 379*t*, 383; accounts receivable turnover ratio (ART) 406;
 - average collection period (ACP) 406;
 - balance sheet 378*t*, 379*t*; cash flow statement 388–9*t*; Chapter 11 bankruptcy protection 410; debt to equity ratio (D/E) 412; dividend payout ratio (DPR) 419;
 - gross profit (operating profit) 414; income statement 373–4*t*; inventory turnover (ITO) 407; market to book ratio 420; net profit margin (NPM) 415; operating profit margin (OPM) 415; return on assets (ROA) 422; return on total assets (ROA) 417; revenue per available seat mile (RASM) 430
- Delta carrier, US 4
- demand deposits 41–2
- Department of Justice (DOJ), US 25
- depreciation 283–94; and abandonment 308–11; aircraft leasing 60; comparison of different approaches 294–7; declining balance 289, 299, 311; definitions / description 284, 567; DirectJet Airline, examples concerning 286, 287, 291, 297, 307, 310; discussion questions and problems 312–14; double declining balance (DDB) 295, 299; and impairment 306–8; income statements 369–70; methodologies 285–94; modified accelerated cost recovery system (MACRS) 290–2, 295, 311, 369, 569; replacement decisions 303–6; requirements for properties 285; salvage value (terminal cash flow) 286, 287, 288, 289, 291, 292, 294, 295, 297, 303, 307, 310, 311; spreadsheet calculations 297–303; straight-line 286–9, 295, 297, 311, 572; sum-of-the-years (SYD) method 292–4, 311
- deregulation: airline industry prior to 2–3; airline industry following 3–6, 27; definitions / description 567
- different-life alternatives, evaluating: annual worth (AW) analysis 155–7; capitalized cost (CC) 133–5; least common multiple (LCM) approach 132–4; mutually exclusive 130–5; study period approach 131, 132*t*; *see also* equal-life alternatives, evaluating
- direct costs 337–8
- direct operating costs (DOC) 355–6
- DirectJet Airline, examples concerning 35;
 - annual worth (AW) analysis 133, 134, 135, 136*t*, 137*t*; annuities / gradient series 65, 71;
 - break-even analysis and payback period 230; decision making 442, 448, 449*t*, 450, 451*t*; depreciation 286, 287, 291, 297, 307, 310; engineering cost estimation 327, 328, 332, 333, 352; financial statements 371, 372*t*, 390; present worth (PW) analysis 125, 131; rate of return (ROR) analysis 169, 170, 174, 175, 180, 181; ratio analysis 403, 409, 416, 424, 426, 427; risk analysis 243, 246, 252, 254, 261, 263, 268, 269
- disbenefits 197, 199
- discounting process 49–50
- discounts: definitions / description 567;
 - discount rate 39, 54, 60; discounted payback period analysis 220, 234–5
- discussion questions and problems: airline / air transport industry 27–8; annual worth (AW) analysis 162–5; annuities / gradient series 88–91; break-even analysis and payback period 238–41; cost estimation, engineering 359–61; decision making 469–71; depreciation 312–14; engineering economics and finance 55–6; financial statements 396–7; interest / interest rates 109–11; present worth (PW) analysis 141–5; public sector projects and benefit to cost analysis 214–17; rate of return (ROR) analysis 191–4; ratio analysis 435–7; solutions to 537–64
- distribution: cumulative distribution function (CDF) 246; exponential 460; inverse cumulative distribution 248*f*; joint probability distribution 246–9; normal *see* normal distribution; Poisson distribution 461; *see also* global distribution systems (GDS)
- dividend discount model (DDM) 334
- dividend payout ratio (DPR) 418–19
- dividend ratios 418–20
- dividend yield 419–20
- DN *see* Do-nothing (DN) alternative
- Don Quijote Airport, Spain 309
- Do-nothing (DN) alternative 128, 550; benefit to cost analysis 203, 204, 206, 207, 214; present worth (PW) analysis 176, 179, 180, 181, 182, 191
- Do-nothing alternative 128
- Doric Nimrod Air France Alpha Limited 126
- double declining balance (DDB),
 - depreciation 295, 299
- dry-leases 60, 567
- Dubai Airshow (2015) 173

- Dubai International Airport 18
- DuPont system ratios 421–5; return on assets (ROA) 422; return on equity (ROE) 422–5
- earnings before interest and taxation (EBIT) 6, 288, 371, 372, 390, 413
- earnings before taxes (EBT) 371–2
- earnings per share (EPS) 288, 372, 421
- earnings statements *see* income statements
- Eastern Airlines 45, 380
- economic and accounting cost, relationship between 334
- economic theory of information 456–7
- economy of scale 567
- economy of scope 567
- Edinburgh Airport (EDI) 18
- effective interest rates: calculating 94–9; continuous compounding 95; description 93; effective rate to nominal rate 97, 99; vs nominal 93–4; nominal annual interest rate for selected effective annual rates 98*t*; nominal interest rate for various effective rates 98*f*; quarterly 95; semi-annual 95
- eighth freedom of the air 567
- Embraer (regional aircraft manufacturer) 13
- Emirates Airlines 23, 173, 283, 296, 369
- engineering cost estimation *see* cost estimation, engineering, DirectJet Airline, examples concerning 327, 328, 332, 333, 352
- engineering economics and finance:
 - definition of engineering economics 32–3;
 - discussion questions and problems 55–6;
 - foundations 31–57; interest rates *see* interest rates; long-term interest rates;
 - short-term interest rates; present value of a single future sum 49–52; rule of 72 53; rule of 100 53–4; single present amount, future value 47–9
- Engineering News-Record (ENR) 346
- Enhanced Equipment Trust Certificates (EETCs) 126
- enplanement 567
- equal-life alternatives, evaluating: annual worth (AW) analysis 155; mutually exclusive 128–30; *see also* different-life alternatives, evaluating
- equal-life service requirement, rate of return 181–3
- equally likely approach, decision making 450–1, 453
- equipment trust certificate (ETC) 567
- equity, cost of 335
- equivalences computation: compounding period compared to payment period 99–105; involving only single-amount factors 99–101; matching compounding period to payment period 99–101; payment period equalling compounding period 101–2; payment period exceeding compounding period 102–4; payment period less than compounding period 104–5; spreadsheet calculations 105–8; using spreadsheets for 84–7
- equivalent annual annuity approach *see* capitalized cost (CC)
- Erlang, Agner Krarup 458
- essential air service (EAS) 567
- European Aeronautic Defence and Space Company (EADS) 12
- European Central Bank (ECB) 42
- European Civil Aviation Conference (ECAC) 569
- event branches 443
- event nodes 443
- Excel functions 84; benefit to cost analysis 212, 213; nominal and effective interest rates 105, 106*f*, 107*f*; present worth (PW) analysis 122–3, 124, 125*f*, 137; rate of return (ROR) analysis 189, 190, 191; variance and standard deviation 254, 255; *see also* spreadsheet calculations
- exclusion 456
- expected monetary value (EMV) 454
- expected return/mean 251–3, 254*t*
- expected value of perfect information (EVPI) 456–7
- Expedia 21
- expenses: accrued 383; operating 369, 570; prepaid 381–2; variable 337; *see also* cost estimation, engineering
- exponential distribution 460
- extended-range twin operations (ETOPS) 13
- EZjET Airlines: amortization schedule 69*t*; annual worth (AW) analysis 160; annuity of a present value 68; depreciation 289; engineering cost estimation 348; ratio analysis 408, 414; risk analysis 269
- F13s (commercial aircraft) 10
- Federal Air Regulation (FAR) 567
- Federal Aviation Act 1958, US 3, 572
- Federal Aviation Administration (FAA) 18, 196, 567
- federal funds rate, US 39, 40*f*, 54
- Federal Reserve Bank, US 39, 40
- Federal Reserve System (FED), US 39
- fifth freedom of the air 568
- financial costs 370–2
- financial institutions 38–9
- financial leases *see* capital (financial) leases
- financial leverage 568
- financial management ratios: debt ratio (D/A) 410–12; debt to equity ratio (D/E)

- 412–13; times interest earned ratio (TIE) 413
- financial risk 568
- financial statements 363–97; balance sheet *see* balance sheet; cash flow *see* cash flow statement; common size 390; description 363; DirectJet Airline, examples concerning 371, 372*t*, 390; discussion questions and problems 396–7; income statements *see* income statements; preparation for fiscal year 284; tax rates for 2016 397–8
- financing activities, cash flow from 387, 388*t*, 389*t*
- first come, first served (FCFS) 463
- first cost (P) 113–14
- first freedom of the air 568
- Fisher, Irving 33
- Fitch 45
- fixed asset turnover ratio (FAT) 408
- fixed assets 378, 382–3; *see also* depreciation; long-term assets
- fixed base operator (FBO) 219, 568
- fixed costs (FC) 319–20, 568; average fixed cost (AFC) 325, 328–9; *see also* cost estimation, engineering; variable costs (VC)
- fixed direct operating costs (FDOC) 356
- Flydubai Airline 283
- foreign bank accounts, US 378–9
- Form 41 data 568
- foundations of engineering economics and finance *see* engineering economics, foundations
- fourth freedom of the air 568
- Francis, David 112
- free cash flow (FCF), measuring 390, 392–5, 393–5*t*
- freedom of the air: first 568; second 572; third 573; fourth 568; fifth 568; sixth 572; seventh 572; eighth 567
- freight 568
- freight forwarders and integrators 22–3
- freight ton-mile 568
- frequent flyer programs 568
- future value: annuities 61–4, 568; arithmetic gradient series 78–9; geometric gradient series 81–2; single present amount 47–9; unequal cash flow series 52; *see also* present value
- Galileo (European GDS system) 20
- Garcia, Marisa 8–9
- Gatwick airport (LGW) 18
- GE Capital Services (GECAS) 16, 17, 265
- General Electric (GE) 15
- geometric gradient series 79–82; annuity of 81; future value 81–2; present value 79–81; *see also* arithmetic gradient series
- Glasgow Airport (GLA) 18
- global air transportation networks 198
- global airline industry 8
- global distribution systems (GDS) 19–22; bargaining power 22; functional components, integrated 20*f*; online travel agents (OTA) 21–2
- global financial crisis (2008) 7
- goodwill 568
- graveyards, plane 309
- Greece, debt crisis 43
- gross domestic product (GDP) 568
- gross profit margin (GPM) 414–15
- gross profit (operating profit) 368–9
- ground handling 17–18
- half-life 568
- Házy, Steven 16, 17
- Heathrow Airport Holdings 32
- heavy maintenance visit (HMV) 566
- hedging 568
- Horizon Airlines 429
- hub-and-spoke (HS) airline networks 4, 24, 568
- IATA *see* International Air Transport Association (IATA)
- impairment 306–8
- implied cost 334
- income statements 364–77, 366*t*, 569; common size 390, 391*t*, 392; cost of goods sold (COGS) 367–8; depreciation 369–70; financial costs 370–2; gross profit (operating profit) 368–9, 370; net income 372–3; operating expenses 369, 570; revenue, sales or income 365
- income taxes 371
- independent projects (IP), evaluating: annual worth (AW) analysis 157–8; present worth (PW) analysis 128, 135–7; public sector projects 208–10; rate of return (ROR) analysis 184–6
- indirect costs 338; indirect cost estimation, engineering and allocation 352–3, 354*t*
- indirect operating costs (IOC) 358
- inflation 33
- information, economic theory 456–7
- insolvency 565
- integrators 22–3
- Intercontinental Exchange (ICE) 40
- interest expense 370
- interest/interest rates 92–111; accrued interest 54; categorization of interest rates 38; compound interest *see* compound interest; connection between interest rates and their determinations 38–46; continuous compounding 97; discussion

- questions and problems 109–11; effective rates *see* effective interest rates; equal to zero, break-even analysis 223–5, 225–6; equivalences computation *see* equivalences computation; greater than zero, break-even analysis 226–8; long-term debt and interest rates 42–6; nominal interest rates *see* nominal interest rates; short-term interest rates 39–42; simple interest 33, 34–6; and time value of money 33–8; in United States 39
- internal rate of return (IRR) 168, 189–90, 569; *see also* rate of return (ROR) analysis
- Internal Revenue Service (IRS) US, 284, 290, 291, 311, 369
- International Accounting Standards (IAS) 284
- International Aero Engines (IAE) 15
- International Lease Finance Corporation (ILFC) 16
- in-transit passengers 569
- inventories 381
- inventory turnover (ITO) 407–8
- inverse cumulative distribution 248*f*
- investing activities, cash flow from 387
- IOUs 42
- IP projects *see* independent projects (IP), evaluating
- jet engine manufacturers 14–15
- jet fuel service stations 183
- JetBlue 45
- Joint Aviation Authorities (JAA) 569
- joint probability distribution 246–9
- jumbo jets 10–11
- junk bonds 252
- Junkers (German aircraft manufacturer) 10
- Just in Time (JIT) 569
- Kahn, Alfred 3
- Keynes, John Maynard 244
- Knight, Frank 244
- L-1011 TriStar program, Lockheed 11, 317
- landing fees 154
- last come, first served (LCFS) 463–4
- last in, first out (LIFO) 463, 464
- launch customers 569
- lean production 569
- learning curves 339–41, 358
- leases: capital 60, 566, 567; cargo facilities 65; definitions/description 569; dry-leases 60, 567; evaluation of lease contract 60; financial 567; implicit interest rates 94; sale and leaseback 572; secure 60; wet-leases 60, 573; *see also* Air Lease Corporation (ALC); aircraft leasing companies
- least common multiple (LCM) approach 132–3; annual worth (AW) analysis 147, 148, 155; capitalized cost (CC) 133–4
- legacy carriers 4, 24, 27
- lessees 569
- lessors 569
- leverage 569
- LF *see* load factor (LF)
- LIBOR *see* London Interbank Offered Rate (LIBOR)
- life cycle costing (LCC) 138–40, 141
- liquidation 565
- liquidity, definitions/description 569
- liquidity ratios 434–5; current ratio (CR) 402–3, 435; quick ratio (QR) 404–5, 435
- load factor (LF): airline break-even load factor 229–31, 433–4, 565; airline industry following deregulation 4–5; average 353; definitions/description 569; passenger load factor 8, 571; ratio analysis 433; revenue passenger load factor 571
- loans 25, 39; car loans 60, 87; commercial 88, 93, 94; consumers 40; corporate bonds 45; credit cards 40, 93; Federal Reserve Bank 39; governmental 44*t*; interest on 97; longer-term 38; payday 36, 93; payments on 68; real estate 34; simple interest 36; small 36; small business 40; twelve month 41*f*
- Lockheed 10, 11, 317, 428
- London Heathrow (LHR) 18, 23; expansion project 32–3
- London Interbank Offered Rate (LIBOR) 40–1, 54
- long-term assets 383; depreciation 283, 284; *see also* fixed assets
- long-term debt and interest rates 42–6; mortgages 55, 58, 68, 70, 87, 88, 93; municipal bonds 44; revenue bonds 44; Treasury Bonds (T-Bonds) 43, 55; Treasury Notes (T-Notes) 43, 55, 573
- Lorenzo, Frank 45
- loss, production at 330
- low-cost carriers (LCCs) 4, 5, 8, 21, 24
- Lufthansa Group 21, 296; ratio analysis 411, 421, 430
- Maintenance, Repair and Overhaul (MRO) services 17, 469
- major airlines 569
- managers, air transport 5
- marginal costs (MC) 324–5
- market to book ratio 420
- market value ratios 420–1
- marketable securities (short-term investments) 380
- Markowitz, Harry 267

- MARR *see* minimum acceptable rate of return (MARR)
- maturity date 43
- maximax approach, decisions under uncertainty 448–9, 452
- maximin approach, decisions under uncertainty 449–50, 452
- Maximum Certificated Take-Off Weight (MCTOW) 570
- maximum zero-fuel 570
- McDonnell Douglas 10–11, 304, 317
- MD 80s 304
- MD-11 (commercial aircraft) 11
- ME projects *see* mutually exclusive (ME) projects
- mean (expected return) 251–3, 254*t*
- mergers 4, 5, 11*f*, 25
- Mesaba 380
- Messerschmitt-Bolkow-Blohm 10
- Microsoft Excel *see* Excel functions
- minimax regret approach, decision making 450, 453
- minimum acceptable rate of return (MARR) 159; benefit to cost analysis 200, 202, 203, 208; break-even analysis 227; payback period analysis 234, 235, 236; present worth (PW) analysis 116, 119, 125, 128, 131, 140; rate of return (ROR) analysis 169, 171, 173, 176, 178, 179, 181, 182, 186, 191; *see also* rate of return (ROR) analysis
- Mitsubishi Regional Jet (MRJ) 13, 14
- Model 40A aircraft, Boeing 11
- modified accelerated cost recovery system (MACRS) 290–2, 295, 311, 369, 569
- modified rate of return analysis (MROR) 188–9, 191
- Modigliani, Franco 31
- money, time value 33–8; *see also* interest/interest rates
- Monte Carlo analysis (simulation) 275–7, 278*f*, 279
- Moody's Investors Service 45
- mortgages 55, 58, 68, 87, 88, 93; real estate 70
- Mossin, Jan 267
- MRO (Maintenance, Repair and Overhaul) services 17
- municipal bonds (munis) 44
- mutually exclusive different-life alternatives, evaluating 130–5; capitalized cost (CC) 133–5; least common multiple (LCM) approach 132–4; study period approach 131, 132*t*
- mutually exclusive (ME) projects: annual worth (AW) analysis 154–7; benefit-based public sector projects 204–8; cost-based project alternatives 178–9; different-life alternatives, evaluating 130–5; equal-life alternatives, evaluating 128–30; incremental cash flows 176–8; interest greater than zero 228–9; present worth (PW) analysis 127, 128–35; public sector projects 202–8; rate of return 175–83; *see also* project alternatives
- narrow-body aircraft 375
- National Plan of Integrated Airport Systems (NPIAS) 564, 570
- net annual cash flows (NACF) 114; annual worth (AW) analysis 149, 153–4, 160, 162; break-even analysis 232–3; payback period analysis 232, 233
- net income 372–3, 374*t*, 375–6
- net operating loss (NOL) 371
- net present value (NPV) 570
- net profit margin (NPM) 415–16, 570
- New Orleans airport 154
- Newnan, D.G. 168
- Next Generation Air Transportation System (NextGen) 159
- Next-Generation aircraft (Boeing 737–800) 283, 287, 307
- 9/11 terrorist attacks 7
- nodes, decision trees 442–3
- noise, airport 199
- nominal interest rates: vs effective 93–4; effective annual interest rate for selected rates 96*t*; effective interest rate for various rates 96*f*; effective rate to nominal rate 97, 99
- non-operating costs (NOC) 358
- non-pecuniary cost 334
- nonscheduled service 570
- normal distribution 258–65; cumulative 248*f*; probability, calculating using spreadsheets 260; standard 260–3, 264*f*, 265
- Northwest Airlines 5
- oil crisis, 1973 5, 438
- oil prices 319
- online travel agents (OTA) 21–2
- operating activities, cash flow from 386–7
- operating costs (OC) 354–8, 358–9; direct 355–6; fixed direct operating costs (FDOC) 356; indirect 358; and non-operating costs (NOC) 358; variable direct operating costs (VDOC) 356–8
- operating expenses 369, 570
- operating leases 60, 570
- operating leverage 570
- operating profit margin (OPM) 415, 570
- operating revenues 570
- opportunity costs 332–3

- optimistic approach (maximax), decision making 448–9, 452
 options: Black-Scholes option pricing model 244; call options 565; definitions/description 570; put options 571
 overbooking 245, 570
 oversale 570
 Owen, John 168

 paid-in capital 385
 par value (face value) 385, 571
 passenger load factor 8, 571
 passenger miles per gallon (PMPG) 438
 Passenger Name Record (PNR) 571
 Passenger Revenue per Available Seat Mile (PRASM) 571
 passenger yield 431–2
 payback period analysis 219, 231–5, 570; comparing with compounding periods 99–105; conventional analysis 219–20, 232–4, 237; discounted analysis 220, 234–5; discussion questions and problems 238–41; minimum acceptable rate of return (MARR) 234, 235, 236; net annual cash flows (NACF) 232, 233; with non-uniform net annual cash flows 233–4; payback period equalling compounding period 101–2; payback period exceeding compounding period 102–4; payback period less than compounding period 104–5; payback period matching to compounding period 99–101; with uniform net annual cash flows 232–3; *see also* break-even analysis
 payday loans 36, 93
 payment period (PP): comparing compounding period to 99–105, 109; equalling compounding period 101–2; exceeding compounding period 102–4; less than compounding period 104–5; matching to compounding period 99–101
 payoff tables 448–9
 Pegasus 21
 People Express Airlines 5
 perpetuity 571
 Philip, Prince (Duke of Edinburgh) 1
 Phoenix Aviation 392; income statements 393
 Poisson distribution 461
 pollution, air 199
 portfolios 265–7; portfolio beta 266–7
 PP *see* payment period (PP)
 Pratt & Whitney 15, 265
 preferred stocks 385, 571
 prepaid expenses 381–2
 present value (PV): annuities 64–8; arithmetic gradient series 73–9; benefit to cost analysis 200; geometric gradient series 79–81; of a single future sum 49–52; *see also* future value
 present worth (PW) analysis 112–45; accept and reject criteria 117*t*; bond valuation 120–6; and break-even analysis 227, 228; calculation of present worth 49; DirectJet Airline, examples concerning 125, 131; discussion questions and problems 141–5; Do-nothing (DN) alternative 176, 179, 180, 181, 182, 191; independent projects (IP), evaluating 135–7; least common multiple (LCM) approach 132–4; life cycle costing (LCC) 138–40, 141; minimum acceptable rate of return (MARR) 116, 119, 125, 128, 131, 140; mutually exclusive different-life alternatives 130–5; mutually exclusive equal-life alternatives 128–30; mutually exclusive projects 127, 128–35; project alternatives, types 127–8; project cash flows, measuring 113–16; rate of return analysis compared 169; single projects, evaluating 116–20; spreadsheets, using for computation 137, 138*f*; study period approach 131, 132*t*; uses of 112; *see also* annual worth (AW) analysis; future value; valuation
 price-earnings ratio (P/E ratio) 421
 Priceline 21
 primary markets 571
 prime rate 40, 54, 571
 privatization of airlines, UK 6, 18
 probabilistic simulation 276
 probability: and business decisions 244–9; cumulative distribution 246; normal distribution probability, spreadsheet calculation 260
 producer price index (PPI) 346
 profit and loss statements *see* income statements
 profitability, airline 6–8, 377; gross profit (operating profit) 368–9, 370; production at a profit 330
 profitability ratios 413–18, 571; gross profit margin (GPM) 414–15; market value ratios 420–1; net profit margin (NPM) 415–16, 570; operating profit margin (OPM) 415, 570; return on equity (ROE) 418; return on total assets (ROA) 417–18
 project alternatives: cost-based 127, 178–9; Do-nothing alternative 128; mutually exclusive different-life alternatives, evaluating *see* different-life alternatives, evaluating; mutually exclusive equal-life alternatives, evaluating *see* equal-life

- alternatives, evaluating; revenue-based 127, 180–1; types 127–8; *see also* mutually exclusive (ME) projects; project cash flows, measuring; projects
- project cash flows, measuring 113–16; first cost (P) 113–14; net annual cash flows (NACF) 114; terminal cash flow (TCF) 114–15
- projects: independent *see* independent projects (IP), evaluating; mutually exclusive *see* mutually exclusive (ME) projects; permanent, annual worth of 159–60; project alternatives *see* project alternatives; public sector *see* public sector projects; single *see* single projects, evaluating; *see also* project alternatives; project cash flows, measuring
- property, plant and equipment (PP&E) 283, 382
- proration 571
- public sector projects and benefit to cost analysis 195–218; benefit-based mutually exclusive alternatives 204–8; characteristics of public projects 197; cost-based mutually exclusive alternatives 202–4; disbenefits 197, 199; discussion questions and problems 214–17; independent projects (IP), evaluating 208–10; ownership of projects 213; partnerships between public and private sectors 195; public sector vs private sector projects 196–200; single projects, benefit-cost analysis 200–2
- put options 571
- PV *see* present value
- PW *see* present worth (PW) analysis
- Qantas Airways 23, 116, 411, 425
- Qatar Airways 296, 369
- queuing theory 458–68; customer arrivals 459–62; first come, first served (FCFS) 463; last come, first served (LCFS) 463–4; last in, first out (LIFO) 463, 464; queue discipline 463; waiting line characteristics 462–4
- quick ratio (QR) 404–5, 435
- railroads, US 2
- randomization, Monto Carlo simulation 276
- rate of return (ROR) analysis 167–94; accept and reject criteria 170*t*, 177*t*, 189*t*; advantages over PW and AW analysis 169; bonds 173–5; calculating 168–73; complications 186–9; cost-based alternatives 178–9; definitions/description 168; DirectJet Airline, examples concerning 169, 170, 174, 175, 181; discussion questions and problems 191–4; equal-life service requirement 181–3; incremental cash flows 176–8; independent projects (IP), evaluating 184–6; interpretation of values 170; minimum acceptable rate of return (MARR) 169, 171, 173, 176, 178, 179, 181, 182, 186, 191; modified rate of return analysis (MROR) 188–9, 191; multiple rates of return scenario 187, 188*f*; mutually exclusive projects, evaluating 175–83; nominal and effective interest rates 108; reinvestment rate assumption 188; revenue-based alternatives 180–1; spreadsheets, using for rate of return computation 189–90; trial and error method 170–3; *see also* minimum acceptable rate of return (MARR)
- rating agencies 45, 334
- ratio analysis 400–17; asset management ratios 405–9, 410*f*; and credit rating 400; DirectJet Airline, examples concerning 403, 409, 416, 424, 426, 427; discussion questions and problems 435–7; dividend ratios 418–20; DuPont system ratios 421–5; financial management ratios 510–13; key financial ratios for airline industry 428–33; liquidity ratios 402–5, 434–5; market value ratios 420–1; price-earnings ratio (P/E ratio) 421; profitability ratios 413–18; uses of 400; window dressing 425–7
- real estate mortgages 70
- regional aircraft manufacturers 13–14
- reinvestment rate assumptions, rate of return 188
- reneging 462
- rental cars 34
- replacement chain approach 132–3
- replacement decisions 303–6
- Reservation Systems Services 20
- retained earnings 385
- return on assets (ROA) 422
- return on equity (ROE) 418, 422–5
- return on invested capital (ROIC) 4, 5, 6, 413–14; vs weighted average cost of capital (WACC) 335–6, 337*t*
- return on investment (ROI) 571
- return on total assets (ROA) 417–18
- revenue bonds 44
- revenue management 571
- revenue passenger enplanement 571
- revenue passenger load factor 571
- revenue passenger mile (RPM) 353, 429, 571
- revenue per available seat mile (RASM) 230, 430–1, 571
- revenue per revenue passenger mile (R/RPM), passenger yield 431–2
- revenue ton mile (RTM) 571

- revenue-based project alternatives: present worth 127; rate of return (ROR) analysis 180–1
- risk analysis 242–82; capital asset pricing model (CAPM) 267–72, 279; and certainty 244, 249; cumulative distribution function (CDF) 246; decision making under risk 454–6; DirectJet Airline, examples concerning 243, 246, 252, 254, 261, 263, 268, 269; hazards, types 242; joint probability distribution 246–9; mathematical measurement of risk 244; normal distribution 248*f*, 258–65; portfolios 265–7; probability and business decisions 244–9; risk and reward, measuring 249–56; risk aversion 572; risk free assets 572; risk premium 572; sensitivity analysis 272–8; simulation 275–7, 278*f*, 279; standard deviation *see* standard deviation (σ); variability of outcome 455; variables 247; variance *see* variance (σ^2), risk analysis; *see also* decision making
- risk of default 45
- Rohn, Jim 441
- ROIC *see* return on invested capital (ROIC)
- Rolls-Royce 15
- ROR *see* rate of return (ROR) analysis
- round nodes 443
- rule of 72 53
- rule of 100 53–4
- Ryanair 21, 114, 403, 409; ratio analysis 421, 430
- Sabre reservation system 21
- safety stocks 572
- sale and leaseback 572
- salvage value (terminal cash flow): annual worth (AW) analysis 146, 148, 149, 152, 160, 161, 162, 163, 165; break-even analysis and payback period 232, 236; depreciation 286, 287, 288, 289, 291, 292, 294, 295, 297, 303, 307, 310, 311; present worth (PW) analysis 113, 114–15, 130, 131, 141, 142, 143, 144; rate of return (ROR) analysis 180, 188
- sample space 250–1
- seat pitch 572
- Seattle monorail 201
- second freedom of the air 572
- secondary market 572
- secure leases 60
- Securities and Exchange Commission (SEC) 572
- security market line (SML) 269–72, 279
- sensitivity analysis 272–8
- service facility characteristics 464–6
- service providers 17–18
- 707 (Dash 80), Boeing commercial airliner 11–12
- seventh freedom of the air 572
- Sharpe, William F. 267
- shifted cash flow series 82–4
- short-term interest rates 39–42, 54; certificates of deposit (CD) 41–2; discount rate 39; federal funds rate 39, 40*f*, 54; London Interbank Offered Rate (LIBOR) 40–1, 54; prime rate 40, 54, 571; Treasury Bill (T-Bill) 41, 55, 573
- shutdown operations 330
- simple interest 33, 34–6, 37, 54; rule of 100 53–4
- simulation *see* Monte Carlo analysis (simulation)
- Singapore Airlines 283, 297; ratio analysis 403, 411, 417, 433
- Singh, Manmohan 195
- single present amount, future value 47–9
- single projects, evaluating: annual worth (AW) analysis 148–54; break-even analysis 223–5; interest equal to zero 223–5; interest greater than zero 226–8; present worth 116–20; public sector, benefit-cost analysis 200–2
- sinking fund 63
- sixth freedom of the air 572
- SkyWest Airlines 373
- small certificated air carrier 572
- Smith, Adam 92
- Sonic Cruiser (747X), Boeing 154
- Southampton airport (SOU) 18
- Southeast Asian Airlines, Abacus developed by 20–1
- Southwest Airlines (LUV), US 4, 27, 304, 319; financial statements 373, 387, 389*t*; ratio analysis 409, 417, 421
- Spirit Airlines 433
- spreadsheet calculations: annual worth (AW) analysis 160–1; benefit-cost analysis, public sector projects 212–13; depreciation computations 297–303; effective interest rates 105–8; equivalences computation 84–7, 105–8; Excel functions 84, 105, 106*f*, 107*f*, 122; present worth computation 137, 138*f*; rate of return (ROR) analysis 189–90; variance and standard deviation 254–6, 257*f*, 257*t*
- square nodes 442
- square root of the variance *see* standard deviation (σ) 253
- SSJ-100 (Russian aircraft model) 13–14
- stakeholders' equity 384–5
- Standard & Poor 45, 334
- standard deviation (σ) 253–4, 279; spreadsheet calculations 254–6, 257*f*, 257*t*; *see also* variance (σ^2), risk analysis
- Stanstead airport (STL) 18

- start-up airlines 5
- States West Airlines 371
- stockless production 569
- Stoddart, Paul 116
- Stonecipher, Harry 22
- straight-line (SL) depreciation 286–9, 295, 297, 311, 572
- stranded costs *see* sunk costs
- strike price 565
- substitute services, threat of 22–3
- Sukhoi Superjet (SSJ-100) 13
- sum-of-the-years (SYD) method 292–4
- sunk costs 331–2, 572; sunk cost fallacy 332
- supersonic commercial jet era 3
- supplemental air carrier 572
- Sutter, Joseph F. 14
- swaps 572

- Taleb, Nassim Nicholas 242
- TAROM (Romania's national carrier) 20
- taxation: depreciation methods 283, 285; earnings before interest and taxation (EBIT) 6, 288, 371, 372, 390, 413; earnings before taxes (EBT) 371–2; financial statements, preparation for fiscal year 284; income taxes 371; tax rates for 2016 397–8
- Taylor, Ron 59
- terminal cash flow (TCF) *see* salvage value (terminal cash flow)
- terminal nodes 443
- terminal value (TV) 444
- terrorist attacks 7, 24
- Thatcher Government, UK 6, 18
- third freedom of the air 573
- Thomson Reuters 40–1
- time value of money 33–8
- times interest earned ratio (TIE) 413
- total asset turnover ratio (TAT) 408–9, 410*f*
- total cost (TC) 228, 230, 321–3
- total liabilities 383
- total operating cost (TOC) 429
- total revenue (TR) 230
- traffic 199
- Trans World Airlines (TWA) 410
- transparency 456
- Transport Act 1981, UK 18
- travel agents, online 21–2
- TravelSky (Chinese GDS system) 21
- Treasury Bills (T-Bills) 41, 55, 573
- Treasury Bonds (T-Bonds) 43, 55
- Treasury Notes (T-Notes) 43, 55, 573
- trial and error method, rate of return (ROR) analysis 170–3
- TripAdvisor 21
- trunk airlines, US 2
- Tyler, Tony 6

- unbundling of ticket prices 6
- uncertainty, decisions under 447–54; conservative approach (maximin), 449–50, 452; equally likely approach, 450–1, 453; minimax regret approach, 450, 453; optimistic approach (maximax), 448–9, 452; when arising, 244
- unit cost method 344–6
- United Airlines (United Continental Holding) 20, 126, 229; ratio analysis 409, 411, 417, 433
- United carrier, US 4
- United Continental 45
- United Kingdom: privatization of airlines 6, 18; Thatcher Government 6, 18; Transport Act 1981 18; *see also* Heathrow Airport Holdings; London Heathrow (LHR)
- United States: Air Force 147; Airline Deregulation Act 1978 3, 5, 27, 567; airport infrastructure 38; bankruptcies of airlines 24, 27; Bureau of Air Commerce 2; Civil Aeronautics Administration 2; Civil Aeronautics Authority 2; Civil Aeronautics Board (CAB) 2, 3, 25–6; Department of Justice (DOJ) 25; Department of Transportation 25; deregulation of airline industry *see* deregulation; evolution of airline industry, 26*f*; Federal Air Regulation (FAR) 567; Federal Aviation Act 1958 3, 572; Federal Aviation Administration (FAA) 18, 196, 567; federal funds rate 39, 40*f*, 54; Federal Reserve Bank 39, 40; Federal Reserve System (FED) 39; foreign bank accounts 378–9; government debt 38; interest rates 39; Internal Revenue System (IRS) 284, 290; major airlines 25; Navy 147; railroads 2; trunk airlines 2; *see also specific airlines and aircraft manufacturers*
- United States Department of Transportation 25
- US Airways 5; merger with American Airlines 25
- U.S. Flag Carrier 573

- valuation: of bonds 120–6; *see also* future value; present worth (PW) analysis
- variable costs (VC) 320–1, 573; average variable cost (AVC) 325, 329–30; indirect 338; non-linear, break-even analysis 222, 223*f*; *see also* cost estimation, engineering; fixed costs (FC)
- variable direct operating costs (VDOC) 356–8
- variance (σ^2), risk analysis: example, 254; spreadsheet calculations, 254–6, 257*f*,

- 257*t*; square root of the variance *see*
standard deviation (σ); *see also* standard
deviation (σ)
- Virgin America Airlines 376
- WACC *see* weighted average cost of capital
(WACC)
- wages, salaries and benefits (WSB) 355
- waiting line characteristics: line length 462–3;
number of lines 463; queue discipline 463
- walk away costs 573
- warrants 574
- weather monitoring 182, 183
- weight of aircraft 154
- weighted average cost of capital (WACC) 5,
335–6, 337*f*, 573
- wet-leases, 60 573
- white tail 573
- wide-body aircraft 573
- window dressing 425–7
- working capital 574
- Worldspan 21
- yield 574
- zero coupon bonds 574